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# THE PROBLEM OF ALIEN IMMIGRATION INTO GREAT BRITAIN, ILLUSTRATED BY AN EXAMINATION OF RUSSIAN AND POLISH JEWISH CHILDREN.

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## PART III.

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### (vii) *The Special Eye Examination.*

A. *Characters recorded.* We have already seen that the vision of the Alien Jewish Children is poor. We propose in this section to consider what light our Special Eye Examination of the Alien Jewish Boys throws on this matter\*. Is the poor eyesight of these children produced by any factors of their environment or is it a racial character? Some years ago a memoir was published by our Laboratory† which criticised the view of Cohn and others that myopia, especially the myopia of the Germans, was largely due to environmental factors. We had failed to find any significant correlation of poor sight with environmental conditions, and we were very desirous of testing this matter on the material provided in the present case by our home visitations, and by a more complete ophthalmic examination than is feasible in the usual school medical inspections.

The following characters were recorded:

#### (i) *Visual Acuity.*

(a) *Registration of Record.* The usual method with Snellen's types is unsatisfactory from both the statistical and anthropometric standpoints, as it masses some three-quarters of the population, which undoubtedly have very considerable differences in visual acuity, into one single class, the 6/6 group. We considered it most desirable to break up this group, and accordingly introduced types which gave us the following classes:

A:  $\frac{6}{4}, \frac{6}{5}, \frac{6}{5}, \frac{6}{6}, \frac{6}{6}, \frac{6}{9}, \frac{6}{12}, \frac{6}{18}, \frac{6}{24}, \frac{6}{36}, \frac{6}{60}, \frac{5}{60}, \frac{3}{60}, \frac{1}{60}$ .

We also introduced a notation for those who could read one type correctly and read in part but not wholly correctly a type of the class above. We thus reached the following scheme of categories:

B:  $\frac{6}{4}, \frac{6}{4}, \frac{6}{5}, \frac{6}{5}, \frac{6}{5}, \frac{6}{6}, \frac{6}{6}, \frac{6}{9}, \frac{6}{9}, \frac{6}{9}, \frac{6}{12}, \frac{6}{12}, \frac{6}{12}, \frac{6}{15}, \frac{6}{18}, \frac{6}{24}, \frac{6}{30}, \frac{6}{36}, \frac{6}{60}, \frac{6}{60}, \frac{5}{60}, \frac{3}{60}, \frac{1}{60}$

or as decimals:

C: 1.50, 1.40, 1.30, 1.20, 1.13, 1.06, 1.00, .89, .78, .67, .61, .55, .50, .45, .40, .33, .25, .20, .17, .10, .08, .05, .02.

\* *Annals of Eugenics*, Vol. I, pp. 9 and 53.

† "A First Study of the Inheritance of Vision and of the Relative Influence of Heredity and Environment on Sight," *Eugenics Laboratory Memoirs*, No. V, Cambridge University Press.

The divisions between these categories may then be taken as:

D: 1.45, 1.35, 1.25, 1.167, 1.10, 1.033, .945, .835, .725, .64, .58, .525, .475, .425, .365, .29, .225, .185, .135, .09, .065, .035, .015.

Or, taking the central values in these ranges, we have:

E: 1.50, 1.40, 1.30, 1.208, 1.133, 1.066, .989, .89, .78, .682, .61, .552, .50, .45, .395, .327, .257, .205, .17, .112, .077, .05, .018.

the differences between C and E arising from the inequality of the ranges in C.

But for our sparse numbers the grouping was still somewhat erratic. We accordingly made the following grouping, which gives a reasonably smooth distribution:

Table CXLV. *Frequency of Vision Classes. 507 Boys.*

Snellen Types	Decimal Values	Frequencies				Central Values	
		R. Eye	L. Eye	Total Eyes	Binocular Vision	Monocular	Binocular
6/4	1.50	2	1	3	3	1.50	1.50
6 <sup>-</sup> /4	1.40	9	8	17	7	1.40	1.40
6 <sup>+</sup> /5, 6/5	1.30, 1.20	45	40	85	64	1.285 <sup>-</sup>	1.29
6 <sup>-</sup> /5, 6 <sup>+</sup> /6	1.13, 1.07	105	92	197	127	1.101	1.11
6/6, 6 <sup>-</sup> /6	1.00, .89	120	124	244	107	.91	.91
6 <sup>+</sup> /9, 6/9	.78, .67	59	58	117	42	.755 <sup>+</sup>	.75
6 <sup>-</sup> /9, 6 <sup>+</sup> /12, 6/12	.61, .55, .50	61	53	114	39	.571	.58
6 <sup>-</sup> /12, 6/15, 6/18	.45, .40, .33	43	58	101	27	.37	.37
6/24, 6/30	.25, .20	25	25	50	11	.244	.25
6/36, 6/60	.17, .10	22	28	50	10	.14	.14
5/60	.08	8	10	18	6	.08	.08
3/60, 1/60	.05, .02	8	10	18	4	.04	.04
Totals ...		507	507	1014	447	—	—

The last two columns have been obtained by taking the mean values for the uncompounded subranges to obtain a central value for the final ranges. There is so little difference between the results obtained for monocular vision (Total Eyes) and binocular vision, that it seemed simplest to use the central values of the binocular vision whether we are dealing with monocular or binocular frequencies\*. In the tables it is the central values provided by the last column which will always be given and these have been used for determining variation and correlation constants.

The reader will have little difficulty in interpreting our visual acuity values. For example, if the acuity of an individual be given as 1.16, this means that he can read at 6 metres a Snellen's type of size  $6/(1.16) = 5.172$  or a type which might be interpolated at about 1/6 of the distance from Type 5 to Type 6.

(b) *Comparison of our Method and that of the usual School Medical Examination.* If the reader examines the frequency distribution for 1014 eyes in Table CXLV, he will recognise that 302 eyes in 1014, i.e. 29.8 % or practically 30 % of the children, had greater acuity than 6/6. But all these children cannot be treated as other than belonging to the category 6/6 in the usual method of estimation. Again, of the 325 eyes which could read 6/9 but not 6/6, no less than 92 % read 6<sup>+</sup>/9 or 6<sup>-</sup>/6. Looked at from our standpoint the mean of the 6/6 group was not 1.00, but 1.16 and of the 6/9 group not .67, but .84. In this way we reach for the Medical Examination values for the categories: 6/6, 6/9, 6/12, 6/18, 6/24, 6/36, 6/60 and < 6/60, not centering at 1.00, .67, .50, .33, .25, .17, .10 and .08 respectively but at 1.16, .84, .57, .37, .25, .18, .10 and .06. These applied to the 1723 boys and 1880 girls of the Jews' Free School Medical Examination give mean acuities of:

*Boys:* .722 = 6/8.310, and *Girls:* .674 = 6/8.902,

or the means connote Snellen's types 1.4 and 1.6 respectively smaller than those corresponding

\* If without appealing to experience we had taken merely the central points of the corresponding ranges in D (at the top of this page), we should have had the values 1.50, 1.40, 1.26, 1.10, .94, .74, .56, .38, .24, .14, .08, .04; the divergence is of no practical importance.

to the means found from treating the usual values as the centres of their groups, i.e.  $\cdot 6152 = 6/9\cdot 753$  and  $\cdot 5728 = 6/10\cdot 475$ .

It is clear that however helpful the usual school examination may be as a rough sieve for extracting children with bad vision, the data provided in this way are by no means suited to a scientific investigation of the distribution of acuity of vision in a given population. It may be doubted indeed whether Snellen's types are suited at all to such an investigation although we improve our results very much by the use of additional letters. Galton, who saw the difficulty more than forty years ago, defined visual acuity as the distance in inches at which diamond type could be read, thus in theory getting a perfectly continuous scale. Unfortunately in the practice of his Anthropometric Laboratory\* he used an instrument in which diamond type was set up at a number of definite distances in geometrical progression†. Thus he obtained a distribution with an even more troublesome division of the subranges than we find with the use of Snellen's types. Probably an instrument in which a screw of small pitch could be used to bring a specimen of diamond type nearer and nearer to the eye would be fittest; it would enable us to get a continuous scale and to choose equal subranges.

As we have considerable numbers of the same children recorded under both Medical and Special Examinations we are able to correct the values given by the former and assign to them a rather more accurate measure of vision. We shall thus speak of Medical Examination data and *Corrected* Medical Examination data.

In the Special Examination we have visual acuity of R. and L. eyes separately, and can take when we please vision of "better" eye. We have also binocular visual acuity. Our diagrams in Section C enable us to link up these various ways of regarding vision.

(ii) *Refraction Class*. (Determined without a mydriatic.) Each eye was considered separately and the eyes have been classed as follows:

		<i>Refraction Class</i> ‡.	
		Sphere	Cylinder
$N$ = Normal (Emmetropic)		0 to $+0\cdot 75$ D.	
$H$ = Hypermetropic	$\left\{ \begin{array}{l} (1) + 1 \text{ D. to } +1\cdot 75 \text{ D.} \\ (2) + 2 \text{ D. or over} \end{array} \right.$		$- 0\cdot 75 \text{ to } + 0\cdot 75 \text{ D.}$
$M$ = Myopic	$\left\{ \begin{array}{l} (1) - 0\cdot 25 \text{ D. to } - 0\cdot 75 \text{ D.} \\ (2) - 1 \text{ D. or under} \end{array} \right.$		
$HA$ = Hypermetropic Astigmatism	0 or +	...	+ 1 D. or over.
$MA$ = Myopic Astigmatism	0 or -	...	- 1 D. or under.
Mixed $A$ = Mixed Astigmatism	$\left\{ \begin{array}{l} (1) + \dots \dots \dots \end{array} \right.$	...	With - 1 or under except when sphere > cylinder.
	$\left\{ \begin{array}{l} (2) - \dots \dots \dots \end{array} \right.$	...	With + 1 or over except when sphere > cylinder.

Astigmatism is measured by the cylindrical correction, which is entered on our refraction section of the schedule.

Very little comparative material is available for Refraction Class, partly because various observers record somewhat differently, partly because only children with defective vision are, in most of the school work, tested for refraction errors. Thus it is difficult to find a random sample. The Glasgow measurements by Rowan and the Charity Organisation Society Edinburgh data are

\* The use of a similar instrument renders the visual acuity of the Cambridge Anthropometric Laboratory Records very difficult to reduce and discuss.

† Galton, in doing this, was taking into account Fechner's Law. Actually he set his types at equal angles round an equiangular spiral with the eye at the pole. No one has yet attempted to reduce Galton's data by considering the measure of vision not to be the inches of distance at which diamond type was read, but to be *the angle* at which the type read was set.

‡ It is needful, as most school medical officers have found, to extend the region covered by the word "normal." The word Emmetropic really refers to a *boundary*, and having no range, there would be no frequency. Experience has shown it convenient, however, to include slight divergences from Emmetropia in an Emmetropic or Normal Group. See the Note appended to Section B, p. 140.

fairly comparable, but to compare both with our Jews Myopic (1) from the above table must be added to Emmetropic. To compare the Jews with Thompson's London data both Hypermetropic (1) and Myopic (1) must be added to Emmetropic. Even then Thompson's data, for reasons given elsewhere\*, are very anomalous.

No mydriatic was used in any of these cases. Where it has been used we have found all comparison idle. The use of a single correction for all grades of refraction does not get over the difficulty of the far greater variability of the series where a mydriatic has been used. In Table CXLVI we have rearranged the Jewish Boys in classes as seemed most fitting to the compared data.

Table CXLVI. *Comparative Results for Refraction Class in Jewish and Gentile Children.*

	Emmetropia	Hypermetropia	Hypermetropic Astigmatism	Mixed Astigmatism	Myopic Astigmatism	Myopia
Alien Jews (900 Boys) Edinburgh Data (498 Boys)	69.9 % 44.8 %	7.7 % 30.3 %	7.7 % 15.5 %	1.4 % 4.2 %	4.4 % 3.2 %	8.9 % 2.0 %
Alien Jews (900 Boys) Glasgow Data (220 Boys)	69.9 % 55.5 %	7.7 % 25.0 %	7.7 % 10.5 %	1.4 % 3.5 %	13.3 % 5.5 %	
Alien Jews (900 Boys) London Data (10,416)	75.5 % 92.5 %	2.1 % 1.7 %	7.7 % 1.8 %	1.4 % 1.7 %	13.3 % 2.3 %	

The one clear point which comes out of these returns is the great excess of Myopia among the Jewish lads. It is clear also that the Scottish boys had more manifest Hypermetropia than the Jews or the Londoners, or else the methods of observation were too different for comparison to be made†. Thompson's data based upon examining only the boys with bad vision are very unsatisfactory, and immensely exaggerate the Emmetropic group. We are inclined to believe that it is really needful to add the bulk of the Hypermetropia to the Emmetropia for purposes of comparison, when we find:

	London	Edinburgh	Glasgow	Alien Jews
Emmetropia and Hypermetropia	94.2 %	75.1 %	80.5 %	77.6 %

We should then discard the London data as unreliable, and find the Jewish Boys intermediate between the two Scottish Groups. We think, however, that while the Jewish Boys have markedly more Myopia they may really have less Hypermetropia. Table CXLVI might suggest that this is certainly so, but experience has taught us that personal equation plays a large part in ophthalmic observations on school children. Some additional records were sent to the senior author of this paper some years ago by Dr J. J. Butterworth, Medical Superintendent of Schools, Preston. He determined the refraction class for 391 girls and 99 boys, aged 14 to 16. The eyes were carefully examined by test types and the ophthalmoscope, a mydriatic was not used except in a few cases where it was not possible to get a correction for the vision with lenses. Small errors of refraction not interfering with vision or producing symptoms (probably our Hypermetropia (1)) were reckoned "normal." Hypermetropic and Myopic Astigmatism were included in Hypermetropia and Myopia respectively. The children were drawn from elementary schools and were bursary holders and pupil teachers. Their parents were representatives of the lower middle and upper working classes.

	Normal	Hypermetropic	Myopic	Mixed Astigmatic
Alien Jewish Boys	68.6 %	9.8 %	20.2 %	1.4 %
Preston Boys	75 %	10 %	13 %	2 %
„ Girls	75 %	10 %	12 %	3 %

This comparison we believe to be the best available at present.

\* "A First Study of the Inheritance of Vision, etc.," *Eugenics Laboratory Memoirs*, No. V, Cambridge University Press, pp. 2, 28.

† As to the Hypermetropic totals see our first footnote, p. 115.

(iii) *General Refraction and General Astigmatism*. These were determined by aid of lenses and checked by retinoscopy in doubtful cases\*. The following are the frequencies found for some 900 Jewish boys:

Table CXLVII. *Percentages of General Refraction and Astigmatism. Alien Jewish Boys.*

Percentages of General Refraction	Central Values in Dioptres												Total Nos.
	+ 8 and over	+ 7.5	+ 6.75	+ 6.0	+ 5.25	+ 4.5	+ 3.75	+ 3.0	+ 2.25	+ 1.5	+ .75	0.0	
	.00	.00	.11	.44	.44	.33	1.83	1.17	3.61	2.78	20.78	49.17	
Percentages of General Refraction	- .75	- 1.5	- 2.25	- 3.0	- 3.75	- 4.5	- 5.25	- 6.0	- 6.75	- 7.5	- 8 and under		900
	9.33	3.94	2.83	.78	.89	.00	.78	.11	.44	.00	.22		

Percentages of General Astigmatism	Central Values in Dioptres										Total Nos.
	- 3.75	- 3.0	- 2.25	- 1.5	- .75	0.0	+ .75	+ 1.5	+ 2.25		
	.00	.00	.17	.44	6.46	74.17	11.09	2.32	3.15		
Univ. Coll. Students	.30	.44	1.41	1.63	8.14	41.87	30.99	7.40	4.07		
Percentages of General Astigmatism	+ 3.0	+ 3.75	+ 4.5	+ 5.25	+ 6.0	+ 6.75	+ 7.25	+ 8.0			906
	.77	.99	.33	.17	.00	.00	.00	.00			
Univ. Coll. Students	.89	.74	.44	.59	.15	.30	.00	.59			676

The only non-selected material that we can find for comparison is provided by the students who come to the Galton Anthropometric Laboratory; but there are difficulties here: (i) we have an average age of 19 to 20 instead of 11 to 12, (ii) a considerable number of the students are Jews, and (iii) an attempt is made to record to a quarter dioptre. The frequency distribution for the General Refraction of the Jewish Boys indicates that a similar attempt was not successful in their examination, the quarters being given far less frequently than the halves. (iv) Now that it is known that eye-testing goes on in the Laboratory, a considerable number of students come suspecting their eyes to be wrong and desiring to know whether they ought to consult an ophthalmologist†. Thus, as there is no compulsion on students to come, we do not get for sight a fair sample of the general student population. All that the comparison shows is that the Jewish Boys, 11 to 12 average age, have less Astigmatism than the young men of 19 to 20, and that the small grades of this may be really due to finer measurement of the latter. We think there is no doubt that the Jewish Boys are more myopic than the Gentiles; whether they are less astigmatic requires an independent examination more free than ours can claim to be from differences of age, method of determination and personal equation. We believe, however, that such an examination will show the Jew to be less astigmatic than the Gentile. What is essentially needed is the examination of an entire school population for ocular characters without the least selection.

(iv) *Corneal Refraction and Corneal Astigmatism* were found by aid of the ophthalmometer.

(a) For the measure of Corneal Refraction, we have taken the refractive power of the eye in the horizontal meridian; or in the cases where the axes are oblique, the power in the principal meridian nearer to the horizontal.

Corneal Astigmatism has been measured by the difference in the refractive powers of the

\* Without a mydriatic this does not allow of a fine determination of Hypermetropia, and it is possible that the Emmetropic totals have been swollen at the expense of the Hypermetropic.

† All students whose eyes are defective are warned and told to consult an ophthalmologist.

cornea in the two principal meridians. The Astigmatism is counted positive when the curvature is greater in the vertical than in the horizontal meridian.

Both Corneal Refraction and Astigmatism were measured in dioptries and an attempt was made to measure to a quarter dioptre accuracy. This was not found possible and accordingly we have clubbed together every two or three  $\cdot 25$  dioptre successive ranges. The following table gives the frequency distributions of some 1000 individual eyes in children of all ages.

Table CXLVIII. *Corneal Refraction and Astigmatism (Boys).*

Refraction					Astigmatism				
Refractive Power Central Values	R. Eye	L. Eye	Both	%	Dioptries Central Values	R. Eye	L. Eye	Both	%
38-25	1	—	1	0-10	- 2-25	—	1	1	0-10
38-75	—	—	—	0-00	- 1-5	4	3	7	0-70
39-25	2	2	4	0-40	- .75	9-5	9	18-5	1-84
39-75	5	2	7	0-70	0	213	217	430	42-74
40-25	13	7	20	2-00	+ 75 <sup>1</sup>	194-5	193	387-5	38-52
40-75	10	11	21	2-10	+ 1-5	39	36	75	7-46
41-25	24	26	50	4-99	+ 2-25	26	26	52	5-17
41-75	22	27	49	4-89	+ 3-0	8	11	19	1-89
42-25	38	52	90	8-98	+ 3-75	3	5	8	0-80
42-75	57	38	95	9-48	+ 4-5	2	1	3	0-30
43-25	65	55	120	11-98	+ 5-25	3	1	4	0-40
43-75	62	70	132	13-17	+ 6-0	1	—	1	0-10
44-25	66	69	135	13-47					
44-75	49	47	96	9-58					
45-25	30-5	35	65-5	6-54					
45-75	21	30	51	5-09					
46-25	27	21	48	4-79					
46-75	6-5	6	12-5	1-25					
47-25	1	2	3	0-30					
47-75	1	1	2	0-20					
Totals ...	501	501	1002	100-01	Totals ...	503	503	1006	100-02

<sup>1</sup> The plus sign as with refraction measured subjectively signifies here "with the rule," i.e. convex correcting cylinder has axis nearer to vertical than horizontal.

The above table shows that we may combine either for Corneal Refraction or for Corneal Astigmatism our observations for both eyes. We shall see in Section B that in the case of the Jewish Boys observed there was extremely little change in either Corneal Refraction or Corneal Astigmatism with Age.

In the case of Corneal Astigmatism we have no comparative English data. The only comparative material available is that due to Steiger\* for 1502 schoolboys aged 8 to 16 in Berne. Steiger's frequencies run with admirable smoothness. Our Jews' Free School recorders found it difficult to determine the refraction to a quarter dioptre and we had to make adjustments in grouping to get comparable series. The following table provides the best results we could obtain for comparison with 1006 Jewish cases.

It will be seen that the Alien Jewish Boys have greater percentages of the higher degrees of Corneal Astigmatism both with and against the rule than the Swiss Boys, although as a race the Germanic is markedly defective in sight. The Swiss Boys, according to the data, have a mean of  $\cdot 68$  D. as against the Jewish  $\cdot 54$  D. and tend to be more astigmatic with the rule than the Jewish Boys.

\* "Studien über die erblichen Verhältnisse der Hornhautkrümmung," *Zeitschrift für Augenheilkunde*, Bd. xvi, S. 236.

Table CXLIX. *Corneal Astigmatism in Jewish Boys and Swiss Boys.*

Dioptres	Alien Jewish Boys	Swiss Boys (Germanic Race)
Up to -1.25	0.60 %	—
-1.25 to -.75	0.80 %	—
-.75 to -.25	1.79 %	0.77 %
-.25 to +.25	38.32 %	12.94 %
+.25 to +.75	25.25 %	48.70 %
+.75 to +1.25	18.44 %	28.10 %
+1.25 to +2.25	9.34 %	7.09 %
+2.25 to +3.00	3.68 %	1.60 %
Above +3.00	1.79 %	.80 %
No. of cases on which based	1006	1502

Steiger's data for 882 Berne schoolboys (10–16) enable us to form some comparison between the Corneal Refraction ("horizontal" meridian) for our Alien Jewish children and the boys of German race. We are, however, compelled to adopt relatively few classes.

Table CL. *Corneal Refraction. (Jewish and Swiss Boys.)*

Dioptres	Alien Jewish Boys		Swiss Boys (Berne)	
	Totals	%	Totals	%
Up to 40.125	25	2.5	21	2.4
40.125 - 41.125	60	6.0	60	6.8
41.125 - 42.125	128	12.7	180	20.4
42.125 - 43.125	206	20.5	279	31.6
43.125 - 44.125	263	26.1	194	22.0
44.125 - 45.125	180	17.9	120	13.6
Above 45.125	144	14.3	28	3.2
Totals ... ..	1006	—	882	—
Means... ..	43.353 ± .032		42.801 ± .029	
Standard Deviations ...	1.497 ± .023		1.275 ± .021	
Coefficients of Variation	3.453 ± .079		2.979 ± .063	

The differences between the constants may be all considered significant; thus the corneal curvature\* of the Jewish child is significantly greater than that of the German child, and it is sensibly more variable, i.e. less concentrated about its mean value.

(b) *Personal Equation of Observers using the Ophthalmometer.* Three trained observers used the ophthalmometer in succession. The frequency distributions for both Corneal Refraction and Astigmatism of *A* and *B* were in reasonable agreement, but there were considerable differences between those of *C* and the distributions of *A* and *B* together. A difficulty therefore arises as to whether this difference is due to personal equation or to a different class of boys having been submitted by the teachers to *A* and *B*†. Unfortunately we could not control the personal equation of the earlier and later observers because no observations had been made on the same boys; we did not *a priori* anticipate any considerable difference between the results and only at the reduction of the observations 10 years later was such a difference discovered. Even had it been discovered in 1914 many of the boys would have left the school and for those still in it there would have been a difference of 3 to 4 years in age between the two investigations.

We endeavoured to compare *A* and *B* with *C* in other ocular characters to test whether the

\* The mean radius of curvature (of the "horizontal" meridian) of the Jewish boys is 7.773 mm. and of the Swiss boys 7.874 mm.

† *A* and *B* observed in 1910 and 1911, *C* in 1914. The death of *A* prevented his completing the task he had undertaken and it was only after some years that we were able to find an ophthalmologist to continue the work. *B* was a laboratory assistant of *A*, and therefore fairly well trained in his methods.

divergence was due to personal equation or to differentiation in the groups of boys dealt with. The Acuity of Vision results showed that *C* distributed the population he observed more widely, especially making more use of the additional Snellen's types we had introduced, but on the whole we could not say that the visual acuity results showed that he was dealing with a sensibly divergent population. We came to the conclusion therefore that it was probably a matter of personal equation. In order to cover both possibilities however we have for both Corneal Refraction and Astigmatism worked out in many cases our results for the data of *A*, *B* and *C* pooled and for the data of *A* and *B* only. This distinction will be met with especially in Sections D, F and G, where nevertheless it rarely leads to any sensibly divergent conclusions.

(v) *Distance of Near Point*. This was also determined in a fair number of cases.

Besides the optical characters we have just considered, a certain number of head measurements were taken in order to test whether vision is in any way related to shape of head. These were maximum length (*L*) from glabella to occipital point, maximum parietal breadth (*B*), auricular height (*OH*) from the auricular axis as centre line of the auricular passages to the point of the head in the sagittal plane vertically above the auricular axis, when the head is facing straight-forward, the perpendicular distance from the nasion to the auricular line (*n*), the perpendicular distance of the most advanced point of the frontal sinus above the left eye to the auricular axis (*s*), the perpendicular distance from the centre of the anterior surface of the left eyeball\* to the auricular axis (*c*), and the interpupillary distance (*i.p.*). From these measurements five indices were determined, namely:

- (i) The First Cephalic Index ...  $100 B/L$ ,
- (ii) The Second Cephalic Index ...  $100 OH/L$ ,
- (iii) The Third Cephalic Index ...  $100 OH/B$ ,
- (iv) The Interpupillary Index ...  $100 i.p./B$ ,
- (v) The Index of the Sunken Eye ...  $100c/\frac{1}{2}(n + s)$ .

Clearly the larger this latter index is the less sunk is the eye†.

In addition to these lengths on the head we determined the colour of hair and eyes and the appearance of the fundus in order to see whether pigmentation was in any way related to ocular characters.

For details of measurement of the hair and eye colour characters we must refer the reader to pp. 19–22 of the already published portion of our paper in *Annals of Eugenics*, Vol. I.

#### B. *Change of Ocular Characters with Age.*

(i) (*a*) *Visual Acuity and Age*. This is the only ocular character for which we have an adequate number of girls, and this only for the general Medical, not for our Special Eye Examination.

We obtained the following constants for the Medical Examination data:

Mean Age	...	...	Boys: 11·427 yrs.	Girls: 11·030 yrs.
Standard Deviation Age	...	„	1·86845 yrs.	„ 2·0291 yrs.
Mean Visual Acuity	...	„	·6152 = 6/9·753	„ ·5728 = 6/10·475
Standard Deviation Vision	...	„	·2858	„ ·2600

Correlation of Visual Acuity and Age found by the product moment method:

$$\text{Boys: } + \cdot 1282 \pm \cdot 0160 \quad \text{Girls: } + \cdot 0925 \pm \cdot 0154.$$

\* This included the thickness of the eyelid which was drawn down over the cornea when the distance was measured.

† Τῶν δ' ὀφθαλμῶν οἱ μὲν μεγάλοι, οἱ δὲ μικροί, οἱ δὲ μέσοι, οἱ μέσοι βέλτιστοι. καὶ ἡ ἐκτὸς σφόδρα, ἡ ἐντός, ἡ μέσως. τούτων οἱ ἐντὸς μάλιστα δξυωπέστατοι ἐπὶ παντὸς ζώου, τὸ δὲ μέσον ἥθους βελτίστου σημεῖον. ARISTOTLE, *De Animalibus Historiae*, I, x.

Table CLI. *Visual Acuity and Age (Monocular). Medical Examination.*

Central Ages (years)	Boys' Visual Acuity									Girls' Visual Acuity									Totals
	6/6	6/9	6/12	6/18	6/24	6/36	6/60	< 6/60	Totals	6/6	6/9	6/12	6/18	6/24	6/36	6/60	< 6/60	Totals	
6-208	—	—	—	—	—	—	—	—	—	—	2	2	—	—	—	—	—	—	4
6-708	—	2	—	2	—	—	—	—	4	19	7	11	1	—	—	2	—	—	40
7-208	3	12	9	4	—	—	—	—	28	26	11	15	9	5	—	—	—	—	66
7-708	—	22	11	8	3	2	2	—	48	6	16	23	7	3	1	—	—	—	56
8-208	1	38	32	10	2	1	2	—	86	8	33	44	21	13	—	3	—	—	122
8-708	22	32	27	26	3	2	2	—	114	7	45	38	28	11	5	2	—	—	136
9-208	12	23	15	8	4	6	2	—	70	7	18	17	16	9	1	4	2	—	74
9-708	14	22	12	12	5	4	3	2	74	6	26	23	15	4	10	2	—	—	86
10-208	18	24	21	13	1	4	2	1	84	2	30	28	28	9	5	2	—	—	104
10-708	15	26	22	8	3	4	4	—	82	4	27	38	25	8	10	—	—	—	112
11-208	19	30	34	16	7	6	2	—	114	18	19	30	12	11	4	—	—	—	94
11-708	20	22	12	4	3	7	2	1	71	33	49	38	9	7	7	5	—	—	148
12-208	122	65	55	33	22	16	15	4	332	58	69	52	34	23	22	7	—	—	265
12-708	98	72	34	23	21	19	11	—	278	68	76	63	17	14	17	4	6	—	265
13-208	68	36	22	11	12	11	—	—	160	49	39	28	19	10	2	3	—	—	150
13-708	38	24	11	10	12	8	4	1	108	36	41	20	16	6	7	—	2	—	128
14-208	14	15	10	4	4	3	2	—	52	4	13	4	5	2	2	1	1	—	32
14-708	3	9	1	3	—	—	1	1	18	—	2	—	—	—	—	—	—	—	2
15-208	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	2
Totals	467	474	328	195	102	93	54	10	1723	353	523	474	262	135	93	35	11	—	1886

Both these correlation coefficients are significant, and show that vision improves with age. From them might be calculated the regression lines of visual acuity on age. But the slightest investigation of the data shows that children about 8 and 12 years have been selected for examination, so that the correlation coefficient is of small importance. If we compute the correlation ratio of vision on age we surmount this difficulty. We find:

$$\eta'^2_{v.A} = .027,953, \quad \bar{\eta}^2 = .008,706 \pm .002,134, \quad \eta'_{v.A} = .1672 \text{ for boys, and}$$

$$\eta'^2_{v.A} = .067,137, \quad \bar{\eta}^2 = .007,953 \pm .001,968, \quad \eta'_{v.A} = .2591 \text{ for girls.}$$

Both these values are significant and indicate that the correlation is skew.

The following are the mean visual acuities at the several ages:

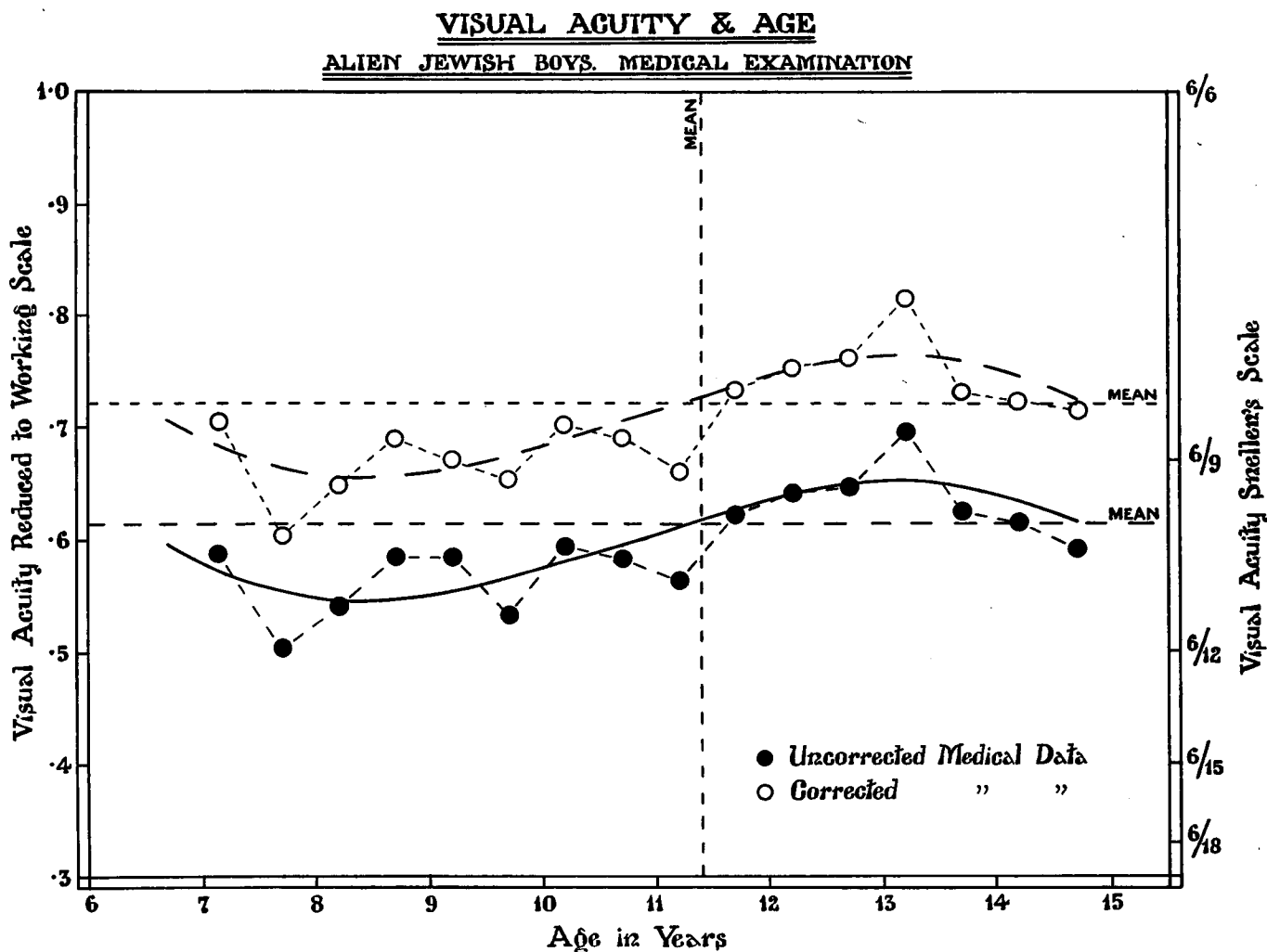
Table CLII. *Visual Acuities at Several Central Ages. Medical Examination.*

Boys		Girls	
Central Ages (years)	Visual Acuity <sup>1</sup>	Central Ages (years)	Visual Acuity <sup>1</sup>
7-146	.5894 { .7059 }	6-663	.7286 { .8541 }
7-708	.5035 { .6046 }	7-208	.6832 { .7959 }
8-208	.5422 { .6500 }	7-708	.5616 { .6612 }
8-708	.5861 { .6905 }	8-208	.5130 { .6016 }
9-208	.5861 { .6719 }	8-708	.5088 { .6014 }
9-708	.5339 { .6539 }	9-208	.4827 { .5649 }
10-208	.5956 { .7030 }	9-708	.4973 { .5867 }
10-708	.5840 { .6904 }	10-208	.4677 { .5499 }
11-208	.5645 { .6629 }	10-708	.4736 { .5538 }
11-708	.6230 { .7362 }	11-208	.5651 { .6580 }
12-208	.6439 { .7524 }	11-708	.6165 { .7293 }
12-708	.6490 { .7619 }	12-208	.5722 { .6712 }
13-208	.6976 { .8169 }	12-708	.6151 { .7254 }
13-708	.6266 { .7325 }	13-208	.6569 { .7717 }
14-208	.6169 { .7262 }	13-708	.6367 { .7531 }
14-708	.5917 { .7156 }	14-292	.5742 { .6864 }

<sup>1</sup> The numbers in brackets are the "corrected" values: see p. 112.

These mean values for the arrays are indicated in Diagrams 47 and 47 bis. We fitted them with

cubics\* which give a very reasonable representation of the facts observed. We see that the young children start with the best vision, that it falls rapidly and reaches its minimum at about  $8\frac{1}{2}$  years in Boys and  $9\frac{1}{2}$  in Girls. Acuity then improves, and reaches a maximum about  $13\frac{1}{4}$  years in Boys and  $13\frac{1}{2}$  years in Girls, or somewhere about puberty, after which it begins to fall again.



Now when a remarkable result of this kind is reached it seems desirable to set to work and criticise it and find if possible weak points in its deduction. The first of these we have already (p. 111) referred to, i.e. the defective centering and bad grouping of the Snellen type testing. We accordingly adopted the scale of centering determined on p. 112 (b), and found the following constants, those for age of course being unaltered:

Mean Visual Acuity	...	Boys: .7239 = 6/8.288	Girls: .6737 = 6/8.906
Standard Deviation	...	„ .3435	„ .3172

\* Boys:  $V.A. = .6410 + .0220 (A. - 12.208) - .0080 (A. - 12.208)^2 - .00192 (A. - 12.208)^3$ .

Girls:  $V.A. = .5445 + .0556 (A. - 11.208) + .0026 (A. - 11.208)^2 - .00440 (A. - 11.208)^3$ .

Correlation of Visual Acuity and Age found by the product moment method:

	Boys: $\cdot1142 \pm \cdot0160$	Girls: $\cdot0925 \pm \cdot0154$
Correlation Ratio	$\eta'^2_{V.A} = \cdot022,016$	$\eta'^2_{V.A} = \cdot062,948$
	$\bar{\eta}^2_{V.A} = \cdot008,706 \pm \cdot002,124$	$\bar{\eta}^2_{V.A} = \cdot007,953 \pm \cdot001,968$
	$\eta'_{V.A} = \cdot1484$	$\eta'_{V.A} = \cdot2509$

The general result of this change is merely to increase the mean visual acuities. The variabilities are also increased, but the Girls have still a worse vision, and one more concentrated than the Boys\*.

### VISUAL ACUITY & AGE

#### ALIEN JEWISH GIRLS. MEDICAL EXAMINATION

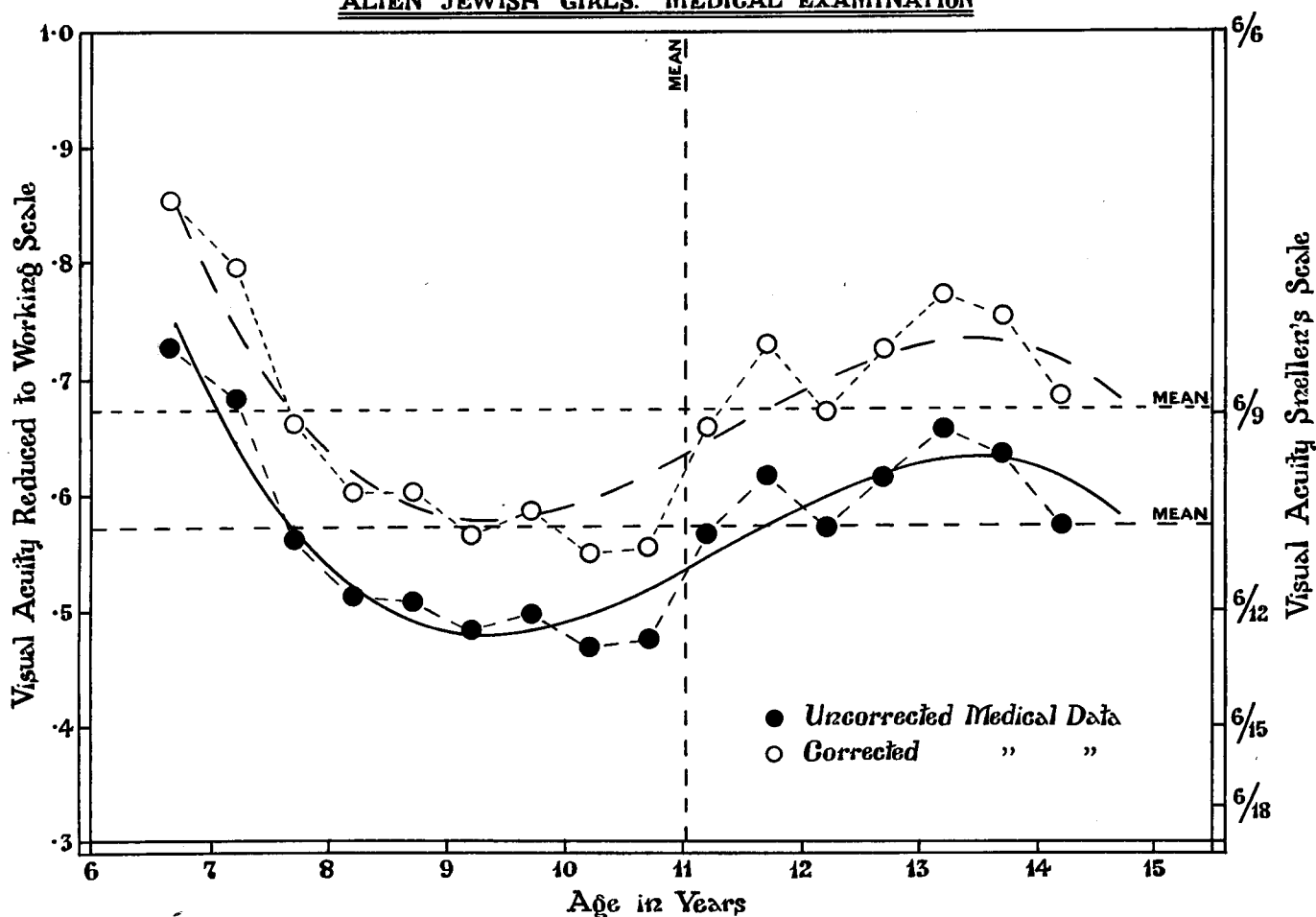


Diagram 47 bis.

The correlation constants are practically unmodified; the distributions are still skew and visual acuity shows a moderate if quite significant correlation with age.

The acuities observed at the several central ages will be found recorded in Table CLII in brackets. It will be seen by comparing the array means and the upper curves on the Diagrams 47 and 47 bis, that it is for practical purposes adequate to add for Boys  $\cdot11$  and for Girls  $\cdot10$  to the Acuity of Vision as found from the Snellen type testing. The old scale curves are practically shifted

\* This is of some importance for the problem of whether excessive Hebrew study is the source of bad sight among the Jewish Boys.

vertically upwards to obtain the new scale curves\*, and we have not modified the phenomenon of minimum acuity in the eighth to ninth years.

Now it was not possible to test for vision all the younger children in the school, but we had hoped that the choice made of them would be a random one. If it was, the dip in the eighth to ninth years would have physiological importance, and it should repeat itself in our own more leisurely Special Eye Examination, where we were confident of having allowed the teachers on the whole series to make no selection of the children†.

(b) *Visual Acuity and Age, Special Eye Examination.* The following table gives the data for monocular visual acuity in 1010 eyes, namely 505 Right and 505 Left Eyes.

Table CLIII. *Monocular Visual Acuity and Age. Boys. Special Examination.*

Visual Acuity Central Values	Central Ages in Years																Totals
	7·708	8·208	8·708	9·208	9·708	10·208	10·708	11·208	11·708	12·208	12·708	13·208	13·708	14·208	14·708	15·208	
1·50	—	—	—	—	—	—	—	—	—	—	1	—	2	—	—	—	3
1·40	—	—	—	—	—	—	—	—	1	—	8	5	—	3	—	—	17
1·29	1	—	—	4	1	6	2	12	5	17	7	17	9	4	—	—	85
1·11	2	1	4	17	17	7	9	19	22	22	29	21	19	6	2	—	197
·91	3	1	1	6	4	16	10	15	7	26	21	43	50	26	5	8	242
·75	—	2	2	5	5	5	5	14	8	17	20	9	17	4	—	—	117
·58	—	—	5	1	8	1	12	9	10	12	12	19	17	4	3	1	114
·37	—	2	2	3	6	5	3	8	6	3	19	16	16	6	3	3	101
·25	—	—	—	1	3	3	1	2	5	2	4	15	5	4	3	—	48
·14	—	—	—	1	—	4	1	1	8	6	3	8	8	7	2	1	50
·08	—	—	—	—	—	1	1	2	2	2	5	2	2	—	—	1	18
·04	—	—	—	—	2	—	—	2	—	3	3	1	3	4	—	—	18
Totals	6	6	14	38	46	48	48	84	74	110	132	156	148	68	18	14	1010

The constants deduced from this table are the following:

Mean Vision:  $\cdot7814 = 6/7\cdot679$ .

Standard Deviation of Visual Acuity:  $\cdot3588$ .

Mean Age: 12·2568 yrs.

Standard Deviation of Age: 1·5511 yrs.

Correlation Coefficient:  $-\cdot0867 \pm \cdot0211$ .

Correlation Ratio:  $\eta'^2_{V,A} = \cdot017,827$ ,  $\bar{\eta}^2_{V,A} = \cdot011,881 \pm \cdot003,967$ ; or  $\eta'_{V,A}$  is not really significant; it equals  $\cdot1335$ .

Comparing these results with those for the general Medical Examination we see that the latter under-estimated the visual acuity, and over-emphasised the relation of Vision and Age. The vision falls to about the ninth year, then remains singularly steady till the thirteenth year, and falls smartly at puberty. The data suggest a cubic rather than a straight line:

$$V.A. = \cdot7876 - \cdot002,222 (A. - 12\cdot208) - \cdot005,364 (A. - 12\cdot208)^2 - \cdot003,424 (A. - 12\cdot208)^3.$$

The means of the arrays are:

Central Ages (years)	Visual Acuity	Central Ages (years)	Visual Acuity
8·362	·8073	12·208	·8405
9·208	·9295	12·708	·7876
9·708	·7661	13·208	·7701
10·208	·7842	13·708	·7593
10·708	·7600	14·208	·7259
11·208	·8307	14·708	·5916
11·708	·7458	15·208	·6564
		14·927	·6200

\* The shifted cubics give quite good graduation curves, i.e.

$$\text{Boys: } V.A. = \cdot7497 + \cdot0220 (A. - 12\cdot208) - \cdot0080 (A. - 12\cdot208)^2 - \cdot00192 (A. - 12\cdot208)^3.$$

$$\text{Girls: } V.A. = \cdot6456 + \cdot0556 (A. - 11\cdot208) + \cdot00260 (A. - 11\cdot208)^2 - \cdot00440 (A. - 11\cdot208)^3.$$

† The medical examination was designed to cover all the "entrants" and "leavers," but it seems possible that the teachers sent up with the former class certain children not entrants about whose sight they wanted information.

Diagram 48 shows that the graduation is quite good, but it removes the drop in the ninth year. It would seem reasonable to suggest that there had been in the excess frequency in the Medical Examination at 8·5 to 9 years an inclusion of boys with bad sight; see totals column, Table CLI, p. 119.

From our curve we obtain the following system of visual acuity values to be added to the observed visual acuities at the stated ages in order to reduce the visual acuity to age 12.

### VISUAL ACUITY & AGE

#### SPECIAL EYE EXAMINATION. ALIEN JEWISH BOYS

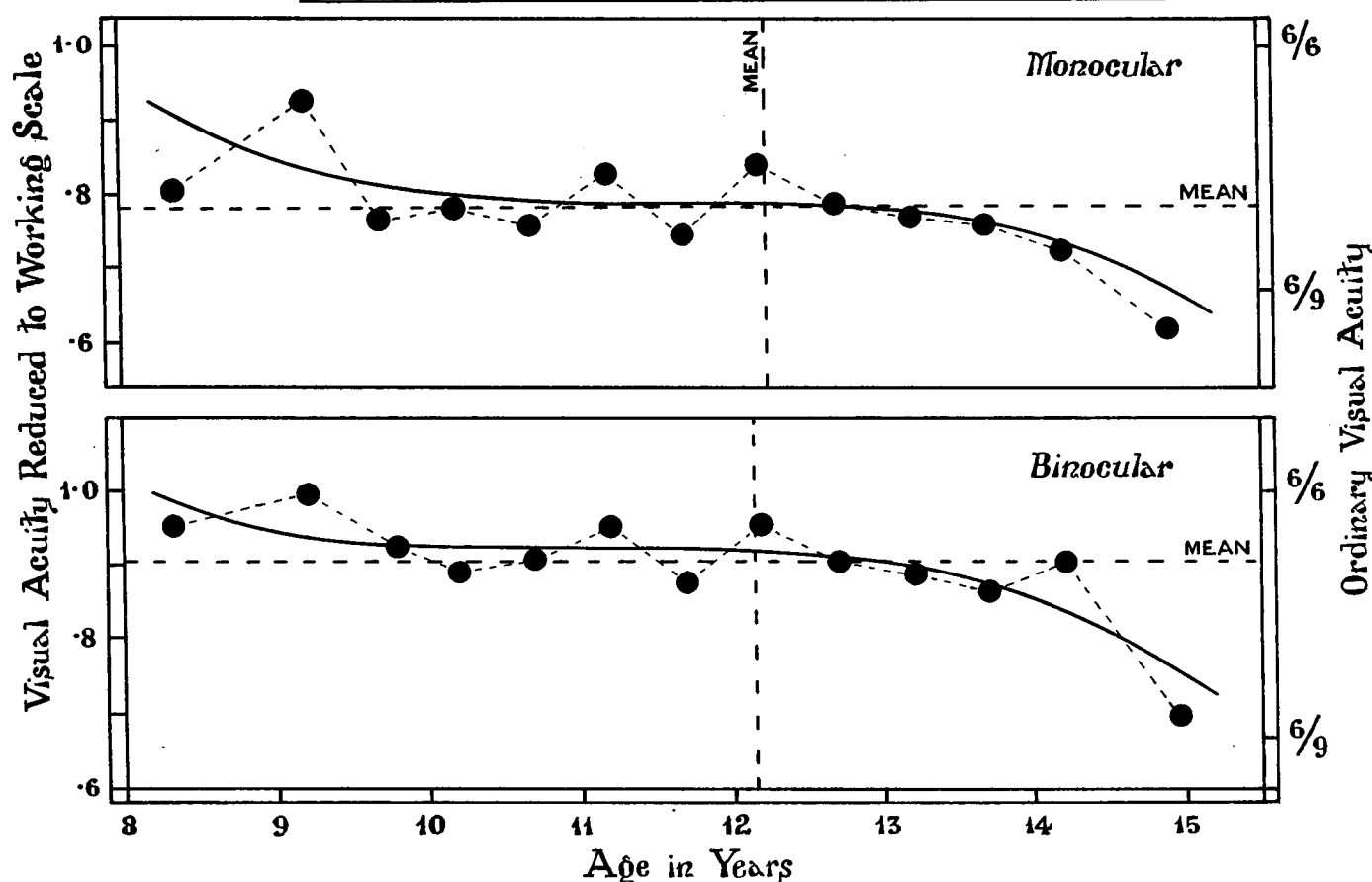


Diagram 48.

#### *Reduction of Visual Acuity to Standard Age 12 years. Special Examination.*

Age (years)	Reduction	Age (years)	Reduction	Age (years)	Reduction
7·5-8	-·21	10·5-11	-·00	13-13·5	+·01
8-8·5	-·14	11-11·5	-·00	13·5-14	+·03
8·5-9	-·09	11·5-12	-·00	14-14·5	+·05
9-9·5	-·05	12-12·5	+·00	14·5-15	+·09
9·5-10	-·03	12·5-13	+·00	15-15·5	+·15
10-10·5	-·01				

It will be seen that from 10·5 to 13 no correction is needed, and these would be the best years for comparing the sight of different classes of children, or children in different schools or different localities. We have not made the above age corrections when comparing the influence on vision of various environmental and physical factors, as except for age 7 to 8 they are of small importance. They were, however, used in the correlations dealing with Hebrew study in Section E.

(c) *Age and Vision of the Left and Right Eyes separately.*

Although the visual acuity of Left and Right Eyes is very closely the same, it seems desirable to place on record the data for the two eyes separately.

Table CLIV. *Age and Visual Acuity. Right and Left Eyes. Special Examination.*

Central Visual Acutities	Central Ages in Years																Totals
	7-708	8-208	8-708	9-208	9-708	10-208	10-708	11-208	11-708	12-208	12-708	13-208	13-708	14-208	14-708	15-208	
1-50 {	L.	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
	R.	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	2
1-40 {	L.	—	—	—	—	—	—	—	—	—	4	2	—	2	—	—	8
	R.	—	—	—	—	—	—	—	1	—	4	3	—	1	—	—	9
1-29 {	L.	—	—	—	2	—	3	1	2	9	3	7	4	2	—	—	40
	R.	1	—	—	2	1	3	1	3	8	4	10	5	2	—	—	45
1-11 {	L.	1	1	1	9	9	3	4	8	11	9	14	11	8	2	1	92
	R.	1	—	3	8	8	4	5	11	11	13	15	10	11	4	1	105
·91 {	L.	2	—	1	3	2	7	4	9	3	14	12	21	25	13	3	123
	R.	1	1	—	3	2	9	6	6	4	12	9	22	25	13	2	119
·75 {	L.	—	1	1	2	2	3	5	8	5	8	10	2	10	1	—	58
	R.	—	1	1	3	3	2	4	6	3	9	10	7	7	3	—	59
·58 {	L.	—	—	3	1	4	—	5	3	5	6	4	11	8	2	1	53
	R.	—	—	2	—	4	1	7	6	5	6	8	8	9	2	2	61
·37 {	L.	—	1	1	1	4	3	2	5	2	2	11	10	9	3	2	58
	R.	—	1	1	2	2	2	1	3	4	1	8	6	7	3	1	43
·25 {	L.	—	—	—	—	1	2	1	—	3	1	2	8	3	2	1	24
	R.	—	—	—	1	2	1	—	2	2	1	2	7	2	2	2	24
·14 {	L.	—	—	—	1	—	2	1	—	5	3	2	4	4	5	1	28
	R.	—	—	—	—	—	2	—	1	3	3	1	4	4	2	1	22
·08 {	L.	—	—	—	—	—	1	1	1	1	—	3	1	1	—	—	10
	R.	—	—	—	—	—	—	—	1	1	2	2	1	1	—	—	8
·04 {	L.	—	—	—	—	1	—	—	1	—	3	1	1	1	2	—	10
	R.	—	—	—	—	1	—	—	—	—	2	—	2	2	—	—	8
Totals ...	{ 3	3	7	19	23	24	24	42	37	55	66	78	74	34	9	7	505
	3	3	7	19	23	24	24	42	37	55	66	78	74	34	9	7	505

Visual Acuity Mean, Left Eye ... .. ·7627 = 6/7·867      Right Eye ... ·8001 = 6/7·499.  
 „      Standard Deviation, Left Eye ·3628      Right Eye ... ·3539.

It would appear from these results that the Right Eye has on the average the better vision\*, but that there is very little difference between the variability of the two eyes.

The correlations between age and either eye's acuity of vision are:

Left Eye ... — ·0885 ± ·0298      Right Eye ... — ·0849 ± ·0298,

the mean of which, — ·0867, is precisely what we have found by combining the two series. The difference of mean acuity in the two eyes thus produces no sensible difference in correlation. This justifies combining Right and Left Eyes in a single table.

(d) *Binocular Visual Acuity and Age.* Table CLV (p. 125) embraces our data for binocular vision and age.

This table provides us with the following constants:

Mean Binocular Visual Acuity: ·9055.      Standard Deviation of Visual Acuity: ·3273.

Mean Age: 12·1589 yrs.      Standard Deviation of Age: 1·5887.

Correlation Coefficient, Visual Acuity and Age: — ·0929 ± ·0317.

$\eta'^2_{V.A} = \cdot021,858$ , but  $\bar{\eta}^2_{V.A} = \cdot026,966 \pm \cdot007,215$ .

\* This is true for nearly every age array, i.e. ten out of the thirteen. Galton stated that his data showed no difference between the two eyes, *Journal of Anthropological Institute*, Vol. xiv, p. 286. This has been confirmed by a more elaborate recent rediscussion of his material.

$\eta'^2_{V.A}$  is less than  $\bar{\eta}^2_{V.A}$  and therefore we could not predict from the data that  $\eta'_{V.A}$  had any significance. The small but significant value of the correlation coefficient suggests that the value of  $\eta'_{V.A} = .1478$  may be of some small importance.

Table CLV. *Binocular Visual Acuity and Age. Boys. Special Examination.\**

Binocular Visual Acuity Central Values	Central Ages in Years																Totals
	7-708	8-208	8-708	9-208	9-708	10-208	10-708	11-208	11-708	12-208	12-708	13-208	13-708	14-208	14-708	15-208	
1.50	—	—	—	—	—	—	—	—	—	—	1	1	1	—	—	—	3
1.40	—	—	—	—	—	—	—	—	1	—	3	2	1	—	—	—	7
1.29	1	—	1	2	2	3	3	10	4	11	6	12	6	3	—	—	64
1.11	1	1	2	11	8	8	5	9	10	16	21	12	14	8	1	—	127
.91	1	1	1	2	4	5	8	10	3	13	12	19	15	8	—	4	106
.75	—	—	1	2	1	2	4	3	5	5	4	6	8	—	1	—	42
.58	—	—	1	1	4	1	3	2	2	3	6	5	9	1	1	—	39
.37	—	1	—	—	1	2	1	2	4	—	4	6	2	2	2	—	27
.25	—	—	—	1	—	1	—	—	1	—	—	4	2	1	—	—	10
.14	—	—	—	—	—	1	—	1	1	3	1	1	1	—	—	1	10
.08	—	—	—	—	—	—	—	—	—	1	3	1	1	—	—	—	6
.04	—	—	—	—	—	—	—	1	—	—	1	—	1	1	—	—	.4
Totals	3	3	6	19	20	23	24	38	31	52	62	69	61	24	5	5	445

The following are the mean visual acuities for the central ages provided.

Central Age (years)	Visual Acuity	Central Age (years)	Visual Acuity	Central Age (years)	Visual Acuity
8.333	.9542	11.208	.9558	13.708	.8615
9.208	.9968	11.708	.8765	14.208	.9017
9.708	.9270	12.208	.9571	14.958	.6960
10.208	.8917	12.708	.9040	Whole Population }	.9055
10.708	.9088	13.208	.8874		

It will be clear that at any age binocular vision is better than monocular vision (see p. 122). Diagram 48 (lower curve) shows changes of visual acuity with age. We have fitted a cubic to the results, i.e.:

$$V.A. = .92007 - .01130 (A. - 12.208) - .00932 (A. - 12.208)^2 - .00272 (A. - 12.208)^3.$$

Finally the corrections for reducing binocular vision at each age to standard age 12 years are as follows:

Age (years)	Reduction	Age (years)	Reduction	Age (years)	Reduction
7.5-8	— .11	10.5-11	— .00	13.5-14	+ .05
8-8.5	— .08	11-11.5	— .00	14-14.5	+ .08
8.5-9	— .04	11.5-12	— .00	14.5-15	+ .13
9-9.5	— .02	12-12.5	+ .00	15-15.5	+ .19
9.5-10	— .01	12.5-13	+ .01		
10-10.5	— .01	13-13.5	+ .02		

Here the only really significant corrections are at age 14.5 to 15.5. As our data for these categories are very spare, no age corrections have been made except in the case of Hebrew study in Section E.

\* As the bulk of the tables to follow are based on the Special Examination, we omit these words hereafter, only stating when we are dealing, as we do exceptionally, with the Medical Examination data.

(ii) *Refraction Class and Age.* The following table provides the observed data:

Table CLVI. *Refraction Class and Age. Boys.*

Refraction Class	Central Ages in Years																Totals
	7-708	8-208	8-708	9-208	9-708	10-208	10-708	11-208	11-708	12-208	12-708	13-208	13-708	14-208	14-708	15-208	
Normal ... ..	6	3	10	30	28	27	32	43	34	62	77	87	85	30	6	7	567
Hypermetropia ...	—	1	—	—	—	5	2	4	4	8	4	13	17	6	—	3	67
Hypermetropic Astigmatism	—	—	—	—	6	2	5	7	6	3	11	10	8	5	2	4	69
Mixed Astigmatism ...	—	—	—	—	—	—	—	1	4	1	3	2	1	1	—	—	13
Myopic Astigmatism ...	—	—	—	—	1	1	3	5	—	3	5	8	7	4	2	—	39
Myopia ... ..	—	—	—	—	3	5	4	16	6	17	20	26	24	14	4	2	141
Totals ... ..	6	4	10	30	38	40	46	76	54	94	120	146	142	60	14	16	896

The mean ages of the several refraction classes are:

Normal ... ..	12-1254	Mixed Astigmatism ...	12-5160.
Hypermetropia ... ..	12-7829	Myopic Astigmatism ...	12-7211.
Hypermetropic Astigmatism ...	12-4692	Myopia ... ..	12-7296.

Whole population ... .. 12-3277.

Standard Deviation of Age ... 1-4899.

Correlation Ratio of Age on Refraction Class:  $\eta'^2 = .013,582$ ,  $\bar{\eta}^2 = .005,580 \pm .002,371$ .

Thus the difference between  $\eta'^2$  and  $\bar{\eta}^2$  is more than three times the probable error, and may be considered as significant. The value of  $\eta'$  is .1165, small, however, if sensible\*.

Diagram 49 indicates the nature of the changes with age, which may be summed up as follows:

- Myopia increases with age.
- Mixed Astigmatism tends to increase up to 11 years and then to decrease and ultimately become steady in amount.
- Hypermetropic Astigmatism probably remains fairly constant after 9-5 years but may tend slightly to increase from 14 to 15.
- Manifest Hypermetropia tends to increase from 9-5 to 13 years, but may remain constant after this.

The general result is that there is a rapid increase in the prevalence of eye anomalies up to 11-5 years of age. From 11-5 to 13-5 there is, however, an almost stationary condition of the anomalies involving Myopia, only Hypermetropia (without Astigmatism) tending to increase. With the oncoming of puberty there is, however, a serious increase in the total percentage of eye anomalies. These new results are in very reasonable accord with the conclusions drawn by Amy Barrington and one of the present authors when discussing the Edinburgh C.O.S. data†. There the amounts of Myopia, Myopic Astigmatism and of Mixed Astigmatism remained nearly constant from 7½ to 12½ years. Then with the onset of puberty Myopia and Hypermetropic Astigmatism markedly increased, while Hypermetropia, if anything, decreased. The increase of normal vision in the Scottish Children from 6 to 10 finds no equivalent in the Jewish Boys. There is, however, a rise in the normal class at 12-5 due to a reduction in the amounts of Mixed and Hypermetropic Astigmatism. Examining Table CLVII it seems highly probable that the 10-0 % of Hypermetropic Astigmatism and 3-8 % of Mixed Astigmatism at age 11 to 12 are merely coincident excesses attributable to random sampling, and that there is actually a continuous fall

\* We note again how a relatively small  $\eta$  shows exaggeratedly on a percentage table.

† Amy Barrington and Karl Pearson: "A First Study of the Inheritance of Vision and of the Relative Influence of Heredity and Environment on Sight," *Eugenics Laboratory Memoirs*, No. V, Cambridge University Press, pp. 30-31.

Table CLVII. *Refraction Class and Age. Percentages of each Refraction Class with Age.*

Age	Normal	Hypermetropia	Hypermetropic Astigmatism	Mixed Astigmatism	Myopic Astigmatism	Myopia	Number Observed
Under 9	95.0	5.0	0.0	0.0	0.0	0.0	20
9-10	85.3	0.0	8.8	0.0	1.5	4.4	68
10-11	68.6	8.1	8.1	0.0	4.7	10.5	86
11-12	59.2	6.2	10.0	3.8	3.8	16.9	130
12-13	65.0	5.6	6.5	1.9	3.7	17.3	214
13-14	59.7	10.4	6.2	1.0	5.2	17.4	288
14 and over	47.8	10.0	12.2	1.1	6.7	22.2	90
All ages	63.3	7.5	7.7	1.4	4.4	15.7	896

in the amount of Emmetropia in the Jewish Boys. In this case these Jewish differ essentially from the Gentile children; they would indicate a continuous increase of eye anomaly with age, and we think this is the general interpretation to be put on Table CLVII. It is to be noted, however, that both Jews and Gentiles show no increase in Myopia and Myopic Astigmatism during the years from 10.5 to 13.5 which might be considered critical years of school life. Our new data confirm the old, in the view that the school cannot be looked upon as the forcing house of Myopia. They accord much better with the view that Myopia is a growth product of the early years of childhood and of the oncoming of puberty. It is these years which are marked by the large increases of Myopia\*.

It must be emphasised here that when we speak of the "increase of Myopia" we mean the increase in the numbers recorded as Myopic, the intensity of the Myopia in the individual is not under consideration: whether that intensity increases or decreases with age will be considered in our section on General Refraction and Age.

(iii) *General Refraction and Age.* Table CLVIII provides our data. For statistical purposes the chief objection to it is that more than 49 % of individuals lie in the zero refraction group, but ill-balanced grouping occurs in nearly all ophthalmological measurements as at present taken; they are not fine enough for anthropometric and statistical purposes.

\* Some of the great increase of Myopia in the early years of childhood may be apparent only, being due to the greater difficulty in measuring the refraction in young children, but we doubt if anything like the whole increase can be due to this cause.

PERCENTAGES IN EACH REFRACTION CLASS &amp; AGE

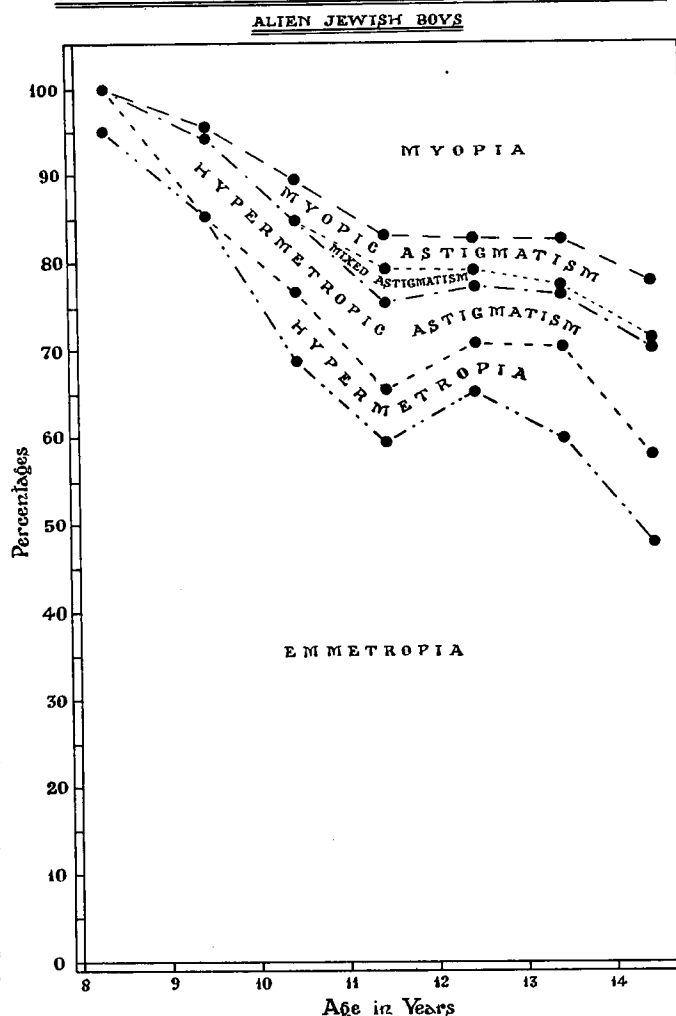


Diagram 49.

The constants of this table are:

Mean Age ... 12.3277 yrs. Mean Refraction ... + .0933 D.  
 Standard Deviation: Age ... 1.5324 yrs. Standard Deviation: Refraction 1.5621 D.  
 Correlation Ratio, Refraction on Age:  $\eta'^2 = .017,605$ ,  $\bar{\eta}^2 = .013,393 \pm .003,662$ .  
 Correlation Coefficient, Refraction and Age:  $r = - .0435 \pm .0225$ .

It is clear from these results that there is, whether judged by  $\eta'$  or  $r$ , no sensible relation between General Refraction and Age. This may appear *a priori* to contradict the results of the last section

Table CLVIII. *General Refraction and Age.*

General Refraction in Dioptres	Central Ages in Years																Totals
	7-708	8-208	8-708	9-208	9-708	10-208	10-708	11-208	11-708	12-208	12-708	13-208	13-708	14-208	14-708	15-208	
+ 6.75	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
+ 6.0	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	2	4
+ 5.25	—	—	—	—	1	—	—	—	—	—	—	1	2	—	—	—	4
+ 4.5	—	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	3
+ 3.75	—	—	—	—	1	0.5	2	—	—	2	1	4	4	—	—	1	15.5
+ 3.0	—	—	—	—	1	0.5	1	2	2	—	—	2	1	—	—	—	9.5
+ 2.25	—	1	—	—	2	3.5	3	4	3	2	2	1	3	6	1	1	32.5
+ 1.5	—	—	—	—	—	1	—	2	7	4	2	3	2	2	1	1	25
+ .75	—	0.5	4	2	3	5.5	—	8	6	13	34.5	44.5	42	18	2	4	187
0	6	2.5	6	28	25	23	35	40	29	54	53	57.5	56	17.5	5	5	442.5
— .75	—	—	—	—	4	1	3	13	5	10	10	14.5	13	9.5	1	—	84
— 1.5	—	—	—	—	—	1	1	3	1	1	7.5	7.5	7.5	3	2	—	34.5
— 2.25	—	—	—	—	—	3	—	1	1	2	4	7	2.5	2	2	—	24.5
— 3.0	—	—	—	—	—	—	—	—	—	—	3	—	4	—	—	—	7
— 3.75	—	—	—	—	—	—	—	1	—	—	2	1	3	—	—	1	8
— 4.5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
— 5.25	—	—	—	—	—	1	—	2	—	1	—	2	—	—	—	1	7
— 6.0	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
— 6.75	—	—	—	—	—	—	—	—	—	3	—	—	1	—	—	—	4
...	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
— 12.75	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
...	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
— 15.75	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	6	4	10	30	38	40	46	76	54	94	120	146	142	60	14	16	896

on Refraction Class, where we found that the relative numbers in each class were affected by age. But while the bulk of the negative refractions correspond to Myopia, and those of the positive refractions to Hypermetropia, this table and Diagram 50 really answer a different problem. There we are considering whether the *number* of myopes increased with age. Here we are considering whether the average value of the refraction in the Jewish Boy population tends to be either more or less negative with age, and the answer is that it does neither. This is clearly indicated on the diagram as well as in the measures of correlation. The two statements are quite compatible, if any one of the following hypotheses be confirmed:

- Manifest Hypermetropia and Myopia both increase in intensity with age and nearly equally.
- Myopia as well as manifest Hypermetropia tend to decrease in intensity with age, but the number of persons with anomalies tends to increase.
- Of the two anomalies one decreases and the other increases with age, but the decreasing one occurs in an increasingly larger number of individuals than the increasing one.

We have investigated these possibilities, and find that it is the last alternative which corresponds to the facts. The intensity of Myopia increases with age, the intensity of Hypermetropia decreases

with age; the numbers of children who have manifest Hypermetropia and Myopia both increase, but the number of Hypermetropic children is not only at every age greater than the number of Myopic, but increases with age at a greater rate. This is clearly indicated in the accompanying table and will be seen graphically in Diagram 50.

Table CLIX.

Age in Years	Number of Boys with Refraction $> +.375$ D.	Mean Intensity of Refraction	Number of Boys with Refraction $< -.375$ D.	Mean Intensity of Refraction
Under 10	16.5	+ 1.977 D.	4	-.75 D.
10-11	18	+ 2.229 D.	10	- 1.80 D.
11-12	34	+ 1.522 D.	27	- 1.417 D.
12-13	62.5	+ 1.182 D.	44.5	- 2.006 D.
13-14	110.5	+ 1.327 D.	63	- 1.726 D.
Over 14 <sup>1</sup>	(39)	+ 1.481 D.	(23.5)	- 2.633 D.
All ages	280.5	+ 1.431 D.	172	- 1.855 D.

<sup>1</sup> The fall in the numbers here is not due to any decrease in the numbers of boys affected, but simply to the fact that the boys begin to leave school at these ages.

## GENERAL REFRACTION &amp; AGE

## ALIEN JEWISH BOYS

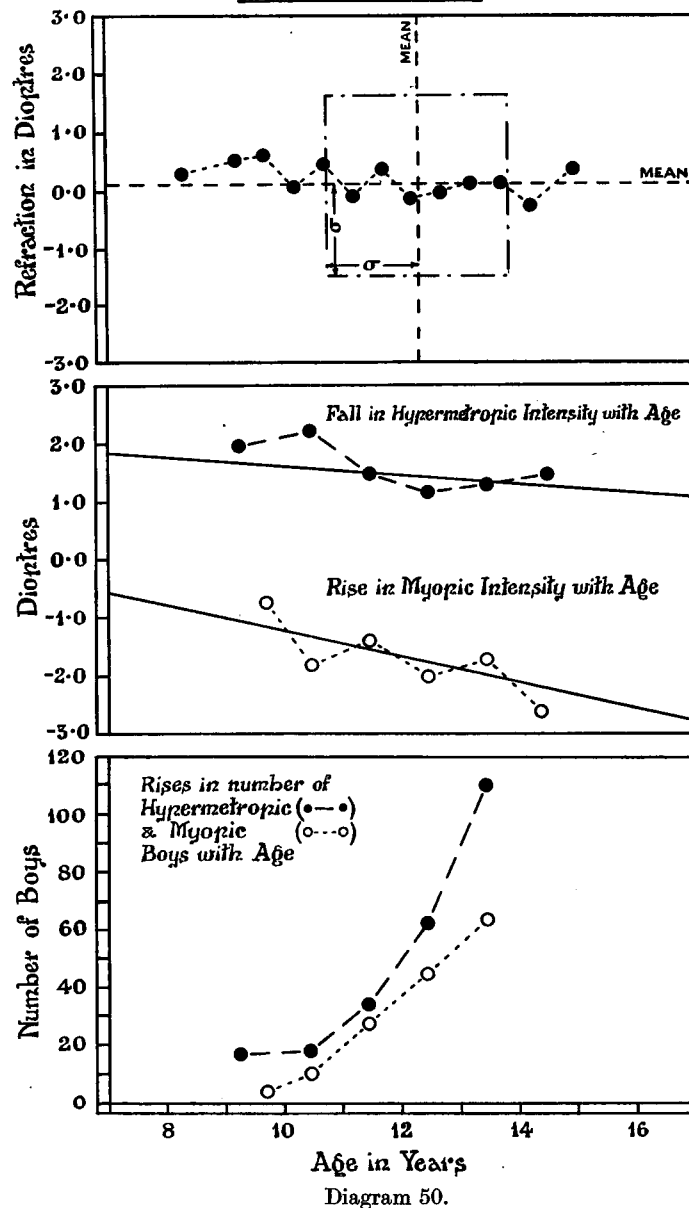


Diagram 50.

The regression of refraction on age is essentially zero, for the increasing number of myopes with an increasing intensity of the anomaly is balanced by a larger increasing number of children with Hypermetropia, who have, however, a decreasing intensity of this anomaly.

We have taken plus refraction roughly to represent the prevalence of Hypermetropia and minus refraction that of Myopia, without making finer distinctions.

(iv) *General Astigmatism and Age.* Table CLX contains our data. It will be seen that, as in the case of General Refraction, the frequency for statistical purposes is inconveniently crowded

Table CLX. *General Astigmatism and Age.*

General Astigmatism in Dioptres. Central Values	Central Ages in Years																Totals
	7-708	8-208	8-708	9-208	9-708	10-208	10-708	11-208	11-708	12-208	12-708	13-208	13-708	14-208	14-708	15-208	
+ 3.0	—	—	—	—	—	0.5	—	—	—	—	—	—	—	—	—	—	0.5
+ 2.25	—	—	—	—	—	0.5	—	—	—	—	—	1	—	—	—	—	1.5
+ 1.5	—	—	—	—	—	—	—	—	—	—	0.5	1.5	2	—	—	—	4
+ 0.75	—	—	—	—	1	1	—	2	3	5	16.5	12.5	13	1.5	—	1	56.5
0.0	6	3	8	30	26	35	33	54	40	72	85	99	109	43	8	9	660
- 0.75	—	1	2	—	5	1	9	12	4	14	10.5	17.5	11	8.5	3	2	100.5
- 1.5	—	—	—	—	2	—	2	—	4	—	3.5	4.5	3	4	1	—	24
- 2.25	—	—	—	—	2	1.5	2	4	2	3	2	5	2	3	2	1	29.5
- 3.0	—	—	—	—	—	0.5	—	2	—	—	—	4	—	—	—	—	6.5
- 3.75	—	—	—	—	—	—	—	2	1	—	2	1	2	—	—	1	9
- 4.5	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	1	3
- 5.25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	6	4	10	30	38	40	46	76	54	94	120	146	142	60	14	16	896

into the zero category. The constants of the distribution are:

Mean ... .. Age: 12.3277 yrs. Astigmatism: - .2193 D.

Standard Deviation ... .. „ 1.5324 yrs. „ .7478 D.

Product Moment Correlation:

$$r = - .0188 \pm .0225.$$

Correlation Ratio, Astigmatism on Age:

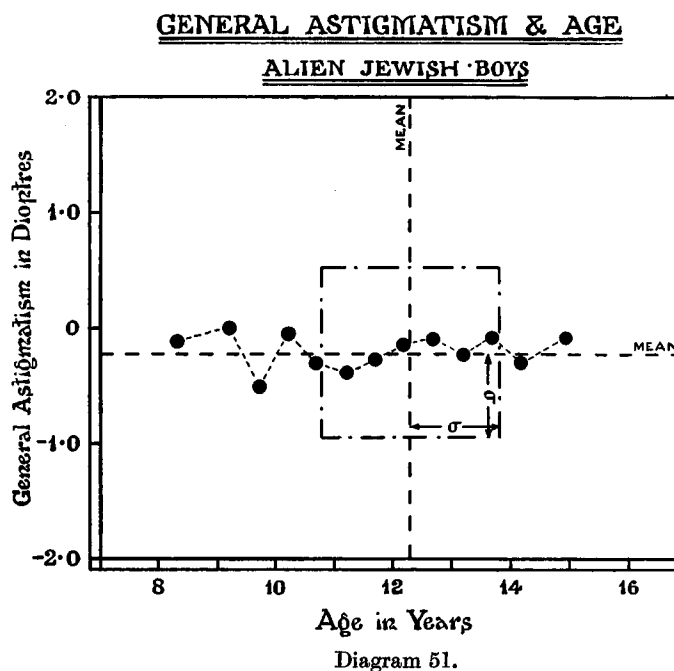
$$\eta'^2 = .029,268,$$

$$\bar{\eta}^2 = .013,393 \pm .003,662.$$

As for General Refraction, the correlation coefficient is zero having regard to its probable error. Diagram 51 confirms this result graphically; at all ages, except 9.71, the mean Astigmatism is below - .5 and never positive. We can conclude therefore that there is practically no change in the general population of Jewish Boys with age. At the same time  $\eta'^2$ , if small, still appears to be significant having regard to  $\bar{\eta}^2$ . The diagram might suggest that the significance of  $\eta'$  depends on the smooth bend of the curve between ages 11 and 13. If, however, we study the means at the various ages, paying attention to their probable errors, we find:

Mean General Astigmatism at each Age.

Age in Years	Mean	Age in Years	Mean
8.31	- .1125 D. $\pm$ .1128 D.	12.21	- .1436 D. $\pm$ .0520 D.
9.21	- .0000 D. $\pm$ .0921 D.	12.71	- .1000 D. $\pm$ .0460 D.
9.71	- .5132 D. $\pm$ .0818 D.	13.21	- .2260 D. $\pm$ .0417 D.
10.21	- .0563 D. $\pm$ .0798 D.	13.71	- .0845 D. $\pm$ .0423 D.
10.71	- .3098 D. $\pm$ .0744 D.	14.21	- .3000 D. $\pm$ .0651 D.
11.21	- .3947 D. $\pm$ .0579 D.	14.97	- .0825 D. $\pm$ .0921 D.
11.71	- .2778 D. $\pm$ .0686 D.		
		All Ages	- .2193 D. $\pm$ .0168 D.



Only the values at central ages 9·71, 11·21 and 13·71 can be considered to differ significantly from the population mean, and as these are irregularly distributed, it does not seem possible to base any statement upon them. We cannot adopt the same method as we applied for General Refraction, because the cases of Astigmatism against the rule are too few to provide reasonable means. We did, however, compute the means for Astigmatism with the rule and found:

Age in Years	Mean Astigmatism with the Rule	Age in Years	Mean Astigmatism with the Rule
Under 10	— 1·6071 D.	12-13	— 1·2107 D.
10-11	— 1·2422 D.	13-14	— 1·4325 D.
11-12	— 1·5726 D.	14 and Over	— 1·6227 D.
General Mean: 1·4395 D.			

It is clear from this table that there is no regular and sensible increase of Astigmatism with the rule with increasing age, and accordingly since General Astigmatism shows no such change,

Table CLXI A. *Corneal Refraction and Age.* (Observers A, B, and C.)

Central Ages in Years	Corneal Refraction, Central Values in Dioptres																			Totals	
	38·125	38·625	39·125	39·625	40·125	40·625	41·125	41·625	42·125	42·625	43·125	43·625	44·125	44·625	45·125	45·625	46·125	46·625	47·125		47·625
7·708	—	—	—	—	—	—	—	—	—	—	2	2	—	—	—	1	1	—	—	—	6
8·208	—	—	—	—	—	—	—	—	—	2	1	—	1	—	2	—	—	—	—	—	6
8·708	—	—	—	—	—	—	2	—	—	3	2	2	1	2	1	1	—	—	—	—	14
9·208	—	—	1	1	—	1	—	—	1	4	8	5	4	3	1	3	3	1	—	—	36
9·708	—	—	—	—	1	2	3	2	6	6	5	3	7	8	2	—	1	—	—	—	46
10·208	—	—	—	—	—	—	—	—	3	4	6	10	12	5	2	2	4	—	—	—	48
10·708	—	—	—	—	—	2	4	1	6	4	3	9	5	5	4	3	2	—	—	—	48
11·208	—	—	1	—	2	—	3	6	11	8	6	11	18	6	7	1	—	2	—	—	82
11·708	—	—	—	—	—	1	3	5	9	5	14	14	7	6	2	—	5	2	1	—	74
12·208	—	—	—	—	2	4	10	3	12	13	14	11	16	14	4	6	2	1	—	—	112
12·708	—	—	1	4	2	3	3	13	11	10	14	21	10	12	11	9	5·5	2·5	—	—	132
13·208	1	—	1	2	7	7	9	6	12	9	26	12	17	14	11	8	8	3	1	—	154
13·708	—	—	—	—	2	1	9	5	12	18	15	20	24	12	9·5	13	9·5	1	1	2	154
14·208	—	—	—	—	3	—	1	6	5	4	3	7	10	7	5	—	5	—	—	—	56
14·708	—	—	—	—	1	—	3	2	2	2	—	2	2	2	2	—	—	—	—	—	18
15·208	—	—	—	—	—	—	—	—	—	3	1	3	1	—	2	4	2	—	—	—	16
Totals	1	—	4	7	20	21	50	49	90	95	120	132	135	96	65·5	51	48	12·5	3	2	1002

Table CLXI B. *Corneal Refraction and Age.* (Observers A and B only.)

Central Ages in Years	Corneal Refraction, Central Values in Dioptres																			Totals	
	38·125	38·625	39·125	39·625	40·125	40·625	41·125	41·625	42·125	42·625	43·125	43·625	44·125	44·625	45·125	45·625	46·125	46·625	47·125		47·625
7·708	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	2
8·208	—	—	—	—	—	—	—	—	—	2	—	—	—	—	2	—	—	—	—	—	4
8·708	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2
9·208	—	—	—	—	—	—	—	—	—	—	—	1	1	3	1	—	—	—	—	—	6
9·708	—	—	—	—	1	—	1	—	—	1	1	—	3	1	1	—	1	—	—	—	10
10·208	—	—	—	—	—	—	—	—	1	1	—	2	1	1	—	1	1	—	—	—	8
10·708	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
11·208	—	—	—	—	1	—	—	—	2	1	1	3	4	5	1	—	—	—	—	—	18
11·708	—	—	—	—	—	—	1	1	3	2	3	5	3	1	—	—	4	1	1	—	24
12·208	—	—	—	—	1	—	1	1	1	2	2	2	6	12	2	1	2	1	—	—	34
12·708	—	—	—	—	—	1	1	4	4	3	8	11	5	7	9	2	2·5	2·5	—	—	60
13·208	1	—	—	—	4	3	1	3	3	4	11	8	9	7	4	5	5	3	1	—	72
13·708	—	—	—	—	—	3	—	—	4	10	12	15	21	7	7	11	8	1	1	2	102
14·208	—	—	—	—	2	—	—	3	4	4	3	5	6	4	4	—	5	—	—	—	40
14·708	—	—	—	—	1	—	—	1	2	2	—	—	—	—	2	—	—	—	—	—	8
15·208	—	—	—	—	—	—	—	—	—	3	1	3	1	—	2	4	2	—	—	—	16
Totals	1	—	—	—	10	4	9	13	24	35	42	57	60	48	35	24	30·5	8·5	3	2	406

we may assume that Astigmatism against the rule also remains constant. We conclude that there is very little, if any, change of Astigmatism in these Jewish Boys from ages 8 to 15.

(v) *Corneal Refraction and Age.* Tables CLXI A and CLXI B (p. 131) give our data for Corneal Refraction and Age. Here to satisfy ourselves as well as the reader we have first treated our material including the three observers A, B, and C, and then we have taken A and B's determinations

### DEGREE OF CORNEAL ASTIGMATISM & CORNEAL REFRACTION & AGE

#### ALIEN JEWISH BOYS

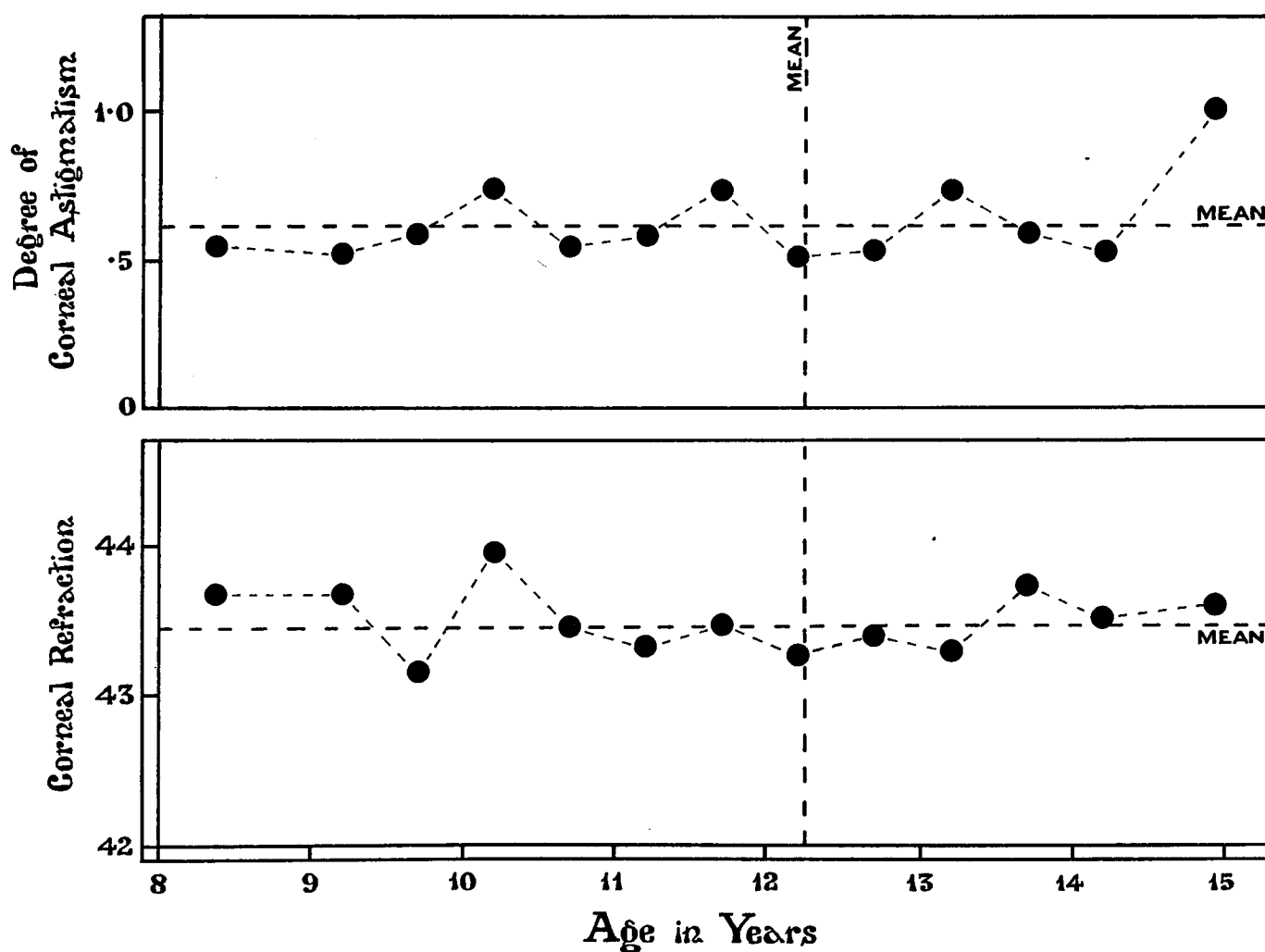


Diagram 52.

*Constants of Tables CLXI A and CLXI B (p. 131).*

	Observations of A, B, and C	Observations of A and B only
Mean Age	12.2542 yrs.	12.9029 yrs.
Standard Deviation for Age	1.5451 yrs.	1.3750 yrs.
Mean Corneal Refraction	43.468 D.	43.885 D.
Standard Deviation for Corneal Refraction	1.5508 D.	1.5168 D.
Product Moment Value of Correlation	.0027 ± .0213	.0342 ± .0334
Correlation Ratio, Corneal Refraction on Age $\left\{ \begin{array}{l} \eta^2 \\ \bar{\eta}^2 \end{array} \right.$	.017,543	.018,934
	.011,976 ± .003,260	.022,167 ± .006,872

only. We place the constants for the two distributions side by side; but they lead to the same conclusion, namely, that Corneal Refraction does not change sensibly with age.

While *C*'s observations were thus for somewhat younger boys, there is very little difference in the two sets of observations in the mean Corneal Refraction or its standard deviation. The correlation coefficient is not significant in either set, and the values for  $\eta'^2$  are not significant relatively to those for  $\bar{\eta}^2$  in both. We are therefore compelled to assert that the mean corneal curvature of the eye remains sensibly constant from 8 to 15 years of age. It would be of great interest to test this by measuring the *same* children during their whole school career.

The constancy of a character for all ages in a given population does not necessarily prove, although it suggests, its constancy in the individual, for individuals may grow in opposite senses, and the combined results give nearly a balance for the whole population.

The lower part of Diagram 52 shows how little Corneal Refraction varies with Age. As far as Corneal Refraction is a part of General Refraction the present results confirm our conclusions for the latter.

(vi) *Corneal Astigmatism and Age.* When we turn to this optical character we see how widely divergent are the observations of *C* from those of *A* and *B*. In no less than 321 out of 596 cases

*Corneal Astigmatism and Age.*

Table CLXII A. (Observers *A*, *B*, and *C*.)Table CLXII B. (Observers *A* and *B* only.)

Age in Years Central Values	Corneal Astigmatism in Dioptres											Totals	Corneal Astigmatism in Dioptres											Totals
	- 2.25	- 1.5	- 0.75	0.0	+ 0.75	+ 1.5	+ 2.25	+ 3.0	+ 3.75	+ 4.5	+ 5.25		+ 6.0	- 1.5	- 0.75	0.0	+ 0.75	+ 1.5	+ 2.25	+ 3.0	+ 3.75	+ 4.5	+ 5.25	
7.708	—	—	—	4	2	—	—	—	—	—	—	—	6	—	—	—	2	—	—	—	—	—	—	2
8.208	—	—	—	2	4	—	—	—	—	—	—	—	6	—	—	1	3	—	—	—	—	—	—	4
8.708	—	—	—	6	5	1	2	—	—	—	—	—	14	—	—	—	1	—	1	—	—	—	—	2
9.208	—	—	1	17	13	2	3	—	—	—	—	—	36	—	1	1	4	—	—	—	—	—	—	6
9.708	—	—	1	26	11	3	3	—	1	1	—	—	46	—	1	3	4	—	—	—	1	1	—	10
10.208	—	—	—	23	16	2	4	2	—	—	1	—	48	—	—	2	2	2	1	—	—	—	1	8
10.708	—	—	—	22	19	5	2	—	—	—	—	—	48	—	—	—	—	—	—	—	—	—	—	0
11.208	—	—	2	46	21	2	5	5	1	—	—	—	82	—	2	6	5	1	1	3	—	—	—	18
11.708	1	1	—	27	27	9	6	1	2	—	—	—	74	—	—	5	10	3	5	—	1	—	—	24
12.208	—	—	1	60	37	8	4	1	—	—	—	1	112	—	1	10	17	2	3	1	—	—	—	34
12.708	—	2	3	56	53	8	8	2	—	—	—	—	132	1	3	15	29	5	6	1	—	—	—	60
13.208	—	—	5	50	69	19	3	4	2	1	1	—	154	—	5	20	37	6	2	—	2	—	—	72
13.708	—	2	4.5	55	71.5	11	7	1	1	1	—	—	154	2	2	29	53	10	3	1	1	1	—	102
14.208	—	2	1	22	25	1	3	2	—	—	—	—	56	2	1	7	24	1	3	2	—	—	—	40
14.708	—	—	—	7	6	3	2	—	—	—	—	—	18	—	—	3	3	2	—	—	—	—	—	8
15.208	—	—	—	7	5	—	—	1	1	—	2	—	16	—	—	7	5	—	—	1	1	—	2	16
Totals	1	7	18.5	430	384.5	74	52	19	8	3	4	1	1002	5	16	109	199	32	25	9	6	2	3	406

*C* made the radii of curvature of the eye equal in the two principal planes. It is possible that having found the eye to have nearly normal vision, he did not observe with adequate accuracy the second radius of curvature. *A* and *B* found only 27 % of equal radii, while *C* found 54 %, i.e. double the number. Some such result as this would be reached, if *C* had neglected his +.5 readings in the neighbourhood of zero, so that they were classified with 0 rather than in the .75 group. It seemed essential accordingly to keep the observations of *A* and *B* apart.

*Constants of the Distributions for Corneal Astigmatism.*

	Observations of <i>A</i> , <i>B</i> , and <i>C</i>	Observations of <i>A</i> and <i>B</i> only
Mean Age	12.2542 yrs.	12.9029 yrs.
Standard Deviation for Age	1.5450 yrs.	1.3750 yrs.
Mean Corneal Astigmatism	.6160 D.	.7592 D.
Standard Deviation for Corneal Astigmatism	.8597 D.	.9343 D.
Product Moment Value of Correlation	+ .0404 ± .0213	— .0467 ± .0334
Correlation Ratio, Corneal Astigmatism on Age $\left\{ \begin{array}{l} \eta'^2 \\ \bar{\eta}^2 \end{array} \right.$	.022,378 .011,976 ± .003,257	.035,432 .022,167 ± .006,880

It will be seen that the inclusion of *C*'s observations reduces the mean Corneal Astigmatism and its standard deviation, because it heaps up the frequency in the zero group. In neither set is the coefficient of correlation of any significance, nor is the correlation ratio significant in the possibly more reliable observations of *A* and *B*. It is just significant when *C*'s observations are included. But an examination of Diagram 52 (upper figure) shows that there is really no continuous change of Corneal Astigmatism with growth when we combine *C* with *A* and *B*. Diagram 53 shows how little change there is with growth even when we use only *A* and *B*'s records. There is thus small doubt that Corneal Astigmatism does not increase between the ages 8 and 15 in the population considered. It would not seem therefore that the curvatures of the cornea are modified by prolonged school work.

### CORNEAL ASTIGMATISM & AGE

#### ALIEN JEWISH BOYS

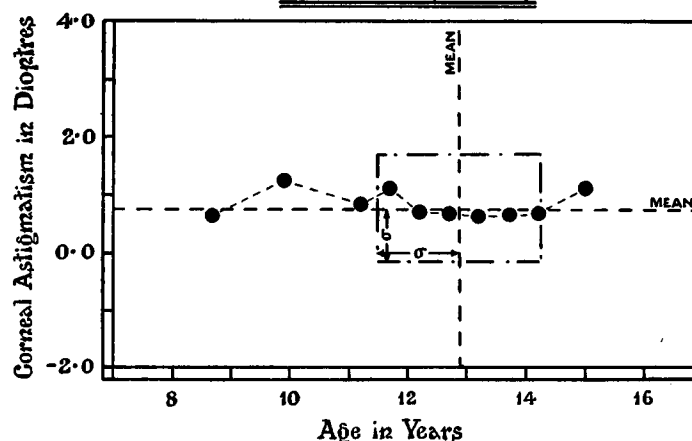


Diagram 53.

(vii) *Near Point and Age*. Our data are presented in Table CLXIII and the constants are as follows:

Mean Near Point\* ... 93.1445 mm. Standard Deviation ... 20.2595 mm.

Mean Age ... 12.0338 yrs. Standard Deviation ... 1.5312 yrs.

Product Moment Correlation ...  $-.1099 \pm .0240$ .

Correlation Ratio, Near Point on Age:  $\eta'^2 = .029,152$ ,  $\eta^2 = .015,625 \pm .004,399$ .

Thus, whether we take the coefficient of correlation or the correlation ratio, there is a significant if not very large association between Near Point and Age. Generally the coefficient of correlation shows that the distance of the Near Point decreases with Age, but the values of the means at each Age as well as their graph suggest that the distance increases up to between 10 and 11 years of

#### *Distance of the Near Point at each Age.*

Age in Years	Near Point in mm.	Age in Years	Near Point in mm.
8.340	94.091	12.208	91.511
9.208	96.842	12.708	92.139
9.708	94.000	13.208	93.459
10.208	96.528	13.708	88.854
10.708	102.826	14.208	91.979
11.208	95.338	14.958	85.000
11.708	90.803		
		All years	93.1445

age and then begins slowly to decrease (see Diagram 54, p. 136). Accordingly the data were fitted with a cubic. Its equation is:

$$N.P.D. = 93.5079 - 2.27710 (A. - 12.208) - .19780 (A. - 12.208)^2 + .08800 (A. - 12.208)^3,$$

where *N.P.D.* is the Near Point Distance in mm. and *A.* the Age in years†.

If we may lay some stress on this graduation the maximum distance of the Near Point would

\* Frequently taken as 110 mm. in Western European Races. Our Anthropometric Laboratory data give for Jewish Students: 126.9 mm.

† Or multiplied out:  $N.P.D. = -68.29418 + 41.89974 A. - 3.42079 A.^2 + .08800 A.^3$ .

Table CLXIII. *Distance of Near Point and Age.*

Central Ages in Years	Central Values																												Totals				
	Distance of Near Point in mm.																																
	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170		175	180	185	190
7-708	—	—	—	—	—	—	2	—	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8-208	—	—	—	—	—	—	—	—	—	2	1	—	—	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8-708	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9-208	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9-708	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
10-208	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
10-708	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
11-208	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
11-708	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
12-208	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
12-708	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
13-208	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
13-708	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14-208	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14-708	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15-208	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	—	2	2	7	15	23.5	50.5	51.5	88.5	81	101	86	63	43	35	28	18	14	21	16	7	5	4	—	1	—	1	1	1	1	1	768

Table CLXIV. *Index of Sunken Eye and Age.*

Central Ages in Years	Central Values																								Totals
	Index of Sunken Eye																								
	76.45	77.45	78.45	79.45	80.45	81.45	82.45	83.45	84.45	85.45	86.45	87.45	88.45	89.45	90.45	91.45	92.45	93.45	94.45	95.45	96.45	97.45	98.45	99.45	
7-708	—	—	—	—	—	—	—	—	—	—	—	—	1 2	—	—	1	—	—	—	—	—	—	—	—	—
8-208	—	—	—	—	—	—	—	—	—	—	—	—	1 2	—	—	1	—	—	—	—	—	—	—	—	—
8-708	—	—	—	—	—	—	—	—	—	—	—	—	1 2	—	—	1	—	—	—	—	—	—	—	—	—
9-208	—	—	—	—	—	—	—	—	—	—	—	—	3 1	2	—	2	—	—	—	—	—	—	—	—	—
9-708	—	—	—	—	—	—	—	—	—	—	—	—	3 1	2	—	2	—	—	—	—	—	—	—	—	—
10-208	—	—	—	—	—	—	—	—	—	—	—	—	2 2	3	4	2	—	—	—	—	—	—	—	—	—
10-708	—	—	—	—	—	—	—	—	—	—	—	—	2 2	3	4	2	—	—	—	—	—	—	—	—	—
11-208	—	—	—	—	—	—	—	—	—	—	—	—	3 3	4	7	5	4	3	3	—	—	—	—	—	—
11-708	—	—	—	—	—	—	—	—	—	—	—	—	3 4	4	7	5	4	3	4	—	—	—	—	—	—
12-208	—	—	—	—	—	—	—	—	—	—	—	—	11 6	7	7	7	4	2	4	2	—	—	—	—	—
12-708	—	—	—	—	—	—	—	—	—	—	—	—	11 6	7	7	7	4	2	4	2	—	—	—	—	—
13-208	—	—	—	—	—	—	—	—	—	—	—	—	10 13	10	9	9	6	6	5	3	—	—	—	—	—
13-708	—	—	—	—	—	—	—	—	—	—	—	—	14 5	7	10	9	5	3	1	—	—	—	—	—	—
14-208	—	—	—	—	—	—	—	—	—	—	—	—	1 1	3	3	4	3	3	2	—	—	—	—	—	—
14-708	—	—	—	—	—	—	—	—	—	—	—	—	1 1	3	3	4	3	3	2	—	—	—	—	—	—
15-208	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	—	—	—	—	—
Totals	1	—	1	—	2	—	10	14	19	35	43	80	69	62	74	38	32	23	6	4	1	1	1	1	518

be reached at about ten years (9.927) and its minimum or the region of horizontality at about sixteen years (15.989), or roughly soon after puberty. The total range in Near Point with Age as indicated by the cubic is from 96.63 at maximum to 86.83 at minimum or 9.8 mm. But the average variation as given by the array standard deviation at each age is 20.1379 mm., the general standard deviation being but little reduced by such a small correlation as .1099. Accordingly the *entire* range of change in the distance of the Near Point with Age is only about one-half of the individual variation, or roughly only one-twelfth of the individual range at any age, and is from the anthropometric standpoint of slight importance.

### POSITION OF NEAR POINT & AGE

#### ALIEN JEWISH BOYS

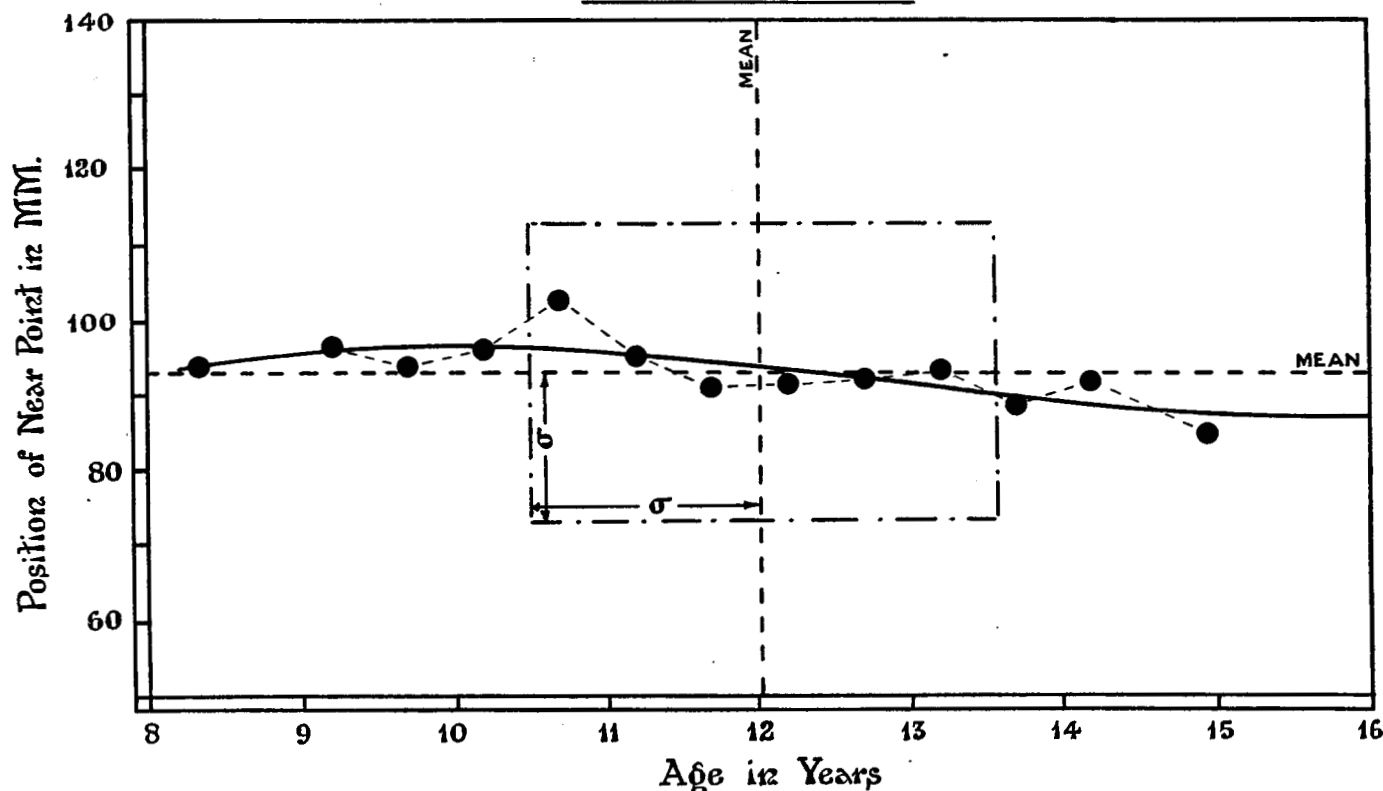


Diagram 54.

(viii) *Index of Sunken Eye and Age.* In our Section G, wherein we deal with the correlation of ocular with certain anthropometric characters, we shall have occasion to deal with several head measurements and their indices. Of these it is known and has been shown elsewhere that the familiar cephalic indices do not change with age and need not therefore be considered here. Among them are two less familiar to the anthropologist, i.e. the index of the Sunken Eye and the Interpupillary Index (for definitions, see our p. 118), and the influence of growth on these will now be considered. Table CLXIV (p. 135) presents our data.

The constants of this table are as follows:

Mean ... ... Age: 12.2730 yrs. Index: 89.6913.

Standard Deviation ... Age: 1.5552 yrs. Index: 2.9480.

Product Moment Coefficient of Correlation:  $-.1375 \pm .0291$ .

Correlation Ratio of Index on Age:  $\eta'^2 = .038,777$ ,  $\bar{\eta}^2 = .023,166 \pm .006,224$ .

The correlation coefficient is definitely significant; the correlation ratio is possibly just significant; we should have for its uncorrected value \*  $\eta' = .1969$ . Diagram 55 shows the change of the Index of Sunken Eye with Age. This diagram and the value of  $\eta'$  may suggest that a cubic (as in the case of the Near Point) showing a rise about 10–11 years of age, then a fall with a final trend to the horizontal after puberty, would best graduate the data. We have contented ourselves, however, with the graduation of the regression straight line, because our numbers are so very inadequate before eleven years of age. The main point to be noted is that though the correlation is significant,

### INDEX OF SUNKEN EYE & AGE

#### ALIEN JEWISH BOYS

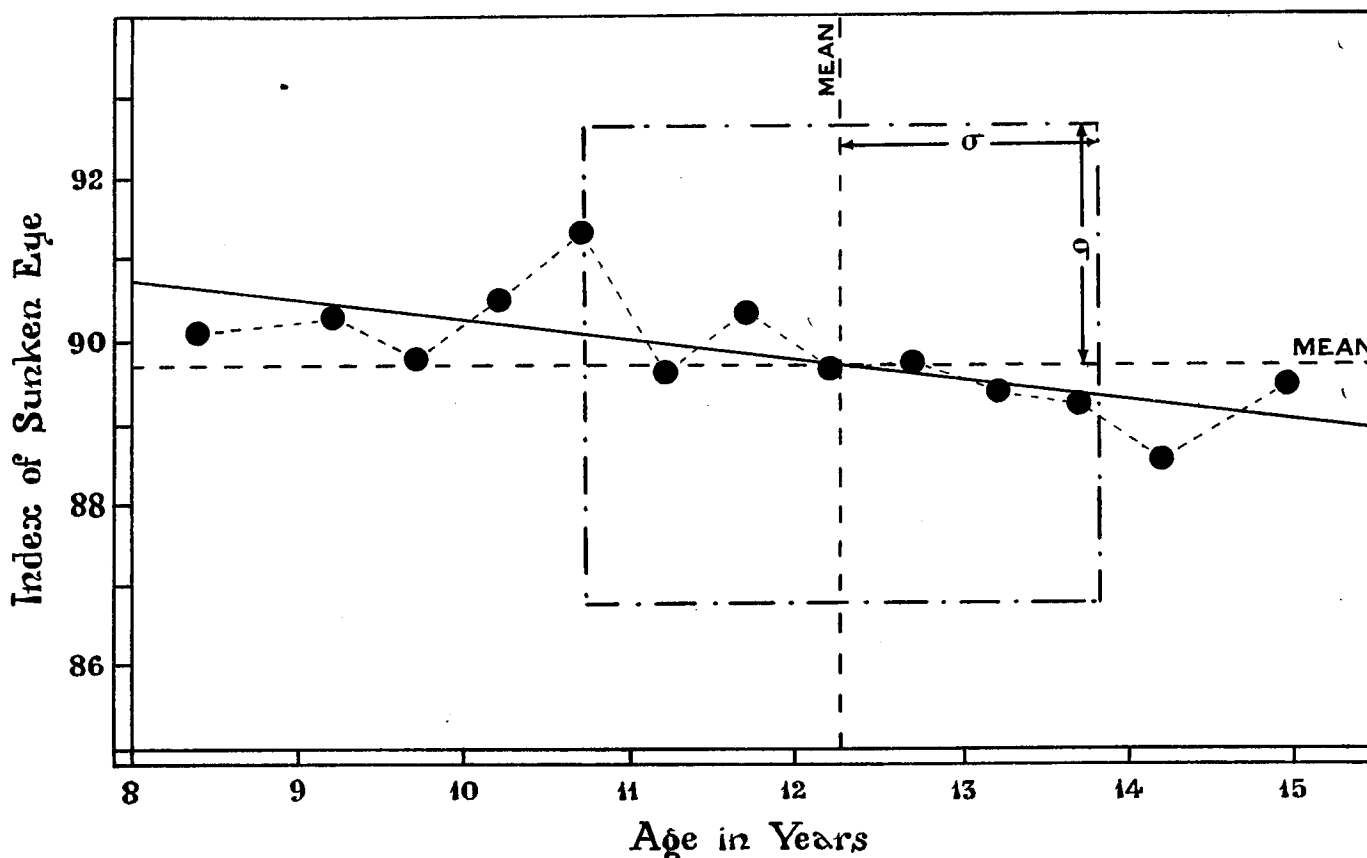


Diagram 55.

it is not of an order which renders correction for age essential. Corrections for age are of small service until a correlation of at least  $.25$  or over is reached. It is the old tale that you cannot learn much of a man's physical or psychical characters from an acquaintance with his second cousin (correlation  $r = .125$  about).

(ix) *Interpupillary Index and Age.* The reader will remember that the Interpupillary Index measures the percentage that the interpupillary distance is of the parietal breadth of the head. The small average value of this in the Jewish race (about 40 %) suggests that relative to their breadth of head the eyes in the Jews are closely set. Table CLXV provides the measurement of the

\* The correction would be very small as  $\eta'$  is based on as many as thirteen arrays.

Table CLXV. *Interpupillary Index and Age.*

Central Ages in Years	Central Values of Interpupillary Index																Totals
	34-45	35-45	36-45	37-45	38-45	39-45	40-45	41-45	42-45	43-45	44-45	45-45	46-45	47-45	48-45	49-45	
7-708	—	1	—	—	1	—	1	—	—	—	—	—	—	—	—	—	3
8-208	—	1	—	—	1	—	1	—	—	—	—	—	—	—	—	—	3
8-708	—	—	1	1	3	—	2	1	—	—	—	—	—	—	—	—	8
9-208	—	1	1	3	3	3	1	3	2	1	—	—	—	—	—	—	18
9-708	—	1	1	1	6	2	4	4	1	2	—	—	—	—	—	—	22
10-208	—	—	4	3	3	1	6	2	3	—	—	1	—	—	—	—	23
10-708	—	—	—	1	1	7	5	5	1	2	—	1	—	—	—	—	23
11-208	—	—	2	5	6	11	7	4	2	2	2	—	1	—	—	—	42
11-708	—	—	1	4	2	13	5	5	3	3	—	—	—	—	—	—	36
12-208	—	1	4	2	13	6	9	10	6	5	—	1	1	—	—	—	58
12-708	2	2	3	10	13	13	11	7	2	1	2	1	1	—	—	1	69
13-208	1	1	2	5	14	16	14	9	9	5	1	—	—	1	—	—	78
13-708	—	1	4	10	22	9	16	7	6	1	—	—	—	—	—	—	77
14-208	—	1	3	7	4	5	6	3	—	2	—	1	—	—	—	—	32
14-708	—	—	3	—	1	1	2	—	1	1	—	—	—	—	—	—	9
15-208	—	—	1	—	2	1	2	1	—	1	—	—	—	—	—	—	8
Totals	3	10	30	52	95	88	92	61	36	26	5	5	3	2	—	1	509

Interpupillary Index on 509 boys. The following are the constants of the frequency distribution:

Mean ... Age: 12.2780 yrs. Interpupillary Index: 39.8115.

Standard Deviation ... Age: 1.5435 yrs. Interpupillary Index: 2.1962.

Product Moment Coefficient of Correlation:  $-.0106 \pm .0299$ .

Correlation Ratio, Index on Age:

$$\eta'^2 = .036,048,$$

$$\bar{\eta}^2 = .023,576 \pm .006,332.$$

It will be seen that there is no significant correlation coefficient, nor can  $\eta'^2$  be considered to differ with definite significance from  $\bar{\eta}^2$ . But Diagram 56 does suggest that there is a continuous change of Index with Age; considering the size of the subrange frequencies, there is considerable smoothness in the run of the observations and they graduate well with the second order parabola:

$$\text{Interpupillary Index} = 26.10944 + 2.39078 A. - .10220 A^2.$$

This indicates that the interpupillary distance grows more rapidly than the parietal breadth from 8 to about 11.5 years (the maximum index at 11.6966 yrs. is equal to 40.0914) and less rapidly than the parietal breadth from 11.5 to 15 years. The parabola would suggest—but it is unwise to lay much stress on extrapolation—that the decrease in the index continues into adolescence. However, we have some rather sparse data from our Anthropometric Laboratory. These give:

English Males	13 to 16 yrs.	(81): 38.99	} 39.46.
„	17 to 24 yrs.	(280): 39.65	
„	25 and over	(48): 39.08	
Jewish Adolescents	17-20 yrs.	(22): 39.00	} 39.06.
Jewish Adults	21 to 25 and over	(13): 39.16	
Hindus, etc. (adults)		(28): 41.94.	

From these we should conclude that the Jewish adult value of the Interpupillary Index is less than that of English adults and both are considerably less than that of Hindus. Further, the Jewish Index continues to decrease after 15 years, but not to such a degree as interpolation from our parabola would suggest. It reaches an adult value slightly over 39 points. This index might have considerable interest for the anthropometry of oriental races.

*Summary of Section B.* The general result of this section may be summed up as follows, namely, that while there are interesting growth changes of the ocular characters with age, none of them are of a substantial order, i.e. greater than  $\cdot 25$ , and many much less than  $\cdot 10$ . Most of these small associations more or less clearly indicate that change with age is not a simply linear relation,

### INTERPUPILLARY INDEX & AGE

#### ALIEN JEWISH BOYS

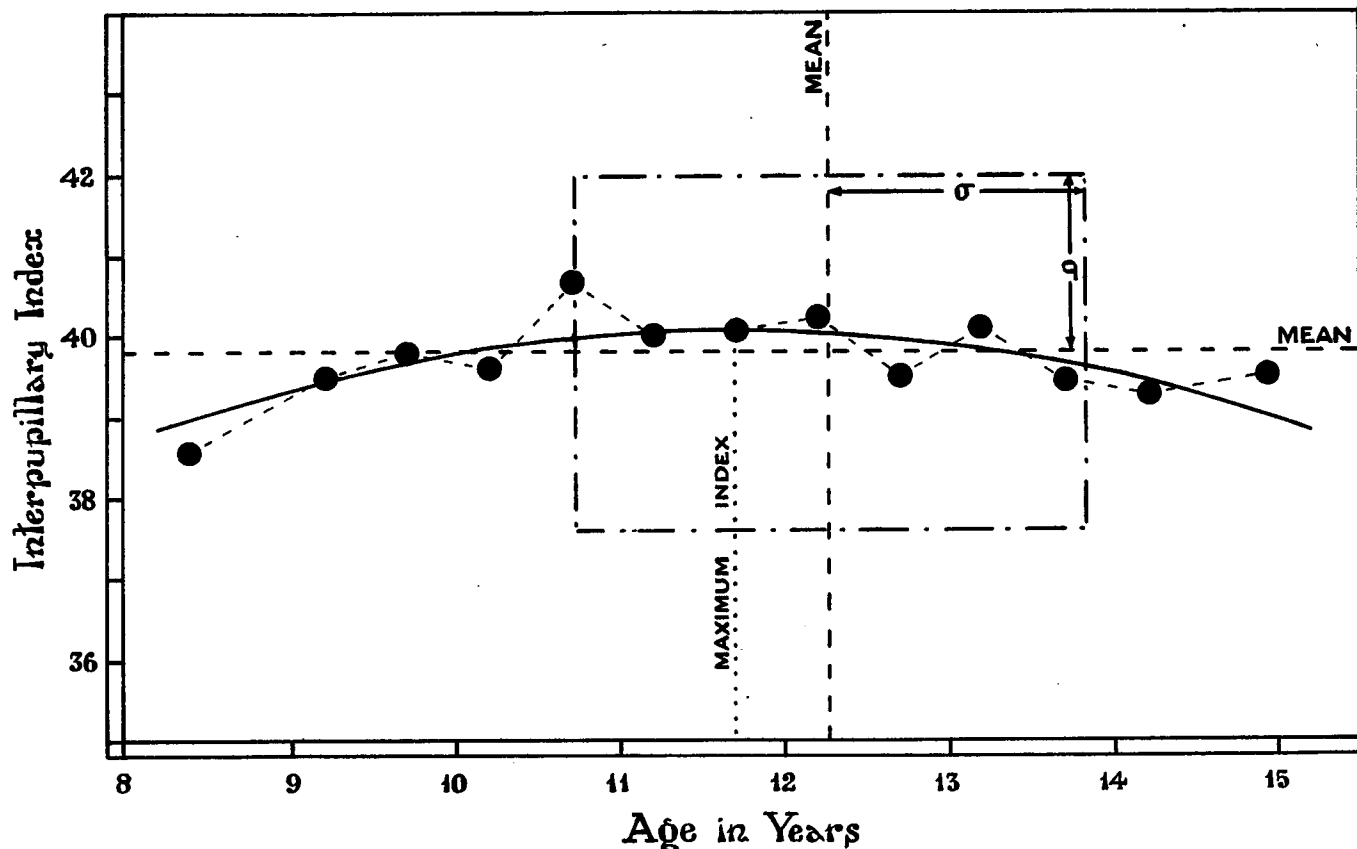


Diagram 56.

but that we have to deal with skew regression curves. The following collected data indicate these points:

Table CLXVI.

Character, the Growth of which is considered	Product Moment Correlation Coefficient	Correlation Ratio of Character on Age (uncorrected)
Monocular Visual Acuity ... ..	$-.0867 \pm .0211$	$\cdot 1335$ (Insignificant)
Binocular " " " "	$-.0929 \pm .0317$	$\cdot 1478$ (Insignificant)
Refraction Class (without mydriatic)	—	$\cdot 1165$ (Significant)
General Refraction ... ..	$-.0435 \pm .0225$	$\cdot 1327$ (Insignificant)
General Astigmatism ... ..	$-.0188 \pm .0225$	$\cdot 1721$ (Significant)
Corneal Refraction ... ..	$\{(A, B, \text{ and } C) + .0027 \pm .0213$	$\cdot 1325$
	$\{(A, B) + .0342 \pm .0334$	$\cdot 1376$ (Both Insignificant)
Corneal Astigmatism ... ..	$\{(A, B, \text{ and } C) + .0404 \pm .0213$	$\cdot 1496$ (Insignificant)
	$\{(A, B) - .0467 \pm .0334$	$\cdot 1883$ (Significant)
Near Point ... ..	$-.1099 \pm .0240$	$\cdot 1707$ (Significant)
Index of Sunken Eye ... ..	$-.1375 \pm .0291$	$\cdot 1969$ (? Significant)
Interpupillary Index ... ..	$-.0106 \pm .0299$	$\cdot 1899$ (? Significant)

It will be apparent that with such values of the correlation coefficients, and with correlation ratios not only often insignificant (or of doubtful significance) but also not substantial when significant, very little can be attempted in the way of age correction that will be of value when we come to deal with the influence of environment on ocular characters, or of ocular characters on intelligence. Unfortunately we were unable to measure General Refraction under atropine, when greater age changes would perhaps have manifested themselves. Later we shall deal with the subject of accommodation wherein the age influence is usually held to be more strongly marked.

*Note to Sections A and B. Refraction Classes.*

We have indicated (footnote, p. 113) that the existing divisions between Refraction Classes are arbitrary and vary from one observer to another. It is certainly desirable that the classification should be standardised. If we do not allow a certain elasticity in our definitions, Myopia, Emmetropia and Hypermetropia become dividing lines, and we are left only with three classes: Myopic, Hypermetropic and Mixed Astigmatism. The reader will grasp this better from the accompanying diagram, which not only provides the classification we have ourselves adopted, but may explain some of the apparent contradictions which arise later when we deal with the interrelations of ocular characters.

**DEFINITION & DISTRIBUTION OF**  
**REFRACTION CLASSES IN THIS MEMOIR**

**ALIEN JEWISH BOYS**

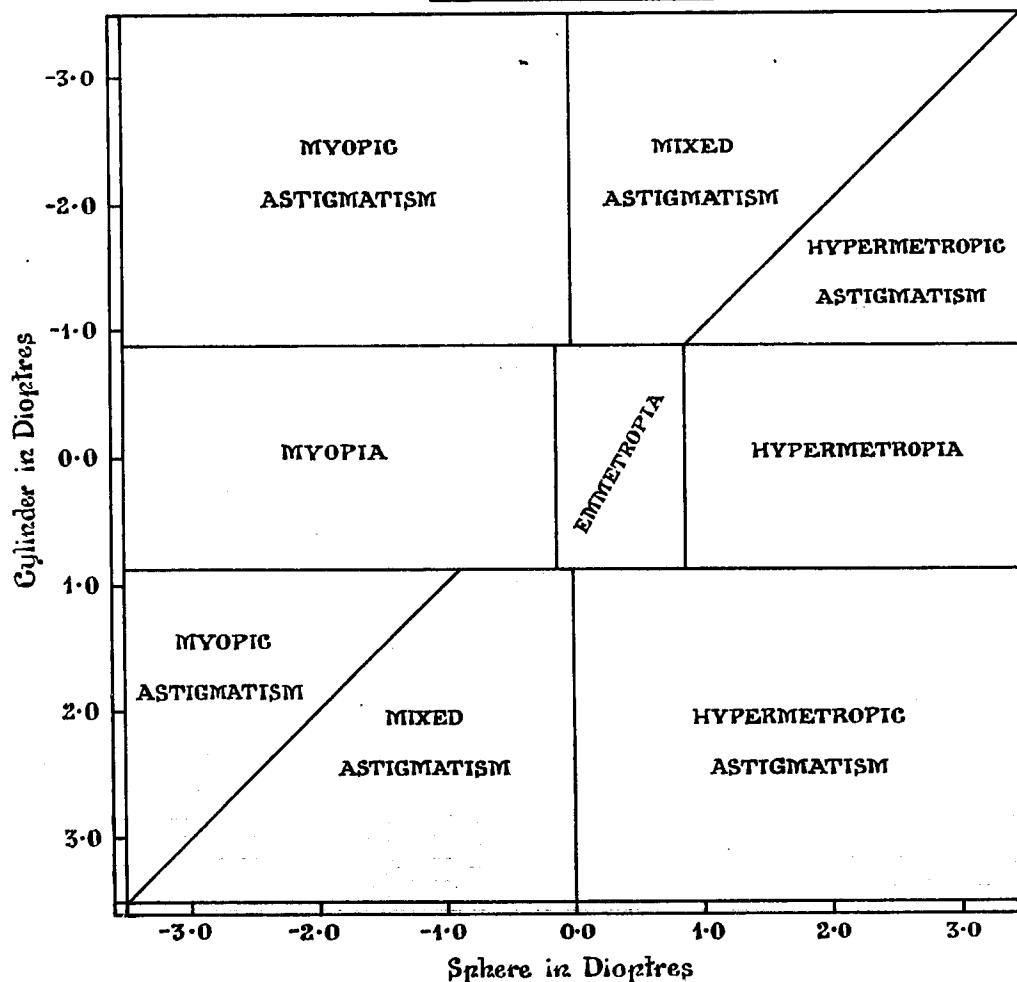


Diagram 57.

# THE PROBLEM OF ALIEN IMMIGRATION

## PART III (*cont.*).

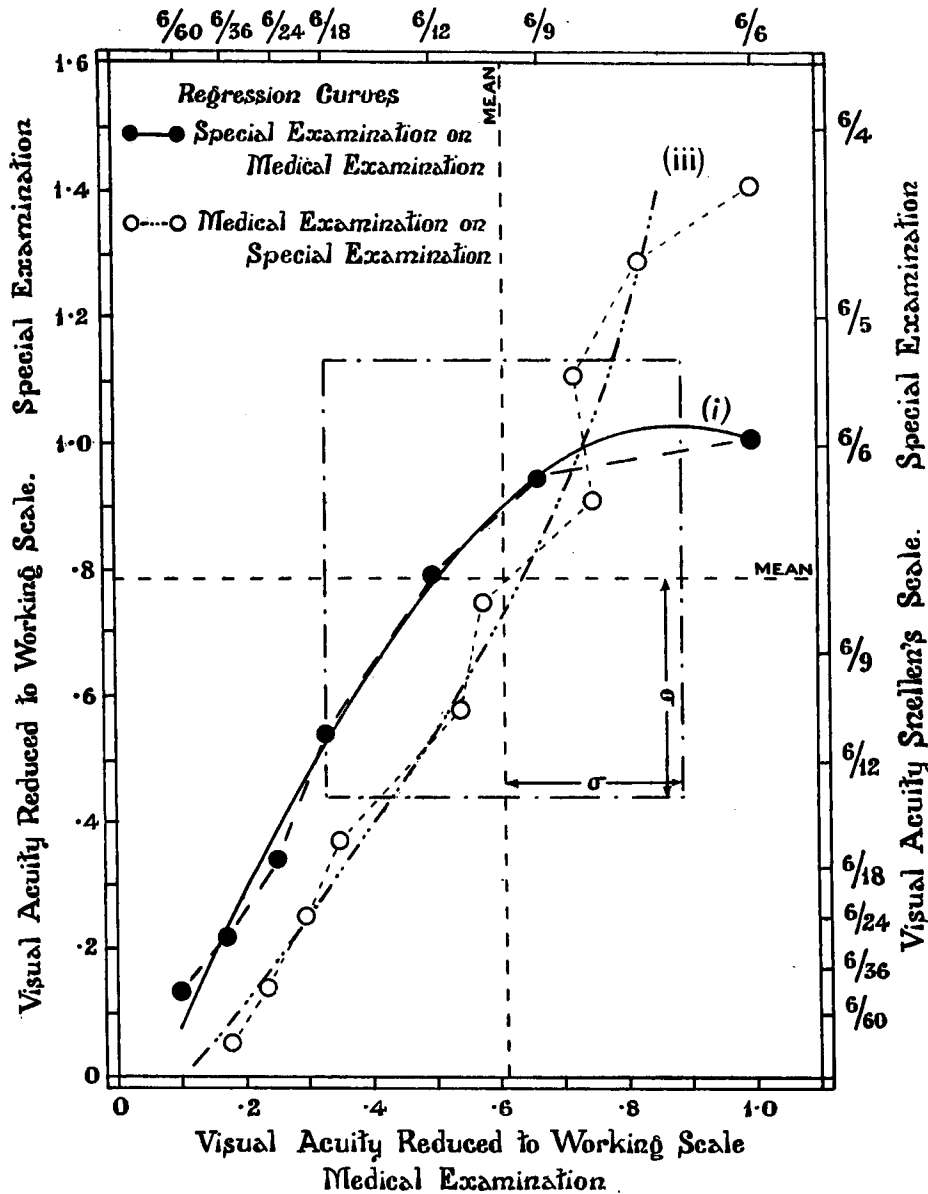
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C. *The Intercorrelation of Ocular Characters.* In this section we propose to deal with the interrelationship of the various ocular characters. So far as we are aware this topic has not hitherto been dealt with by modern statistical methods. For example, the degree of resemblance

(a) *Visual Acuity.* (i) *Different Measures of Visual Acuity.* In Section A it has been pointed

## OF VISUAL ACUITY. ALIEN JEWISH BOYS

**Visual Acuity Snellen's Scale. Medical Examination**

out that we have more than one measure of Visual Acuity. We have (a) the School Medical Examination in the usual Snellen's type categories, (b) the Medical Examination corrected, so that better central values are given to the subranges, and (c) the results of our own special ex-

amination with a system more elaborate than the usual one of type letters, and with a somewhat different method of marking. In Diagrams 58 and 59 we give the curves interrelating these various systems. Taking either (a) or (b) we are able from curves (i) and (ii) to read off for a given value

### RELATIONSHIP BETWEEN THE TWO ESTIMATES

#### OF VISUAL ACUITY. ALIEN JEWISH BOYS

##### MEDICAL EXAMINATION ON CORRECTED DATA

Visual Acuity Snellen's Scale. Medical Examination

$\frac{6}{60}$   $\frac{6}{36}$   $\frac{6}{24}$   $\frac{6}{18}$   $\frac{6}{12}$   $\frac{6}{9}$   $\frac{6}{6}$   $\frac{6}{5}$

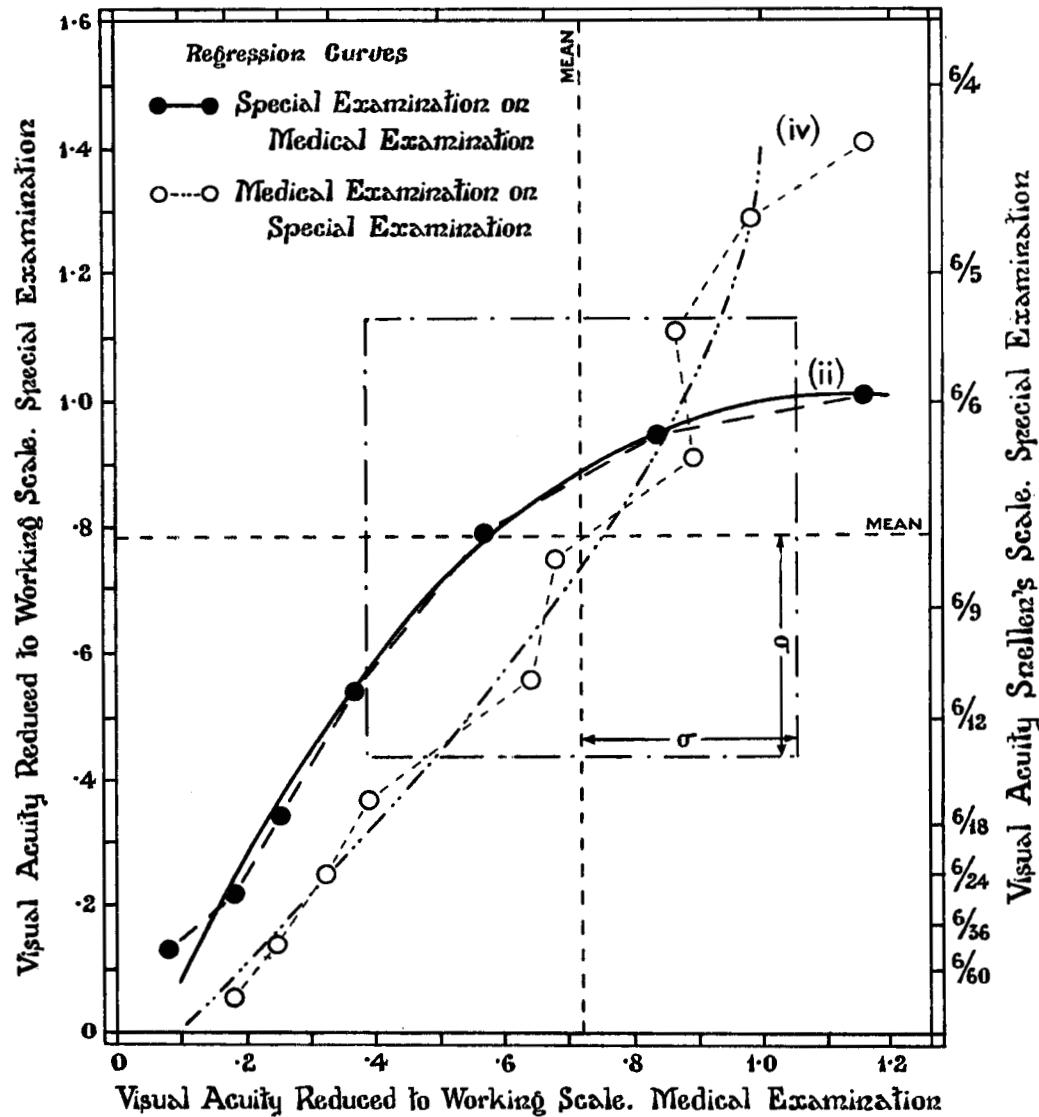


Diagram 59.

the corresponding probable value of (c), or given a value of (c) we are able by curves (iii) and (iv) to determine the corresponding probable values of (a) and (b) respectively.

We were able to do this because we had the measurements for 320 boys or 640 eyes taken in both Medical and Special Examinations. Separate tables were made out for Right and Left Eyes,

of which the former is given below\*. But it was found that there was no essential difference and the two tables could be combined as given in Table CLXVII (to right). The Special Examination scale (*c*) is given at the top of the table; the usual Medical Examination scale (*a*) in the first column on the left, and the corrected Medical Examination scale (*b*) in the mid-column to the left of the combined table.

Table CLXVII. *Acuity of Vision (Monocular) in Special and Medical Examinations (Boys).*

Special Examinations

Medical Examinations	Usual Scale	Right Eye												Totals	Corrected Scale	Right and Left Eyes												Totals
		1.50	1.40	1.29	1.11	.91	.75	.58	.37	.25	.14	.08	.04			1.50	1.40	1.29	1.11	.91	.75	.58	.37	.25	.14	.08	.04	
	1.00	1	5	13	23	32	8	3	—	—	1	—	—	86	1.16	1	10	25	41	65	13	8	1	—	1	—	—	165
	.67	—	—	9	30	28	10	8	1	1	—	—	—	87	.84	—	—	18	62	57	20	19	4	1	—	—	—	181
	.50	—	—	2	15	14	11	15	3	1	—	—	—	61	.57	—	—	3	26	29	25	23	10	3	2	—	—	121
	.33	—	—	1	1	5	7	7	13	4	—	—	1	39	.37	—	—	1	3	8	18	11	32	10	2	—	1	86
	.25	—	—	—	—	—	1	4	8	7	1	—	2	24	.25	—	—	—	1	—	6	15	10	6	1	2	—	42
	.17	—	—	—	—	—	1	1	5	2	5	3	—	17	.18	—	—	—	—	1	2	5	5	14	5	2	—	34
	.10	—	—	—	—	—	—	—	1	—	3	—	2	6	.08	—	—	—	—	—	—	1	1	5	—	4	—	11
	Totals	1	5	25	70	79	38	38	31	15	10	3	5	320	—	1	10	47	133	159	78	69	68	30	30	6	9	640

Considering first the usual medical scale (*a*) we have:

Mean, Medical Examination ... = .6133.

Standard Deviation, Medical Examination = .2762.

Mean, Special Examination ... = .7885.

Standard Deviation, Special Examination = .3459.

Product Moment Correlation Coefficient:  $r = .6722 \pm .0146$ .

$\eta'_{SE,ME} = .7443$ ,  $\eta'_{ME,SE} = .7008$ .

Repeating the work with the corrected Medical scale (*b*) we have:

Mean, Medical Examination ... = .7215.

Standard Deviation, Medical Examination = .3343.

Mean, Special Examination ... = .7885.

Standard Deviation, Special Examination = .3459.

Product Moment Correlation Coefficient:  $r = .6907 \pm .0139$ .

$\eta'_{SE,ME} = .7424$ ,  $\eta'_{ME,SE} = .7187$ .

We see from these results that the corrected Medical scale (*b*) is more highly correlated with the more elaborate Special Examination than the usual Medical scale; it is therefore to be preferred. Further, the regression curve of Special Examination on either Medical Examination (*a*) or (*b*) deviates considerably from linearity, while the regression curves of Medical Examination on Special Examination (whether we use (*a*) or (*b*)) depart less from linearity and may, for practical purposes, be represented by straight lines. The regression curve of Special on Medical Examination has been graduated (see Diagram 58) by aid of the cubic:

$$\tilde{y}_{SE} = - .15196 + 2.37657x_{ME} - .82259x_{ME}^2 - .38888x_{ME}^3,$$

where  $\tilde{y}_{SE}$  is the visual acuity which would most probably result from the Special Examination if the visual acuity  $x_{ME}$  was noted in the Medical Examination (*a*)†. This enables us to find the best available Special Examination equivalent from the rougher Medical Examination.

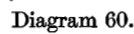
\* Any reader can reproduce the table for the Left Eye, by subtracting the contents of each cell in the table on the left from those of the corresponding cells of the table on the right, if he so desires.

† Conversely  $\tilde{x}_{ME} = .10091 + .85616y_{SE} - .20366y_{SE}^2 - .01550y_{SE}^3$  is the cubic giving the probable Medical Examination result for a given Special Examination value.

Turning to the Corrected Medical Examination data we find for our cubic:

This is shown in Diagram 59 (p. 143). As in the case of the Uncorrected Medical Examination

SPECIAL EYE EXAMINATION. ALIEN JEWISH BOYS



It must, we think, be admitted that with a correlation coefficient of nearly .70 there is some ground for satisfaction that the more hasty School Medical Examination should agree so well

EUGENICS<sup>II</sup>, I & II

with a more elaborate examination made by different investigators with a different scale and, in a large number of cases, after a considerable time had elapsed.

(ii) *Binocular Vision and Vision of Better Eye.* It seems of some interest to know how far Binocular Vision is better than the Vision of the Better Eye, and also how we can obtain when the vision has only been measured for the right and left eyes separately the most probable value of the Binocular Vision. The converse problem, to find from binocular vision the probable vision of the Better Eye, may also be of some interest.

Table CLXVIII. *Binocular Vision and Vision of Better Eye (Boys).*  
Vision of Better Eye

	1.50	1.40	1.29	1.11	.91	.75	.58	.37	.25	.14	.08	.04	Totals
1.50	2	1	—	—	—	—	—	—	—	—	—	—	3
1.40	—	4	—	1	1	—	—	—	—	—	—	—	6
1.29	—	—	48	14	1	1	—	—	—	—	—	—	64
1.11	—	1	3	92	30	1	—	—	—	—	—	—	127
.91	—	1	—	2	71	22	9	—	—	—	—	—	105
.75	—	—	—	—	4	29	6	1	—	—	—	—	40
.58	—	—	—	—	—	2	29	7	—	—	—	—	38
.37	—	—	—	—	—	—	1	23	2	1	—	—	27
.25	—	—	—	—	—	—	—	—	10	—	—	—	10
.14	—	—	—	—	—	—	—	—	—	8	1	—	9
.08	—	—	—	—	—	—	—	—	—	—	6	—	6
.04	—	—	—	—	—	—	—	—	—	1	—	3	4
Totals	2	7	51	109	107	55	45	31	12	10	7	3	439

The accompanying table shows by its mere appearance how strong is the correlation. For its constants we have:

Mean, Binocular Vision ... .9075.  
Mean, Better Eye Vision ... .8675.  
Standard Deviation, Binocular Vision ... .3261.  
Standard Deviation, Better Eye Vision ... .3269.  
Product Moment Correlation Coefficient:  $r = .9485 \pm .0032$ .

We see accordingly that the Binocular Vision is 4.65% higher than the Better Eye Vision, but that the variabilities are about equal. We cannot disregard the fact, however, that the array-means taken for Binocular Vision on Better Eye Vision do not give a linear regression. On the other hand, the Vision of Better Eye on Binocular Vision is given almost exactly by a straight line.

Visual Acuity of Better Eye	Binocular Visual Acuity	Visual Acuity of Better Eye	Binocular Visual Acuity
1.42	1.3467	1.42	1.3356
1.29	1.2794	1.29	1.2363
1.11	1.1321	1.11	1.0665
.91	.9682	.91	.8565
.75	.8242	.75	.7310
.58	.6640	.58	.5503
.37	.4297	.37	.3604
.25	.2700	.25	.2500
.14	.1530	.14	.1333
.07	.0740	.06	.0740
General Population:	.9075	General Population:	.8675

We see that the probable Binocular Visual Acuity is less than that of the Better Eye for the supernormal sights, but becomes better than that of the Better Eye, when we have Better Eye Vision less than 6/6. On the other hand, the probable value of Better Eye Vision is practically throughout less than Binocular Vision. Diagram 60 (p. 145) will enable the reader to determine easily Binocular from Better Eye Vision or the converse. For those who wish to obtain more accurate values we give the cubic equation and the straight line equation for the two regression curves.

$$\hat{y}_{Bi.V} = -.0117 + 1.2839x_{Be.V} - .2473x_{Be.V}^2 + .0198x_{Be.V}^3,$$

$$\hat{x}_{Be.V} = x_{Be.V} = -.0047 + .9508y_{Bi.V}.$$

It might be reasonable to assume that the constant term  $-.0047$  should be neglected or the Better Eye Vision taken to be zero when Binocular Vision ceases.

(iii) *Visual Acuity of Right and Left Eyes.* This association may be worked out either on the data of the Special Examination ( $\alpha$ ), or on the Boys ( $\beta$ ), or Girls ( $\gamma$ ) of the School Examination. The constants of the three distributions are as follows:

(a) *Special Examination (Boys):*

Mean ... ... Right Eye: .8009. Left Eye: .7635.

Standard Deviation ... Right Eye: .3525. Left Eye: .3616.

Product Moment Correlation Coefficient:  $r = .7763 \pm .0119$ .

### VISUAL ACUITY OF LEFT EYE & VISUAL ACUITY OF RIGHT EYE

#### ALIEN JEWISH BOYS. SPECIAL EYE EXAMINATION

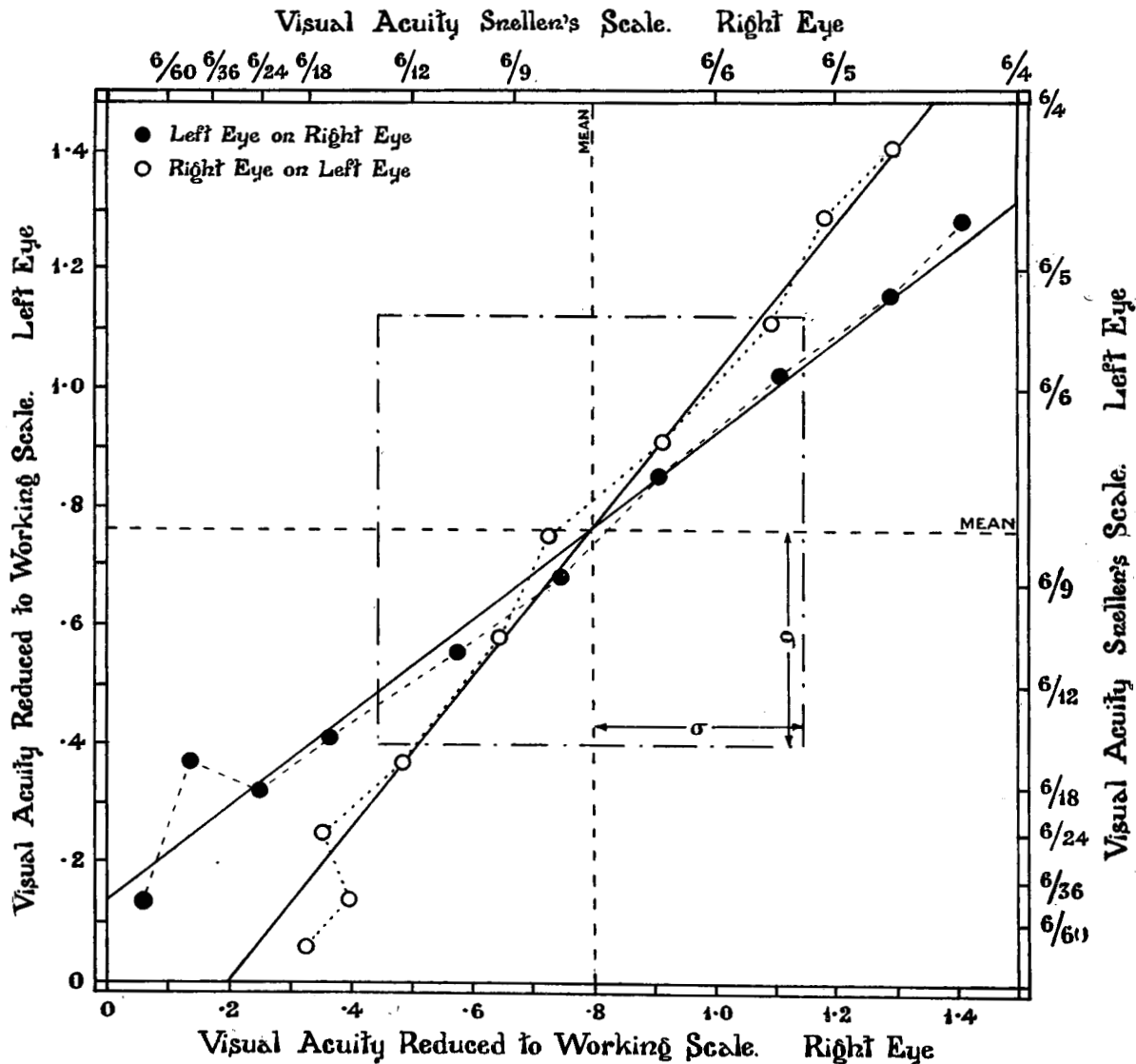


Diagram 61.

Diagram 61 shows how nearly linear is the regression.

No doubt a cubic would give a slightly closer result, but it has not been thought needful to

Table CLXIX. *Visual Acuity of Right Eye and Left Eye. Special Examination. Boys.*

		Right Eye												
Left Eye		1.50	1.40	1.29	1.11	.91	.75	.58	.37	.25	.14	.08	.04	Totals
	1.50	1	—	—	—	—	—	—	—	—	—	—	—	1
	1.40	1	5	—	1	—	—	1	—	—	—	—	—	8
	1.29	—	—	32	5	—	—	—	—	1	2	—	—	40
	1.11	—	3	5	73	9	1	—	1	—	—	—	—	92
	.91	—	1	3	15	91	10	2	—	1	1	1	—	124
	.75	—	—	1	2	9	31	10	4	—	1	—	—	58
	.58	—	—	2	3	2	8	33	3	1	1	—	—	53
	.37	—	—	1	4	3	4	7	28	8	3	—	—	58
	.25	—	—	—	—	2	—	4	4	11	3	1	—	25
	.14	—	—	1	—	2	4	3	3	3	10	—	2	28
	.08	—	—	—	1	2	—	—	—	—	1	5	1	10
	.04	—	—	—	1	—	1	1	—	1	—	1	5	10
Totals	2	9	45	105	120	59	61	43	25	22	8	8	507	

determine it. The following are the equations for finding the probable value of the vision of one eye from that of the other:

$$\tilde{V}_{LE} = .1257 + .7963V_{RE},$$

$$\tilde{V}_{RE} = .1929 + .7568V_{LE}.$$

It will be observed that when the vision of one eye is nil, the vision of the other does not vanish; it is small but finite.

The probable vision of the left eye is less than the observed vision of the right, until it falls to 6/18 vision, when the probable vision of the left

is greater than that observed for the right. The probable vision of the right eye is less than the observed vision of the left, until it falls below 6/9 vision, when the probable vision of the right is greater than that observed for the left. These results are taken from the observed array-means and not from the regression straight lines. See Diagram 61 (p. 147).

We now turn to the Medical Examination. Here we can deal either with the corrected or

Table CLXX. *Visual Acuity of Right Eye and Left Eye. Medical Examination.*

		Boys								Vision of Right Eye								Girls							
Vision of Left Eye	Corrected Scale	Corrected Scale								Totals	Corrected Scale								Totals						
		1.16	.84	.57	.37	.25	.18	.10	.06		1.16	.84	.57	.37	.25	.18	.10	.06							
	Usual Scale	1.00	.67	.50	.33	.25	.17	.10	<.10	1.00	.67	.50	.33	.25	.17	.10	<.10								
	1.16	1.00	201	17	4	4	—	1	—	227	152	16	2	1	—	—	1	—	172						
	.84	.67	19	177	24	10	4	—	3	—	237	18	187	28	10	6	6	1	2	258					
	.57	.50	6	29	101	16	7	5	4	—	168	1	33	178	15	2	2	1	—	232					
	.37	.33	2	3	22	52	8	6	3	—	96	3	7	27	74	16	5	4	—	136					
	.25	.25	3	3	3	10	27	6	—	—	52	1	3	2	16	37	3	2	—	64					
	.18	.17	4	3	5	3	2	22	4	—	43	—	2	2	5	1	28	2	—	40					
	.10	.10	2	—	—	2	—	4	12	—	20	—	2	—	1	2	5	6	—	16					
.06	<.10	—	1	—	—	—	—	—	—	1	1	—	—	1	—	—	—	3	5						
	Totals	237	233	159	97	48	44	26	—	844	176	250	239	123	64	49	17	5	923						

uncorrected scale. The data for Boys and Girls are given in Table CLXX above, and the means of the array of Left Eye Vision for given Right Eye Vision, both for the corrected and uncorrected scales, in Diagrams 62 and 63. The regression curves in all cases are very nearly linear. The curious turn upward with low visual acuity of the Right Eye is, we believe, only due to the small numbers dealt with in the two lowest groups (*Boys*: 26; *Girls*: 17 and 5), but the fact that it occurs in both Boys and Girls must be borne in mind\*. The constants of these tables are given below. We have contented ourselves with giving the regression for Left Eye on Right Eye in our Diagrams 62 and 63 (pp. 149, 150).

\* A similar "turn up" is not traceable in our Special Examination regression curves (Diagram 61), but it would not be unreasonable if some of the low visual acuities of the right eye were due to anomalies or accidents not affecting the left.

The constants of the tables are as follows:

	<i>Boys</i>		<i>Girls</i>	
	Uncorrected	Corrected	Uncorrected	Corrected
Mean Visual Acuity R. Eye ... ..	·6241	·7342	·5740	·6747
Mean Visual Acuity L. Eye ... ..	·6206	·7304	·5745	·6759
Standard Deviation R. Eye ... ..	·2833	·3407	·2610	·3180
Standard Deviation L. Eye ... ..	·2785	·3354	·2577	·3144
Correlation of Visual Acuity in R. and L. Eyes	{ ±·8017	{ ±·7997	{ ±·8333	{ ±·8256
	{ ±·0083	{ ±·0083	{ ±·0068	{ ±·0068

### VISUAL ACUITY OF RIGHT & LEFT EYE

#### ALIEN JEWISH BOYS. MEDICAL EXAMINATION

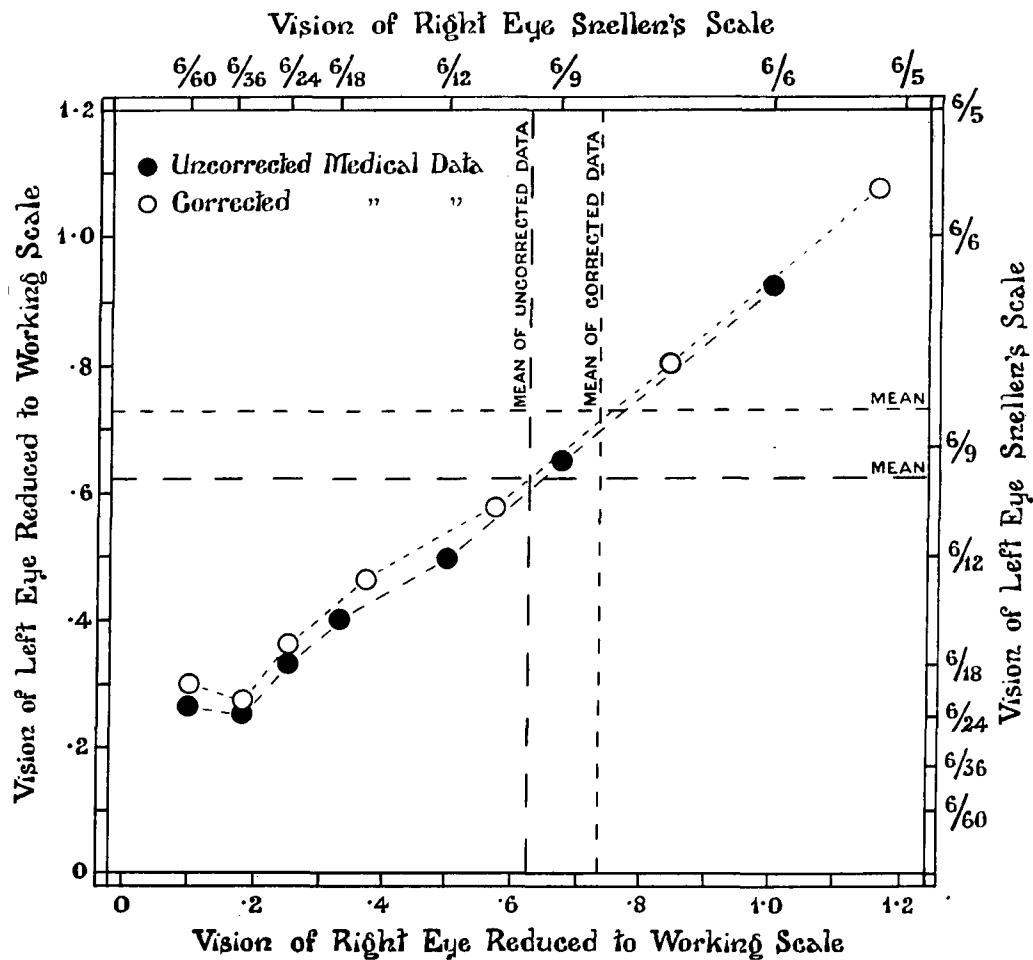


Diagram 62.

The Medical Examination indicates how poor is the visual acuity of the Girls compared with that of the Boys, but it does not show like our Special Examination a definite superiority of acuity in the Right Eye. The Girls are in all cases less variable absolutely than the Boys, but if we take their relative variability as measured by the Coefficient of Variation we find:

Coefficient of Variation	<i>Boys</i>		<i>Girls</i>	
	Uncorrected	Corrected	Uncorrected	Corrected
Right Eye	45·4	46·4	45·5	47·1
Left Eye	44·9	45·9	44·9	46·5

and the Girls, if anything, are slightly more variable than the Boys.

From the above tables 43 Boys (4.85 %) and 78 Girls (7.79 %) recorded "with glasses" were excluded. Their sight without glasses might have modified the lower end of the curves to some extent. The reader will note that the percentage of Girls using glasses is considerably greater than the percentage of Boys. Of course this may be due to the fact that schoolboys dislike more than schoolgirls wearing glasses, but the lesser visual acuity of the Girls is reason enough for the difference.

### VISUAL ACUITY OF RIGHT & LEFT EYE

#### ALIEN JEWISH GIRLS. MEDICAL EXAMINATION

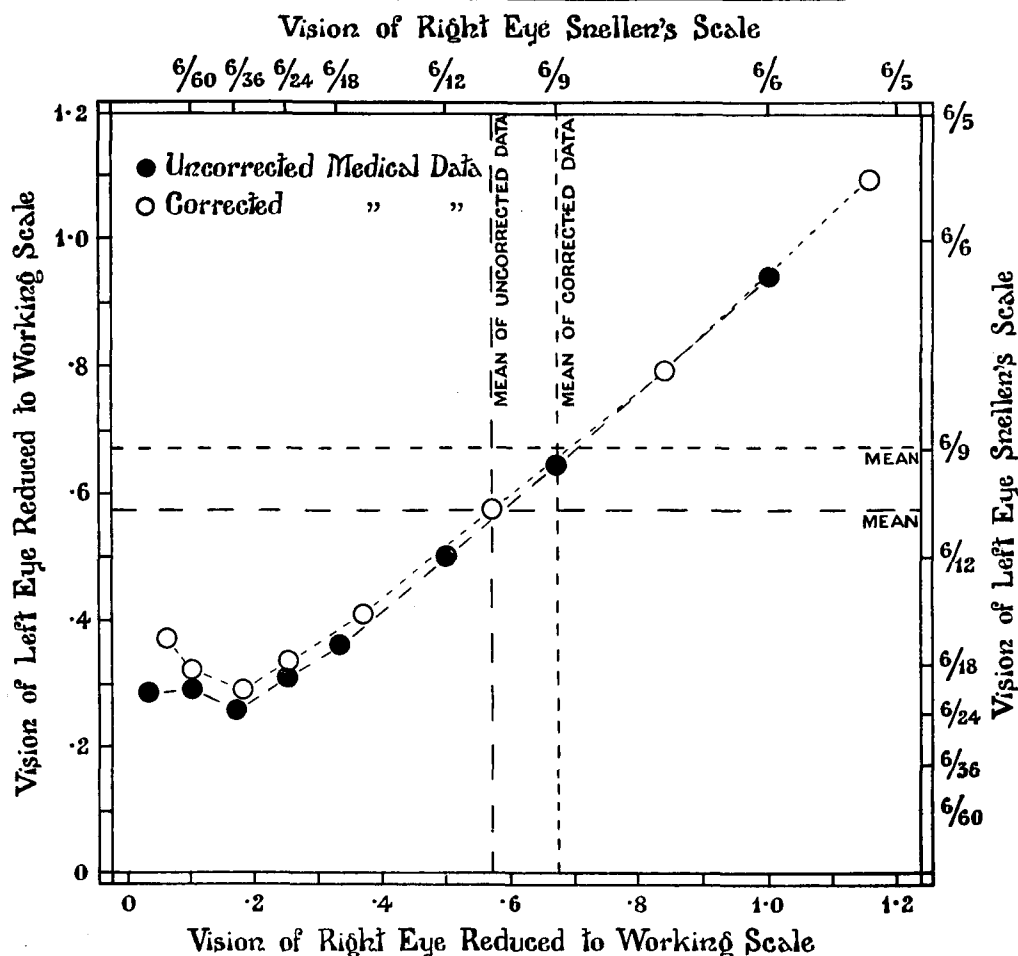


Diagram 63.

The following are the prediction formulae for finding the probable visual acuity of one eye from the other.

*Boys:* Uncorrected (Usual Medical) Scale.

$$\tilde{V}_{LE} = .1287 + .7881 V_{RE},$$

$$\tilde{V}_{RE} = .1180 + .8155 V_{LE},$$

*Girls:* Uncorrected (Usual Medical) Scale.

$$\tilde{V}_{LE} = .1022 + .8228 V_{RE},$$

$$\tilde{V}_{RE} = .0891 + .8440 V_{LE},$$

Corrected Scale.

$$\tilde{V}_{LE} = .1524 + .7873 V_{RE},$$

$$\tilde{V}_{RE} = .1409 + .8123 V_{LE}.$$

Corrected Scale.

$$\tilde{V}_{LE} = .1251 + .8163 V_{RE},$$

$$\tilde{V}_{RE} = .1103 + .8351 V_{LE}.$$

Here  $V$  signifies visual acuity of observed eye and  $\tilde{V}$  the probable value of the visual acuity of the other eye.

(iv) *Visual Acuity and Refraction Class.* This is a point which is of considerable interest. Is it to be anticipated and to what extent that the astigmatic will have worse vision than the simply hypermetropic or simply myopic?

The table giving our data will be found below. The constants of this table are as follows:

Mean Vision: .8179. Standard Deviation: .3400.

Correlation ratio of Vision on Refraction Class:

$$\eta'^2_{V,RC} = .467,747,$$

$$\bar{\eta}^2_{V,RC} = .005,624.$$

Table CLXXI. *Visual Acuity and Refraction Class.*

Refraction Class	Visual Acuity												
	1.50	1.40	1.29	1.11	.91	.75	.58	.37	.25	.14	.08	.04	Totals
Emmetropia ... ..	2	13	68	169	184	76	41	6	2	1	—	—	562
Hypermetropia ... ..	1	5	5	6	29	9	8	4	—	1	1	—	69
Hypermetropic Astigmatism ... ..	—	—	1	1	4	4	19	22	10	5	—	—	66
Mixed Astigmatism ... ..	—	—	—	—	1	—	1	7	1	3	—	—	13
Myopic Astigmatism ... ..	—	—	—	—	2	4	8	3	6	7	3	6	39
Myopia ... ..	—	—	—	9	20	15	18	30	16	22	7	3	140
Totals ... ..	3	18	74	185	240	108	95	72	35	39	11	9	889

$\eta'^2_{V,RC}$  is therefore very definitely significant and  $\eta'_{V,RC} = .6839$ .

### VISUAL ACUITY & REFRACTION CLASS

#### ALIEN JEWISH BOYS

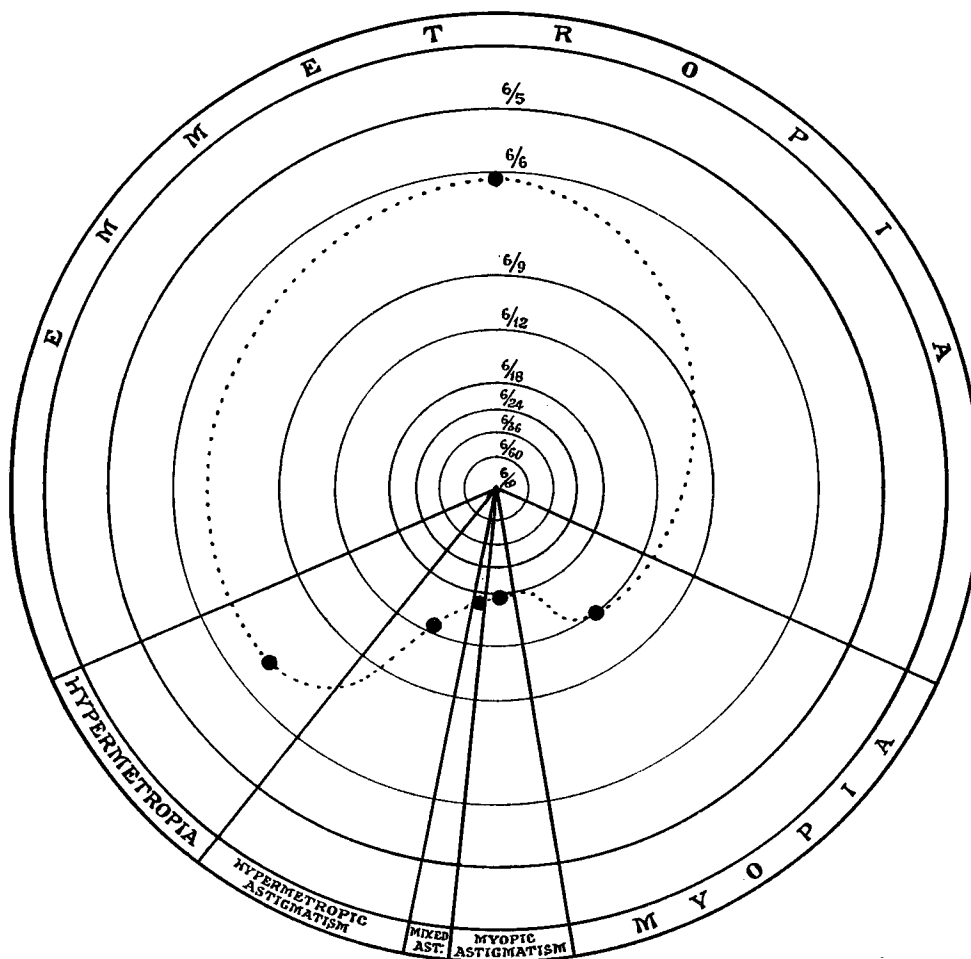
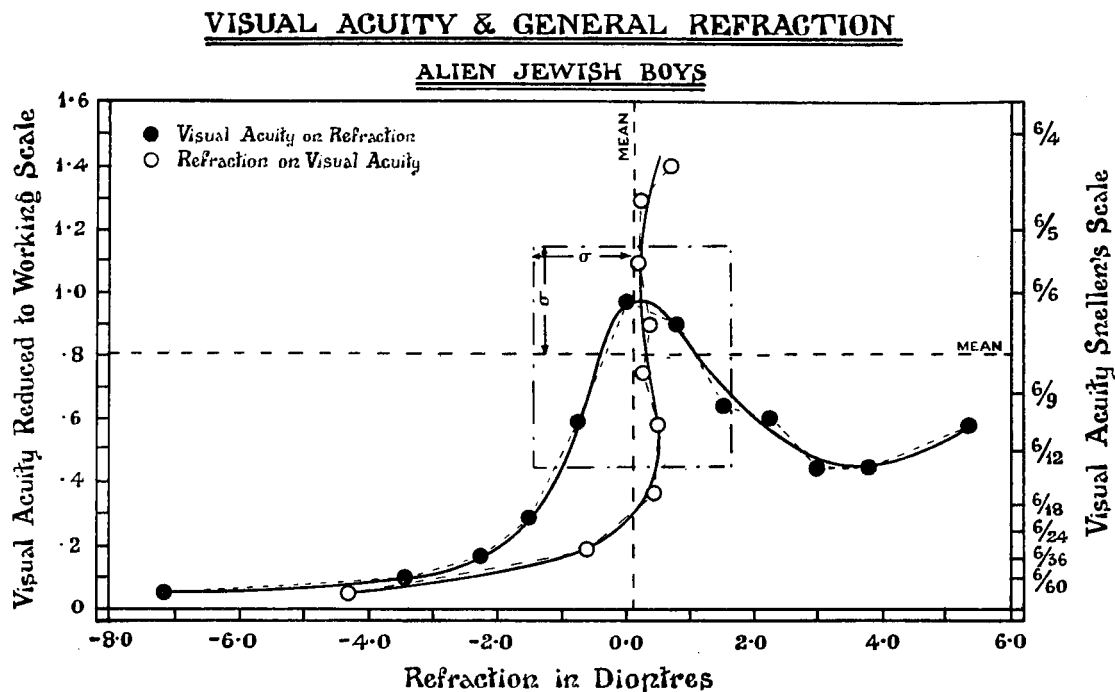


Diagram 64.



(v) *Visual Acuity and General Refraction.* The relation between these two ocular characters is one of great interest to the really enthusiastic statistician, who sees further than the coefficient of correlation. It is also not without interest to the student of evolution. Acuity of vision was essential to primitive man and is highly important even to-day. Not looking at Refraction from the highly artificial scale of modern ophthalmology, but as the power of the compound system of lenses on which sight depends, we recognise how rapidly visual acuity runs down with even



slight deviations from a modal value (see Diagram 66). But the shape of the eye and its parts depends upon the sizes and correlations of a great range of physical characters which probably extend as far as the bony structure of the skull (see our Section G). It is impossible to deny that primitive man with poor visual acuity would stand a slight chance of survival, and we see the selection by visual

Table CLXXIII. *Visual Acuity and General Refraction. Boys.*

General Refraction in Dioptres (Central Values)

Visual Acuity	Central Values	+6.75	+6.00	+5.25	+4.50	+3.75	+3.00	+2.25	+1.50	+0.75	0.00	-0.75	-1.50	-2.25	-3.00	-3.75	-4.50	-5.25	-6.00	-6.75	:	-12.75	:	-15.75	Totals
	1.50	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
1.40	—	—	—	—	—	—	—	—	2	11	4	—	—	—	—	—	—	—	—	—	—	—	—	17	
1.29	—	—	—	—	—	—	—	1	—	17	56	—	—	—	—	—	—	—	—	—	—	—	—	74	
1.11	—	—	—	1	—	—	—	2	1	33	140	4	—	—	—	—	—	—	—	—	—	—	—	181	
.91	—	—	—	1	—	—	—	6	7	66	135.5	11.5	—	—	—	—	—	—	—	—	—	—	—	231	
.75	—	—	—	—	1	—	—	4	2	20	61	16.5	0.5	—	—	—	—	—	—	—	—	—	—	107	
.58	1	1	1	—	—	2	2.5	7.5	2	20	32	20	4	—	—	—	—	—	—	—	—	—	—	94	
.37	—	1	1	1.5	3	2	8.5	5	5	10.5	5	22	10.5	2	—	—	—	—	—	—	—	—	—	72	
.25	—	—	—	—	4.5	2	1	2	2	2.5	1	4	10	6	—	2	—	—	—	—	—	—	—	35	
.14	—	—	—	0.5	2	1	2.5	3	3	—	1	4	7.5	12.5	1	2	—	1	—	—	—	—	—	38	
.08	—	—	—	—	—	1	—	—	—	—	—	—	1	2	3	3	—	3	1	1	—	—	—	15	
.04	—	—	—	—	—	—	—	—	—	—	—	—	2	2	2	1	—	2	—	2	—	1	1	13	
Totals	1	2	4	3	16.5	10.5	32.5	24	183	435.5	82	35.5	24.5	6	8	—	6	1	3	..	1	..	1	880	

acuity modelling all the structures related to the eye as an optical instrument. All this may seem quite familiar and not worth repeating, but still an inspection of Diagram 66 does reinforce the old tale—the tale of natural selection—which now-a-days fails to obtain hearers.

Our data are provided in Table CLXXIII.

The constants of this table are:

Mean, Vision: .8092; Standard Deviation, Vision: .3471.  
Mean, Refraction: .1010 D.; Standard Deviation, Refraction: 1.5268 D.

Product Moment Coefficient of Correlation  $r = .2706 \pm .0211$ .

Correlation Ratio, Visual Acuity on General Refraction:

$\eta'^2_{VA,GR} = .533,037$ ,  $\bar{\eta}^2_{VA,GR} = .012,500 \pm .003,569$ ,  $\eta'_{VA,GR} = .7301$ .

Correlation Ratio, General Refraction on Visual Acuity:

$\eta'^2_{GR,VA} = .329,425$ ,  $\bar{\eta}^2_{GR,VA} = .011,364 \pm .002,408$ ,  $\eta'_{GR,VA} = .5740$ .

and is decidedly significant, if not so large as  $\eta'_{VA,GR}$ .

The pyramidal form of the table is noteworthy, the scatter increasing markedly as we pass from high to low acuity. The following are the values of the array-means:

General Refraction	Mean Visual Acuity	Visual Acuity	Mean General Refraction
5.325 D.	.5875	1.415	.6750 D.
3.75 D.	.4591	1.29	.2027 D.
3.00 D.	.4505	1.11	.1823 D.
2.25 D.	.6174	.91	.3620 D.
1.50 D.	.6546	.75	.2418 D.
.75 D.	.9296	.58	.5386 D.
.00 D.	.9715	.37	.4583 D.
-.75 D.	.5924	.193	-.6062 D.
-1.50 D.	.2899	.061	-4.3393 D.
-2.25 D.	.1727	—	—
-3.43 D.	.1086	—	—
-7.19 D.	.0650	—	—
General Population:	.8092	General Population:	.1010 D.

The two regression curves shown in Diagram 66 are very characteristic. We see that starting with Visual Acuity .06 General Refraction is negative and rises quickly till Visual Acuity is .37, after which it scarcely differs from zero by more than  $\frac{1}{4}$  D. to  $\frac{1}{2}$  D. up to the most acute visions.

For the regression curve of Visual Acuity on Refraction we tried weighted and unweighted cubics unsuccessfully; an unweighted quartic gave a better but not a really satisfactory graduation. A reasonable graduation was obtained by a weighted cubic for the regression of Refraction on Acuity. On the whole it seemed probable that higher order parabolae or even other types of curves would be desirable but in view of the great labour involved already, and what would be needful if other curves were tried, we finally contented ourselves by splining both systems of array-means.

(vi) *Visual Acuity and Corneal Refraction.* Table CLXXIV below provides the data of the Special Examination for recorders *A*, *B* and *C*. Later we shall consider results for *A* and *B* only.

Diagram 67 shows that for low or high Corneal Refraction the Visual Acuity decreases fairly rapidly. The reader will find it of interest to compare this result with that for General Refraction; he will see at once that the curve is nothing like as steep; nor should it be, for Corneal Refraction is only part of the General Refraction upon which fitness of vision depends. The constants of the table are:

Corneal Refraction, Mean: 43.475 D.; Standard Deviation: 1.5523 D.  
Acuity of Vision, Mean: .7911; Standard Deviation: .3555.

Product Moment Coefficient of Correlation:  $r = - .0275$ .

Table CLXXIV. *Visual Acuity and Corneal Refraction.*

Corneal Refraction in Dioptres (Central Values)

Visual Acuity	Central Values	38.125	38.625	39.125	39.625	40.125	40.625	41.125	41.625	42.125	42.625	43.125	43.625	44.125	44.625	45.125	45.625	46.125	46.625	47.125	47.625	Totals
1.50	—	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	—	—	—	—	3
1.40	—	—	—	—	—	—	—	—	—	1	12	18	5	10	3	5	—	0.5	0.5	—	—	17
1.29	—	—	—	—	—	1	2	6	8	13	12	18	5	10	5	2	—	2	—	—	—	86
1.11	—	—	—	—	2	3	8	10	11	19	29	25	27	21	17	9	11	3	—	—	—	195
.91	—	—	—	—	1	1	3	8	8	13	18	31	40	41	24	14	16	17	3	1	2	240
.75	—	—	—	—	1	3	1	5	5	8	5	13	18	16	14	11	9	5	2	—	—	116
.58	—	—	—	—	3	3	11	3	12	12	15	15	13	7	5	5	3	1	1	—	—	109
.37	—	—	4	3	4	—	1	4	6	8	5	9	14	12	9	1	8	5	—	—	—	93
.25	1	—	—	1	2	2	—	3	9	3	4	8	3	7	2	2	2	—	—	—	—	49
.14	—	—	—	—	3	1	4	4	4	1	4	3	6	3	4.5	3	3.5	—	—	1	—	45
.08	—	—	—	—	—	—	1	—	3	1	2	—	3	1	2	2	2	—	—	—	—	17
.04	—	—	—	—	—	1	1	1	1	—	3	1	3	1	2	—	1	—	—	—	—	15
Totals		1	—	4	7	20	21	47	47	89	92	120	129	134	94	65.5	49	47	13.5	3	2	985

VISUAL ACUITY & CORNEAL REFRACTION (OBSERVERS A,B & C)

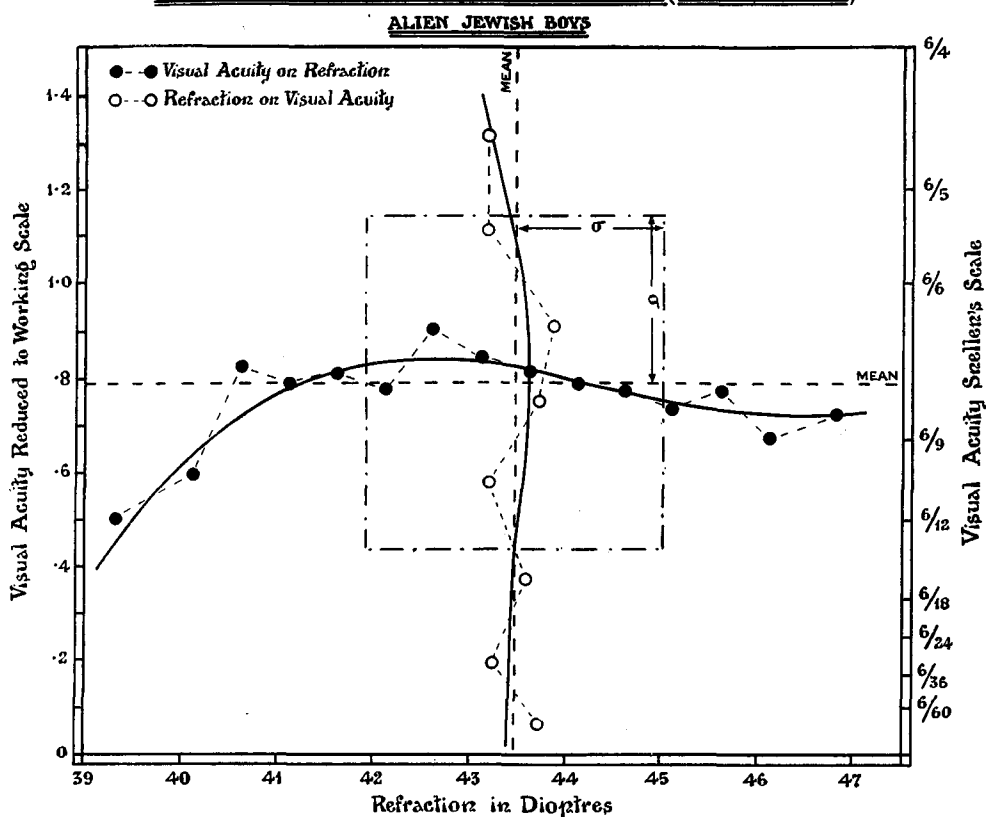


Diagram 67.

This is what we might anticipate for the curve of regression is very nearly parabolic and  $r$  is of no physical value. Turning to the correlation ratio of Visual Acuity on Corneal Refraction we have:

$$\eta'^2_{VA.CR} = .034,974, \quad \bar{\eta}^2_{VA.CR} = .014,213 \pm .003,597,$$

$\eta'^2_{VA.CR}$  is accordingly definitely significant, and we have  $\eta'_{VA.CR} = .1870$ , a value much smaller than that found for Visual Acuity on General Refraction, as might be expected.

We have worked the corresponding values of the distribution constants for *A* and *B*'s records only. Much the same parabolic form is taken by the regression line and we have:

Corneal Refraction, Mean: 43.891 D.; Standard Deviation: 1.5157 D.  
 Acuity of Vision, Mean: .6997; Standard Deviation: .3413.

Product Moment Coefficient of Correlation:  $r = .0491 \pm .0344$ , which is not significant.

Correlation Ratio of Visual Acuity on Corneal Refraction:  $\eta'_{VA,CR} = .2199$ ,

a somewhat higher value than that found for *A*, *B* and *C*'s combined observations and to that extent justifying exclusion of *C*'s material in certain cases of Corneal Variates.

Diagram 67 (p. 155) shows the regression of Visual Acuity on Corneal Refraction for *A*, *B* and *C*'s records, graduated with the cubic

$$\tilde{y}_{VA} = .83485 - .02182 (x_{CR} - 43.125) - .01700 (x_{CR} - 43.125)^2 + .00400 (x_{CR} - 43.125)^3,$$

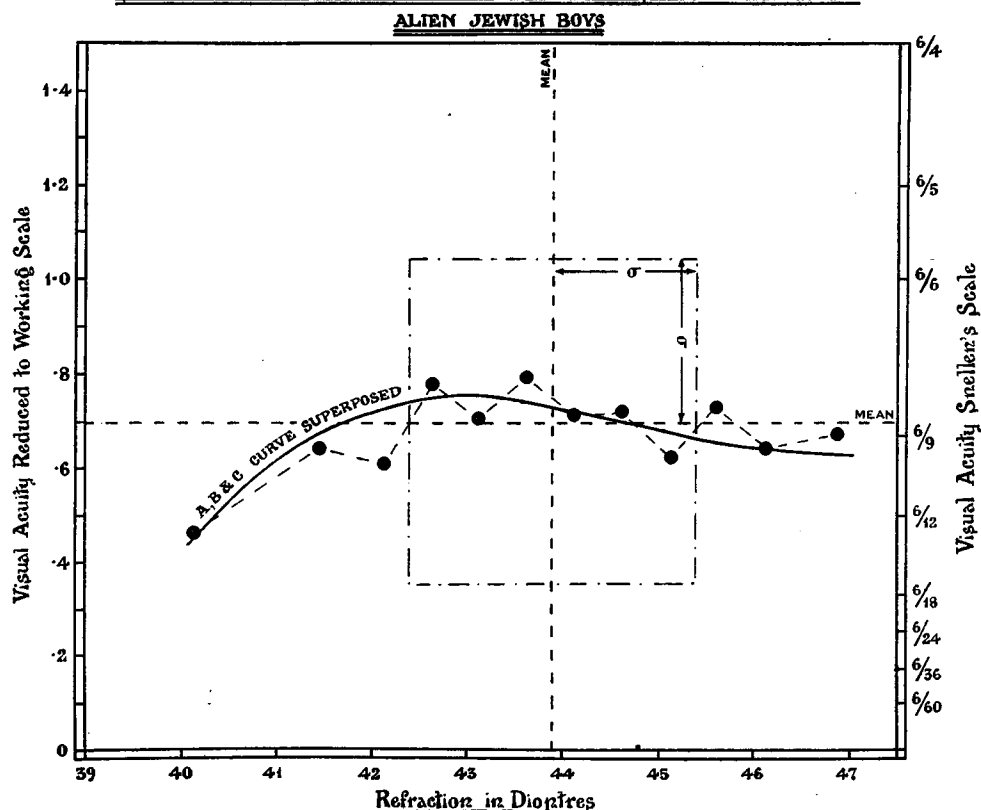
where  $x_{CR}$  is the Corneal Refraction in Dioptres.

If we examine the differences between the observations of *A* and *B* and those of *C*, we find that the chief divergence lies in the mean values. We have

	Mean Visual Acuity	Mean Corneal Refraction
<i>A</i> and <i>B</i>	.6997	43.891 D.
<i>C</i>	.8490	43.211 D.

We have already indicated that we can only account for these divergences (i) on the assumption that *C* had a large personal equation relative to *A* and *B*, or (ii) on the hypothesis that when the ophthalmic work was started the teachers sent boys whose sight was under suspicion to *A* and *B*, but that when *C* started to measure this was impossible, because he had to record the ocular characters of those boys whose school schedules had been already filled in and were waiting only for the optical data. It seemed to us worth while taking the cubic just given and shifting it

VISUAL ACUITY & CORNEAL REFRACTION (OBSERVERS A & B ONLY)



horizontally to the right through  $\cdot416$  D., the difference between the mean Corneal Refraction of  $A + B + C$ 's and  $A + B$ 's records and depressing it through  $\cdot0914$  vertically, the difference between the corresponding visual acuities. This shifted cubic is shown in Diagram 68 and the result is a quite good fit to the observations of  $A$  and  $B$  only. It would thus seem that the divergence is principally a constant difference in the values of Corneal Refraction and Visual Acuity.

Diagram 67 (p. 155) also shows the mean Corneal Refraction for each grade of Visual Acuity. The array-means for Corneal Refraction are:

Visual Acuity Grade	Mean Corneal Refraction	Visual Acuity Grade	Mean Corneal Refraction
1.41	44.163 D.	.58	43.208 D.
1.29	42.962 D.	.37	43.577 D.
1.11	43.169 D.	.197	43.242 D.
.91	43.871 D.	.061	43.719 D.
.75	43.724 D.		
		General Population*:	43.475 D.

The irregularity of these values, as well as a glance at the diagram, suffice to indicate that little can be learnt of Corneal Refraction from a knowledge of Visual Acuity. Notwithstanding, the correlation ratio, if not large, is still significant, for

$$\eta'^2_{CR.VA} = \cdot048,624, \quad \bar{\eta}^2_{CR.VA} = \cdot010,152 \pm \cdot002,152,$$

and the difference is many times the probable error. We have  $\eta'_{CR.VA} = \cdot2205$ , almost the same as  $\eta'_{VA.CR}$ , but in the former case the array-means do not form an orderly sequence as in the latter case. We have graduated them also by aid of the quartic:

$$\tilde{x}_{CR} = 43.40651 - \cdot11100y_{VA} + \cdot95686y_{VA}^2 - \cdot47960y_{VA}^3 - \cdot22670y_{VA}^4.$$

(vii) *Visual Acuity and General Astigmatism.* The regression curve of Visual Acuity on General Astigmatism forms another of the remarkable curves with which we are growing familiar and which are so suggestive from the evolutionary standpoint. If acuity of vision was—as it must have been—essential to primitive man, then there was a strong force tending to keep the surfaces of his lenses spherical. Our data will be found in Table CLXXV.

Table CLXXV. *Visual Acuity and General Astigmatism.*

General Astigmatism in Dioptries (Central Values)

Visual Acuity	Central Values	+2.25	+1.50	+0.75	0.00	-0.75	-1.50	-2.25	-3.00	-3.75	-4.50	-5.25	Totals
1.50	—	—	—	—	3	—	—	—	—	—	—	—	3
1.40	—	—	—	—	17	—	—	—	—	—	—	—	17
1.29	—	—	—	—	73	—	1	—	—	—	—	—	74
1.11	—	—	—	1	171	8	1	—	—	—	—	—	181
.91	—	—	—	13.5	197	19.5	—	1	—	—	—	—	231
.75	—	—	0.5	12.5	71	20.5	1.5	—	—	1	—	—	107
.58	0.5	1	14	46	18	3	9.5	—	—	2	—	—	94
.37	0.5	2.5	6.5	28	14.5	7.5	7.5	3	2	—	—	—	72
.25	—	—	2	14	5	7	2	4	1	—	—	—	35
.14	0.5	—	4	11	12	2	5.5	—	3	—	—	—	38
.08	—	—	—	12	2	—	1	—	—	—	—	—	15
.04	—	—	—	6	1	1	3	—	2	—	—	—	13
Totals	1.5	4	53.5	649	100.5	24	29.5	7	9	2	—	—	880

The array-means are:

Grade of Astigmatism	Mean Visual Acuity
1.70 D.	.4409
0.75 D.	.6422
0.00 D.	.9061
-0.75 D.	.5514
-1.50 D.	.4213
-2.25 D.	.3615
-3.00 D.	.3016
-3.88 D.	.3091

General Population: .8093

The constants of the distribution are as follows:

General Astigmatism, Mean:  $-\cdot2182$  D.;

Standard Deviation:  $\cdot7178$  D.

Visual Acuity, Mean:  $\cdot8092$ ;

Standard Deviation:  $\cdot3471$ .

Product Moment Coefficient of Correlation:  $r = \cdot3665 \pm \cdot0197$ .

As before this coefficient means very little for present purposes.

\* Standard Deviation:  $1.5523$  D.

Correlation Ratio of Visual Acuity on General Astigmatism:

$$\eta'^2_{VA.GA} = \cdot250,430, \quad \bar{\eta}^2_{VA.GA} = \cdot007,955 \pm \cdot002,854,$$

$\eta'^2_{VA.GA}$  is therefore significant and  $\eta'_{VA.GA} = \cdot5004$ .

Working out the regression of General Astigmatism on Visual Acuity we note (see also Diagram 69) that until the Visual Acuity drops to  $\cdot75$  there is little trace of General Astigmatism,

Grade of Visual Acuity	Mean General Astigmatism	Grade of Visual Acuity	Mean General Astigmatism
1.411	0 D.	.58	— .3750 D.
1.29	— .0203 D.	.37	— .6354 D.
1.11	— .0373 D.	.193	— .8836 D.
.91	— .0292 D.	.061	— .7232 D.
.75	— .1051 D.	General Population:	— .2182 D.

after that value it increases rapidly. We have:  $\eta'^2_{GA.VA} = \cdot178,983$ ,  $\bar{\eta}^2_{GA.VA} = \cdot011,364 \pm \cdot003,406$ . Hence  $\eta'^2_{GA.VA}$  is significant and  $\eta'_{GA.VA} = \cdot4231$ .

The regression curves in Diagram 69 are graduated with cubics as follows:

$$\tilde{y}_{VA} = \cdot75269 + \cdot06536x_{GA} - \cdot10789x_{GA}^2 - \cdot02487x_{GA}^3;$$

### VISUAL ACUITY & GENERAL ASTIGMATISM

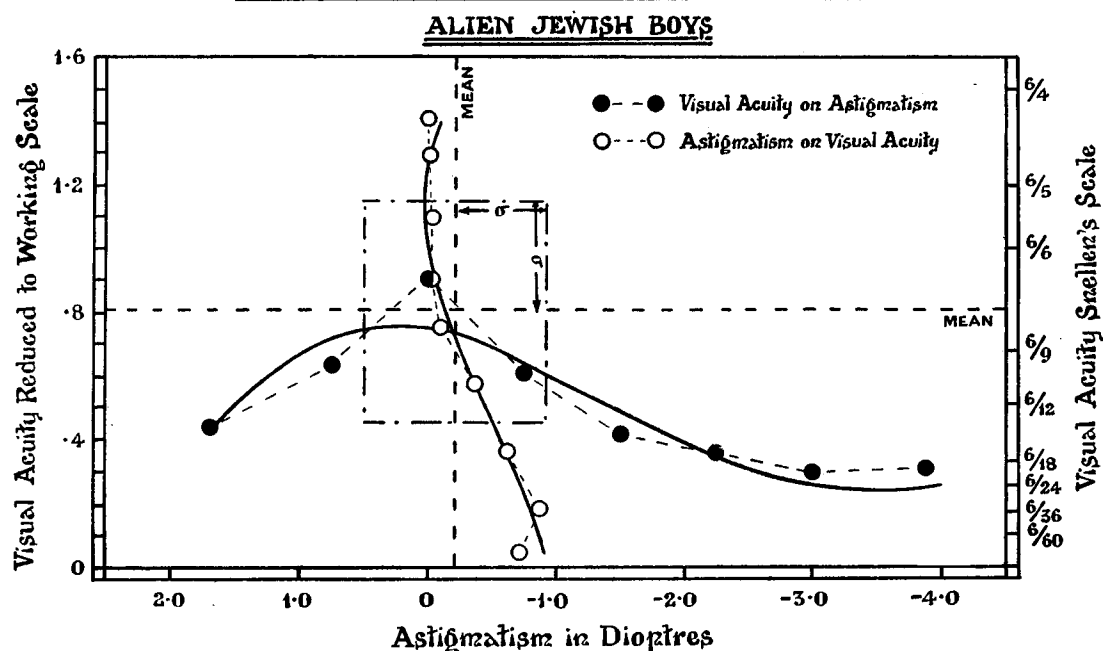


Diagram 69.

this is an unweighted cubic, the weighted cubic practically passing through the peak, owing to the dominant weight of that point, and being a bad fit elsewhere. The other cubic is:

$$\tilde{x}_{GA} = -\cdot95159 + \cdot63498y_{VA} + 1\cdot15327y_{VA}^2 - \cdot84187y_{VA}^3.$$

Probably we should have done better in this case to spline the array-means as they appear to need higher order curves for good graduation.

(viii) *Visual Acuity and Corneal Astigmatism.* We should anticipate here that the correlation would be considerable but not so intense as between Visual Acuity and General Astigmatism. The difference however does not appear to be as great as might be expected. We have indeed:

$$\eta'_{VA.CA} = \cdot4238 \quad \text{while} \quad \eta'_{VA.GA} = \cdot5004.$$

Thus almost as much as to Visual Acuity can be predicted from Corneal as from General Astigmatism. It would seem as if either distortion of the cornea was the fundamental factor in Astigmatism, or the distortion of the remainder of the optical lens system was enforced by that of the cornea, a possibility, if corneal astigmatism be inherited, and therefore congenital. Our data are given in Table CLXXVI.

Table CLXXVI. *Visual Acuity and Corneal Astigmatism. Boys.*

Corneal Astigmatism in Dioptres (Central Values)

Central Values	-2.25	-1.50	-0.75	0.00	+0.75	+1.50	+2.25	+3.00	+3.75	+4.50	+5.25	+6.00	Totals
1.50	—	—	—	2	1	—	—	—	—	—	—	—	3
1.40	—	—	—	6	10	—	—	—	—	—	—	—	17
1.29	—	—	—	64	20	—	—	—	—	—	—	—	84
1.11	—	1	1	119	64	9	2	—	—	—	—	—	196
.91	—	—	5	93	133	7	2	—	—	—	—	—	240
.75	—	1	3	50	43	15	3	1	—	—	—	—	116
.58	1	1	3	32	45	11	11	2	2	1	—	—	109
.37	—	3	2	22	21	19	14	9	2	—	1	—	93
.25	—	—	1	8	18	5	9	6	2	—	—	—	49
.14	—	1	0.5	17	13.5	3	4	1	2	2	1	1	46
.08	—	—	—	8	6	2	2	—	—	—	—	—	18
.04	—	—	1	3	6	3	3	—	—	—	—	—	16
Totals	1	7	17.5	424	380.5	74	50	19	8	3	2	1	987

The array-means of Visual Acuity for each grade of Corneal Astigmatism run as follows:

Grade of Corneal Astigmatism	Mean of Visual Acuity
- 1.59 D.	.5337
- .75 D.	.6943
.00 D.	.8963
+ .75 D.	.8058
+ 1.5 D.	.5807
+ 2.25 D.	.4188
+ 3.0 D.	.3621
+ 4.29 D.	.2993
General Population*:	.7883

The result is shown in Diagram 70 and the observations have been graduated with the un-weighted cubic†:

$$\hat{y}_{VA} = .81363 - .01873x_{CA} - .09687x_{CA}^2 + .01695x_{CA}^3.$$

### VISUAL ACUITY & CORNEAL ASTIGMATISM

(OBSERVERS A, B & C.) ALIEN JEWISH BOYS

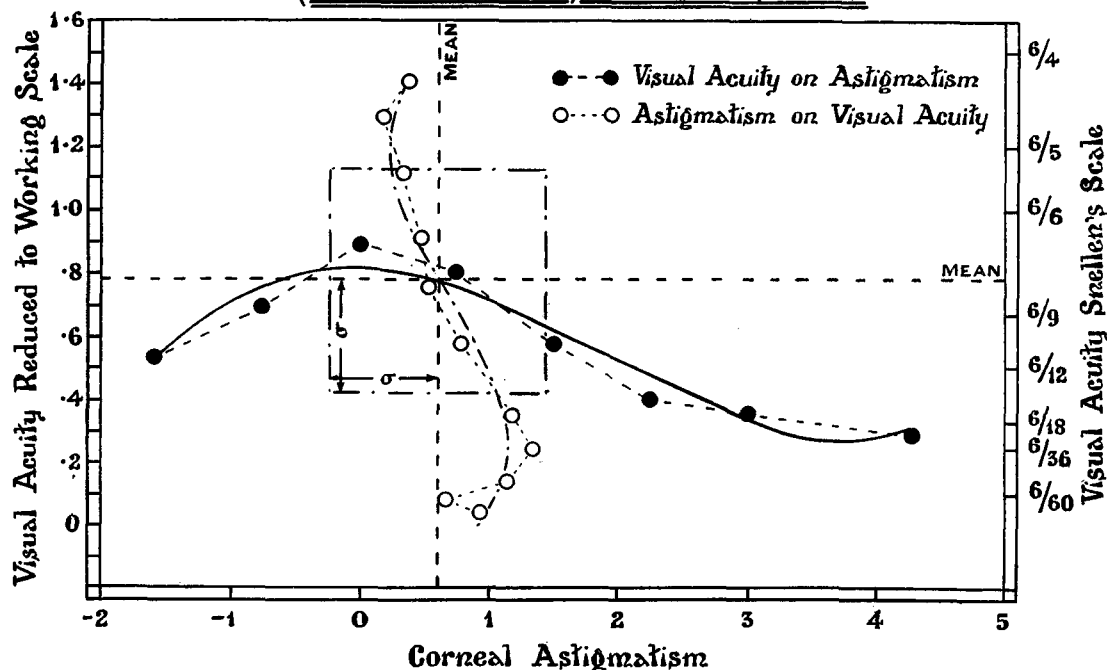


Diagram 70.

\* Standard Deviation: .3567.

† Neither weighted cubic nor quartic was satisfactory owing to the few observations in the terminal arrays, although the array-means gave a reasonably smooth curve.

We have again strong evidence of the evolutionary idea. Let us suppose that it was impossible for a palaeolithic hunter to survive with a visual acuity less than  $\cdot40$ , then in a race such as that of these Jewish Boys 22.5 % would have perished and the Corneal Astigmatism be on the average reduced from  $\cdot6079$  D. to  $\cdot4510$  D. in one generation of selection. The interest of these curves lies in the fact that excess in *either direction* reduces the intensity of a character upon which in early times survival must have largely depended and we obtain an insight into the manner in which natural selection—seconded of course by heredity—would enforce sphericity of the cornea. Returning to our table we find:

$$\eta'^2_{VA.CA} = \cdot179,576,$$

$$\bar{\eta}^2_{VA.CA} = \cdot007,092 \pm \cdot002,814.$$

Thus  $\eta'^2_{VA.CA}$  is definitely significant, and we have  $\eta'_{VA.CA} = \cdot4238$  as our measure of the manner in which Visual Acuity depends on Corneal Astigmatism.

If we turn the problem round and ask what average Corneal Astigmatism occurs for each Grade of Vision we have the following results:

Grade of Vision	Mean Corneal Astigmatism	Grade of Vision	Mean Corneal Astigmatism
1.415	$\cdot3750$ D.	$\cdot37$	$1.1774$ D.
1.29	$\cdot1786$ D.	$\cdot25$	$1.3469$ D.
1.11	$\cdot3253$ D.	$\cdot14$	$1.1413$ D.
$\cdot91$	$\cdot4625$ D.	$\cdot08$	$\cdot6667$ D.
$\cdot75$	$\cdot5237$ D.	$\cdot04$	$\cdot9375$ D.
$\cdot58$	$\cdot7982$ D.	General Population*: $\cdot6079$ D.	

The results have been graduated by the weighted cubic:

$$\tilde{x}_{CA} = \cdot90728 + 2.00037y_{VA} - 4.76859y^2_{VA} + 2.19431y^3_{VA}.$$

The value of the correlation ratio of Corneal Astigmatism on Visual Acuity is given by:

$$\eta'^2_{CA.VA} = \cdot165,366,$$

$$\bar{\eta}^2_{CA.VA} = \cdot010,132 \pm \cdot001,416,$$

and accordingly  $\eta'^2_{CA.VA}$  is significant, and we have  $\eta'_{CA.VA} = \cdot4066$ , a value only slightly less than

**VISUAL ACUITY & CORNEAL ASTIGMATISM WITHOUT REGARD TO SIGN**  
(OBSERVERS A, B & C) ALIEN JEWISH BOYS

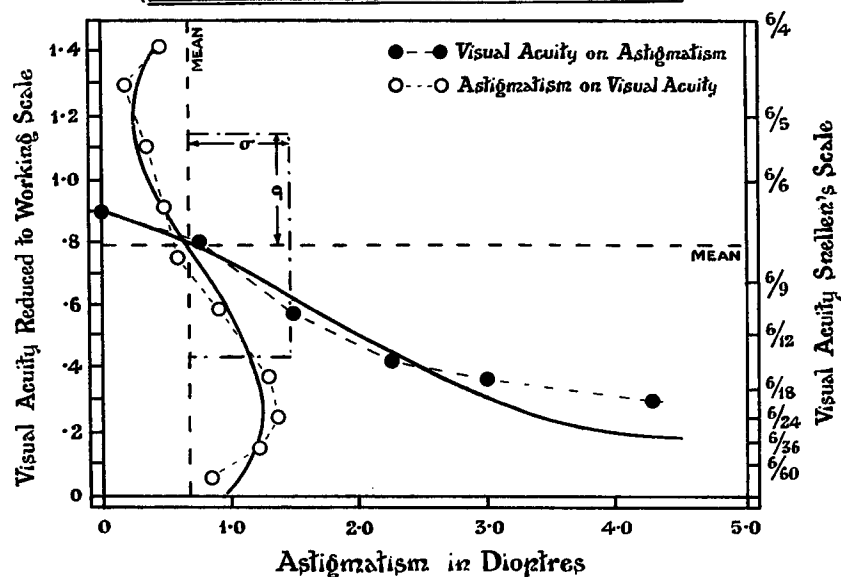


Diagram 71.

that of  $\eta'_{GA.VA}$ †. Studying the regression curve as given in Diagram 70 (p. 159) we see that there is some sign of reduced Corneal Astigmatism for the lowest grades of Visual Acuity. It is possible

\* Standard Deviation:  $\cdot8353$  D.

† i.e.  $\cdot4231$ .

that this is due to the superposition of positive and negative Corneal Astigmatism for these grades of vision; it might also be due to low grades of vision having another source—i.e. in General Refraction rather than in Astigmatism. To test this point a table (CLXXVII) was formed for Corneal Astigmatism without regard to sign\*, and the array-means both ways found:

Grade of Vision	Mean Corneal Astigmatism	Grade of Corneal Astigmatism	Mean Visual Acuity
1.415	.444 D.	.00 D.	.8963
1.29	.179 D.	.75 D.	.8009
1.11	.348 D.	1.5 D.	.5760
.91	.494 D.	2.25 D.	.4220
.75	.588 D.	3.00 D.	.3621
.58	.908 D.	4.286 D.	.2993
.37	1.306 D.		
.25	1.378 D.		
.14	1.223 D.		
.061	.838 D.		
General Population:	.660 D.	General Population:	.7883

It will be seen that when we pay no attention to sign of the Astigmatism the regression of Corneal Astigmatism on Visual Acuity still turns down at low acuity. We conclude accordingly that very low visual acuity has other chief sources than Corneal Astigmatism. The constants of the table are:

Vision, Mean: ... .. .7883;

Corneal Astigmatism, Mean: .6603 D.;

Standard Deviation: .3567.

Standard Deviation: .8235 D.

Table CLXXVII. *Visual Acuity and Corneal Astigmatism* (Astigmatism without regard to sign).

Corneal Astigmatism in Dioptres	Visual Acuity											
	1.80	1.40	1.29	1.11	.91	.75	.58	.37	.25	.14	.08	.04
.0	2	6	64	119	93	50	32	22	8	17	8	3
.75	1	11	20	65	138	46	48	23	19	14	6	7
1.5	—	—	—	10	7	16	12	22	5	4	2	3
2.25	—	—	—	2	2	3	12	14	9	4	2	3
3.0	—	—	—	—	—	1	2	9	6	1	—	—
3.75	—	—	—	—	—	—	2	2	2	2	—	—
4.5	—	—	—	—	—	—	1	—	—	2	—	—
5.25	—	—	—	—	—	—	—	1	—	1	—	—
6.0	—	—	—	—	—	—	—	—	—	1	—	—
Totals	3	17	84	196	240	116	109	93	49	46	18	16
												987

Product Moment Correlation Coefficient:  $r = - .4077 \pm .0179$ .

Correlation Ratio, Visual Acuity on Corneal Astigmatism:

$$\eta'^2_{VA.CA} = .177,587.$$

$$\bar{\eta}^2_{VA.CA} = .005,066 \pm .002,155.$$

$\eta'^2_{VA.CA}$  is thus definitely significant, and we have  $\eta'_{VA.CA} = .4214$ , indicating that the regression of Visual Acuity on Corneal Astigmatism is not so far from linear.

Diagram 71 (p. 160) shows the two regression curves of which the more

important one, that of Visual Acuity on Corneal Astigmatism, has been graduated with the weighted cubic†:

$$\tilde{y}_{VA} = .90285 - .12581x_{CA} - .05404x_{CA}^2 + .01036x_{CA}^3,$$

when Corneal Astigmatism is considered without regard to sign.

Lastly, as we wanted to thrash out the whole question of Corneal Astigmatism and Visual Acuity we investigated the relation between the two on the basis of *A* and *B*'s records only, as we considered these as possibly more exact than those of *C*. Table CLXXVIII (p. 162) provides the data.

\* The sign is after all very arbitrary, for we have taken as the first constituent of the difference the corneal curvature in the principal axis nearest to the horizontal.

† The corresponding weighted cubic for Corneal Astigmatism on Visual Acuity is

$$\tilde{x}_{CA} = .94163 + 2.37854y_{VA} - 5.41543y_{VA}^2 + 2.45889y_{VA}^3.$$

Table CLXXVIII. *Visual Acuity and Corneal Astigmatism (A and B only).*

Corneal Astigmatism in Dioptres (Central Values)

Visual Acuity	Central Values	-2.25	-1.50	-0.75	0.00	+0.75	+1.50	+2.25	+3.00	+3.75	+4.50	+5.25	Totals
1.50	—	—	—	—	2	1	—	—	—	—	—	—	3
1.40	—	—	—	1	5	10	—	—	—	—	—	—	16
1.29	—	—	—	—	—	2	—	—	—	—	—	—	2
1.11	—	—	—	—	4	12	1	—	—	—	—	—	17
.91	—	—	5	45	90	6	2	—	—	—	—	—	148
.75	—	1	3	9	22	5	2	1	—	—	—	—	43
.58	—	—	2	10	18	3	5	1	1	1	—	—	41
.37	—	3	2	9	9	10	6	4	2	—	1	—	46
.25	—	—	1	2	15	2	4	3	2	—	—	—	29
.14	—	1	—	10	5	2	2	—	1	1	—	—	22
.08	—	—	—	2	4	—	1	—	—	—	—	—	7
.04	—	—	1	1	3	2	1	—	—	—	—	—	8
Totals	—	5	15	99	191	31	23	9	6	2	1	—	382

The array-means for Visual Acuity are:

Grade of Corneal Astigmatism	Mean Visual Acuity
-1.50 D.	.4000 ± .1029
-0.75 D.	.6927 ± .0594
0.00 D.	.7411 ± .0231
+0.75 D.	.7773 ± .0167
+1.50 D.	.5361 ± .0407
+2.25 D.	.4278 ± .0480
+3.00 D.	.3956 ± .0767
+4.083 D.	.3389 ± .0767
General Population*:	.6997 ± .0118

Mean, Corneal Astigmatism: .7048 D.

Standard Deviation, Corneal Astigmatism: .9421 D.

We have

$$\eta'^2_{VA.CA} = .141,655,$$

$$\bar{\eta}^2_{VA.CA} = .018,325 \pm .006,536.$$

 $\eta'^2_{VA.CA}$  is thus quite significant, and we have

$$\eta'_{VA.CA} = .3764.$$

This is a somewhat less value than we found for *A*, *B* and *C* together, which *pro tanto* may be taken to confirm *C*'s work as far as Corneal Astigmatism is concerned.

**VISUAL ACUITY & CORNEAL ASTIGMATISM (OBSERVERS A & B ONLY)**  
**ALIEN JEWISH BOYS**

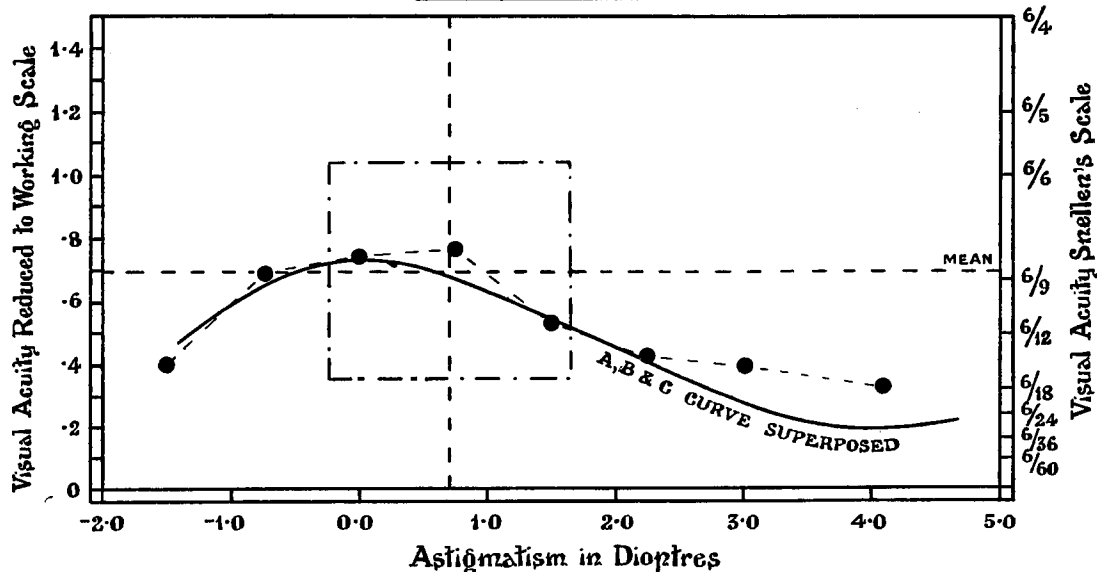


Diagram 72.

Diagram 72 shows the result graphically. It will be seen that it chiefly differs from the result for *A*, *B* and *C* together by the higher values for the array-means of Visual Acuity at  $-.75$  D. and  $+.75$  D. and the lower value at 0 D., a result which would arise if *C* were not very sure of his  $\pm .5$  D. values. We have placed on the Diagram the cubic adapted to *A*, *B* and *C*'s observations

\* Standard Deviation: .3413.

taken in conjunction. It will be seen to give a not unreasonable graduation considering the paucity of terminal observations in *A* and *B*'s series. (Note the probable errors recorded on p. 162.)

(ix) *Visual Acuity and Position of the Near Point.* Our data are provided in Table CLXXIX.

Table CLXXIX. *Visual Acuity and Near Point Distance.*

Distance of Near Point (Central Values in mm.)

Visual Acuity	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175	180	...	200	Totals
1.50	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
1.40	—	—	—	—	—	—	2	6	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11
1.29	—	—	—	—	2	—	3	3	4	6	6	11	17	10	7	2	3	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	77
1.11	—	—	—	1	1	3	4	13	9	15	17	28	23	15	10	13	9	9	4	4	3	1	1	1	—	—	—	—	—	—	—	—	184
.91	—	—	—	—	1	4	3	11	18	25	25	30	23	12	9	3	2	2	2	2	1	1	2	1	—	—	—	—	—	—	—	—	175
.75	—	—	1	—	—	1	3	5	4.5	13.5	9	9	6	9	7	6	6	1	4	4	—	—	—	—	1	—	—	—	—	—	—	—	95
.58	—	—	—	—	—	—	—	2	7	9	12	11	7	6	1	7	5	—	1	8	4	1	1	1	—	—	—	—	—	—	—	—	84
.37	—	—	—	—	—	—	1.5	2.5	1	7	6	7	8	6	5	2	2	2	—	2	3	1	—	—	—	—	—	—	—	—	—	—	57
.25	—	—	—	—	1	2	3	3	2	1	3	—	—	—	4	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	23
.14	—	—	—	1	1	—	2	2	3	3	—	1	2	3	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	25
.08	—	—	—	—	—	1	1	—	1	3	—	3	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	12
.04	1	—	—	—	1	1	—	—	—	2	2	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
Totals	1	—	2	2	7	12	20.5	48.5	51.5	86.5	80	100	86	63	43	34	28	18	14	20	16	7	5	4	—	1	—	1	1	1	..	1	754

The constants of this table are the following:

Vision, Mean: ... .. .8413; Standard Deviation: .3377.

Distance of Near Point, Mean: 93.4284 mm.; Standard Deviation: 20.1580 mm.

The array-means of Visual Acuity for different grades of Near Point are given below and will be found plotted in Diagram 73. It will be seen that from Near Point equal 7 cms. to 12 cms.

### VISUAL ACUITY & POSITION OF NEAR POINT

#### ALIEN JEWISH BOYS

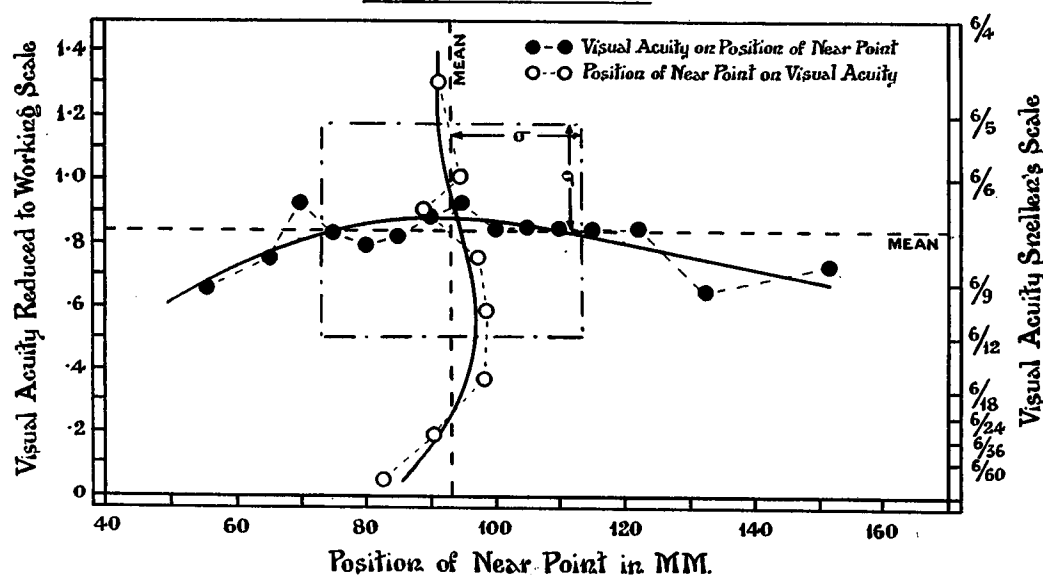


Diagram 73.

there is practically no change in the Visual Acuity. Only where the Near Point is in excess or defect of this range is the Visual Acuity influenced; it then drops from about .78 to .85 to about

·65 to ·74, but not further in the whole range of Near Point found. It is thus clear that the Near Point at any rate in Boys under 16 is not an important factor of Visual Acuity.

Distance of Near Point (mm.)	Mean Visual Acuity	Distance of Near Point (mm.)	Mean Visual Acuity
55·42	·6663	100	·8486
65	·7505	105	·8605
70	·9294	110	·8615
75	·8376	115	·8596
80	·7973	122·19	·8631
85	·8265	132·22	·6633
90	·8867	151·90	·7362
95	·9324		
		General Population*: ·8413	

Determining the Correlation Ratio of Visual Acuity on Distance of Near Point we have:

$$\eta'^2_{VA.NP} = \cdot045,425, \quad \bar{\eta}^2_{VA.NP} = \cdot018,568 \pm \cdot004,687.$$

$\eta'^2_{VA.NP}$  is accordingly probably significant and

$$\eta'_{VA.NP} = \cdot2131,$$

about the lowest value we have found in discussing Visual Acuity with other ocular characters†.

Looking at the matter from the opposite standpoint, let us inquire what is the probable value of the Distance of the Near Point for a given Visual Acuity.

Grade of Vision	Mean Near Point (mm.)	Grade of Vision	Mean Near Point (mm.)
1·42	71·071	·58	98·690
1·29	95·065	·37	98·377
1·11	94·755	·193	90·417
·91	88·800	·064	82·700
·75	97·500		
		General Population‡: 93·428	

These means are somewhat erratic, but the two terminals of the regression curve both indicate a less Distance of the Near Point when the Vision is very good or very bad.

The closeness of the association is measured by the correlation ratio; we have:

$$\eta'^2_{NP.VA} = \cdot062,957, \quad \bar{\eta}^2_{NP.VA} = \cdot010,610 \pm \cdot003,557.$$

Thus  $\eta'^2_{NP.VA}$  is significant and  $\eta'_{NP.VA} = \cdot2456,$

a result slightly larger than that for  $\eta'_{VA.NP}$ §.

Diagram 73 shows these results graphically. It is clear that a small Near Point Distance may be associated either with good or bad extremes of vision, and a low Visual Acuity with either a near or a far Near Point.

As both General Refraction and Near Point Distance are associated with Visual Acuity, it is *a priori* difficult to believe that the current measure of accommodation can be independent of Visual Acuity.

(b) *Refraction Class.* We now take under consideration the relation of Refraction Class to other ocular characters.

(i) We consider first the Refraction Class of Right and Left Eyes. The data for 450 Boys are given in the accompanying table:

\* Standard Deviation: ·3377.

† The equation to the fitted cubic is

$$\tilde{VA} = \cdot87829 + \cdot00177 \left( \frac{NP - 90}{5} \right) - \cdot00294 \left( \frac{NP - 90}{5} \right)^2 + \cdot00012 \left( \frac{NP - 90}{5} \right)^3.$$

‡ Standard Deviation: 20·1580 D.

§ The equation to the fitted cubic is:  $\tilde{NP} = 81·8539 + 65·13180 (VA) - 85·21275 (VA)^2 + 31·14415 (VA)^3.$

Table CLXXX. *Refraction Class of Left Eye and Right Eye.*

Refraction Class. Left Eye	Refraction Class. Right Eye							
		Emme- tropia	Hyp.	H. A.	Mixed A.	M. A.	M.	Totals
	Emmetropia	257	4	8	1	1	8	279
	Hyp.	11	25	1	—	—	1	38
	H. A.	3	2	26	2	—	—	33
	Mixed A.	1	—	1	2	2	1	7
	M. A.	6	—	—	1	10	4	21
	M.	10	—	—	—	6	56	72
	Totals	288	31	36	6	19	70	450

rest. It may be of interest to note that the Right Eye is anormal in 36 % of cases and the Left Eye in 38 %—a difference hardly to be considered significant on our totals. The coefficient of correlation for this fourfold table is  $r_t = .9249 \pm .0130$ ,

a probable error which signifies very little for such a high correlation.

		Right Eye		
		Normal	Anormal	Totals
Left Eye	Normal	257	22	279
	Anormal	31	140	171
	Totals	288	162	450

Secondly, we may treat the table as it stands by the method of mean square contingency, a more reasonable one in this case as it does not suppose continuity of the constituent classes. We find  $\phi'^2 = 2.025,778$ , leading to

a Coefficient of Mean Squared Contingency  $C_2 = .8182$ . Both  $r_t$  and  $C_2$  point to a very high degree of association. But  $r_t$  assumes continuity and  $C_2$  does not. It is not easy to suggest a single continuous variate controlling Refraction Class for this is determined by two characters, namely, General Refraction and General Astigmatism. We content ourselves therefore by saying that the association lies between .8 and .9.

(ii) *Refraction Class and Visual Acuity.* The relation of these two ocular characters has already been fully dealt with (see our pp. 151 *et seq.*).

(iii) *Refraction Class and General Refraction.* Table CLXXXI (p. 166) provides the requisite data and indicates as we might naturally anticipate a very high association. This must necessarily be so because General Refraction in combination with General Astigmatism determines absolutely the Refraction Class.

The constants of this table are as follows:

$$\eta'^2_{GR.RC} = .562,722, \quad \bar{\eta}^2_{GR.RC} = .005,556 \pm .002,362.$$

Thus  $\eta'^2_{GR.RC}$  is definitely significant and we have the high value  $\eta'_{GR.RC} = .7501$ .

The array-means for General Refraction are:

Refraction Class	Mean General Refraction
Hypermetropic Astigmatism	2.5423 D.
Hypermetropia ... ..	1.7206 D.
Emmetropia... ..	.1786 D.
Myopia ... ..	— 1.4714 D.
Myopic Astigmatism ...	— 2.6156 D.
Mixed Astigmatism ...	.9231 D.
General Population* ...	-.0962 D.

It is clear that arranging our Refraction Classes as we have done the one disturbing element is the Mixed Astigmatism Class, which would be best as far as General Refraction is concerned if it were placed between Hypermetropia and Emmetropia. The results are exhibited graphically in Diagram 74 (p. 167), the sectorial areas representing percentages, and the lengths of the rays to the circle 0.0 D the refraction.

\* Standard Deviation: 1.5691 D.

It will be seen that out of the 450 cases only 74 occur in which the two eyes do not fall into the same refraction class, i.e. 16.4 %.

This is accordingly the measure of Anisometropia in our Jewish Boys.

We have reduced this table in two ways:

First, considering it as a fourfold table dividing into normal and the

## PROBLEM OF ALIEN IMMIGRATION

Table CLXXXI. *Refraction Class and General Refraction.*

General Refraction in Dioptres

Refraction Class	+6.75	+6.00	+5.25	+4.50	+3.75	+3.00	+2.25	+1.50	+0.75	0.00	-0.75	-1.50	-2.25	-3.00	-3.75	-4.50	-5.25	-6.00	-6.75	:	-12.75	:	-15.75	Totals
Hypermetropic Astigmatism	1	3	2	2	10.5	6.5	24.5	7	5.5	6	—	—	—	—	—	—	—	—	—	..	—	..	—	68
Hypermetropia ...	—	1	2	1	6	4	7	15	31	1	—	—	—	—	—	—	—	—	—	..	—	..	—	68
Emmetropia...	—	—	—	—	—	—	—	—	143	416	8	—	—	—	—	—	—	—	—	..	—	..	—	567
Myopia ...	—	—	—	—	—	—	—	—	—	11	65.5	31.5	17	6	7	—	5	—	1	..	—	..	—	144
Myopic Astigmatism	—	—	—	—	—	—	—	—	—	7.5	10	4	8.5	1	1	—	2	1	3	..	1	..	1	40
Mixed Astigmatism	—	—	—	—	—	—	1	3	7.5	1	0.5	—	—	—	—	—	—	—	—	..	—	..	—	13
Totals ...	1	4	4	3	16.5	10.5	32.5	25	187	442.5	84	35.5	25.5	7	8	—	7	1	4	..	1	..	1	900

(iv) *Refraction Class and General Astigmatism.* Like Refraction Class and General Refraction the relationship of these two variates is largely produced artificially by the limits attributed by investigators to the various classes. For this reason it is desirable to place not only the Central Values but the subgroup ranges by the side of the table. They explain how the classification into refraction classes has been made.

Table CLXXXII. *Refraction Class and General Astigmatism.*

Refraction Class

General Astigmatism in Dioptres	Central Values	Range	Hypermetropic Astigmatism	Hypermetropia	Emmetropia	Myopia	Myopic Astigmatism	Mixed Astigmatism	Totals
	+ 3.00	+ 3.375 to + 2.625	0.5	—	—	—	—	—	0.5
	+ 2.25	+ 2.625 to + 1.875	1.5	—	—	—	—	—	1.5
	+ 1.50	+ 1.875 to + 1.125	2	—	—	—	1.5	0.5	4
	+ 0.75	+ 1.125 to + .375	6	4	26	14	7	—	57
	0.00	+ .375 to - .375	—	60	491	111	—	—	662
	- 0.75	- .375 to - 1.125	14	4	50	18	13	3	102
	- 1.50	- 1.125 to - 1.875	14	—	—	—	6.5	3.5	24
	- 2.25	- 1.875 to - 2.625	16.5	—	—	—	10	3	29.5
	- 3.00	- 2.625 to - 3.375	5.5	—	—	—	—	1	6.5
	- 3.75	- 3.375 to - 4.125	4	—	—	—	3	2	9
	- 4.50	- 4.125 to - 4.875	3	—	—	—	—	—	3
	- 5.25	- 4.875 to - 5.625	1	—	—	—	—	—	1
Totals	—	—	68	68	567	143	41	13	900

If the reader will turn to our definitions of the Refraction Classes (pp. 113 and 140), he will understand why small amounts of astigmatism, minus or plus, will be found to occur in the emmetropic, hypermetropic and myopic groups. All values for the cylinder lying between - 0.75 D. and + 0.75 D. were not included in the astigmatic groups.

Examining the table as it stands we may find the mean of General Astigmatism, with or without regard to sign, for each Refraction Class. We have:

Refraction Class	Mean Astigmatism	
	Regarding Sign	Disregarding Sign
Hypermetropic Astigmatism	- 1.5662 D.	1.9301 D.
Hypermetropia ...	.00 D.	.0882 D.
Emmetropia...	- .0317 D.	.1005 D.
Myopia ...	- .0206 D.	.1667 D.
Myopic Astigmatism	- 1.1438 D.	1.5000 D.
Mixed Astigmatism	- 1.8462 D.	1.9615 D.
General Population*	- .2192 D.	.3376 D.

These results show us that Astigmatism is least when myopic, greatest when mixed, and intermediary when hypermetropic. In the non-astigmatic groups, the average astigmatism is really negligible, except in the case of myopia, where however it is less than a fifth of a dioptre.

We may further illustrate the distribution of General Astigmatism in the various refraction categories by classifying our astigmatism into larger groups and using rather doubtful percentages.

\* Standard Deviation: Regarding Sign, .7215 D; Disregarding Sign, .6742 D.

Table CLXXXIII a. *Percentages of Refraction Class for each Astigmatism.*

Central Values	Group Range	Hypermetropic Astigmatism	Hypermetropia	Emmetropia	Myopia	Myopic Astigmatism	Mixed Astigmatism	General Population
+ 1.8125 D.	Above + 1.125 D.	66.7 %	0.0 %	0.0 %	0.0 %	25.0 %	8.3 %	0.7 %
+ .75 D.	+ 1.125 D. to + .375 D.	10.0	7.1	46.0	24.8	11.5	0.6	6.3
.00 D.	+ .375 D. to - .375 D.	0.0	9.0	74.1	16.9	0.0	0.0	73.7
- .75 D.	- .375 D. to - 1.125 D.	13.8	3.9	49.3	17.7	12.3	3.0	11.2
- 1.9135 D.	- 1.125 D. to - 2.625 D.	57.0	0.0	0.0	0.0	30.9	12.1	5.9
- 3.6923 D.	Below - 2.625 D.	69.2	0.0	0.0	0.0	15.4	15.4	2.2

This table is easy of interpretation: consider a number like 17.7 % in the myopic column. This signifies that 11.3 % of the population had General Astigmatism between - .375 D. and - 1.125 D. and that of this percentage 17.7 % were myopic. These individuals all had astigmatism

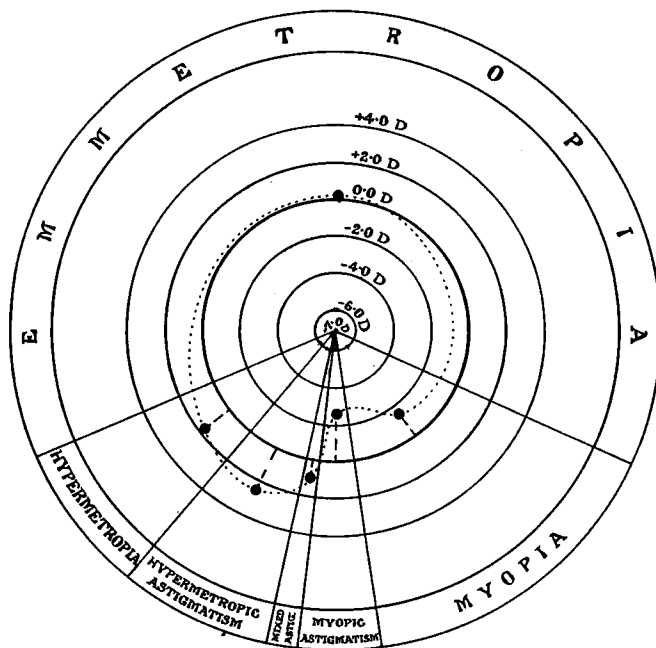
REFRACTION CLASS & GENERAL REFRACTIONALIEN JEWISH BOYS

Diagram 74.

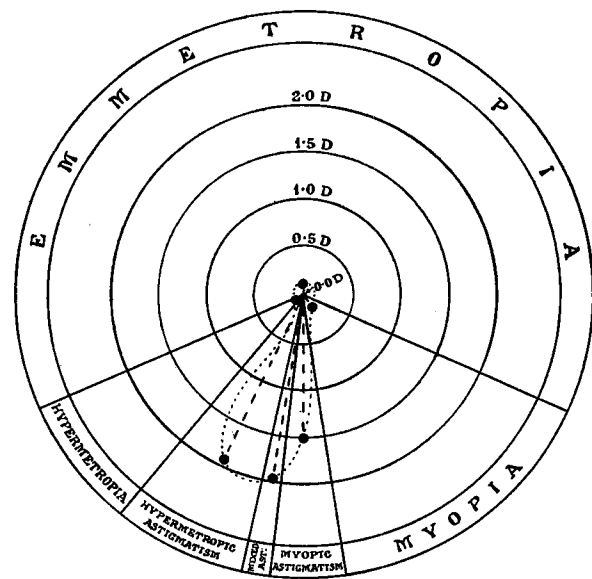
REFRACTION CLASS & GENERAL ASTIGMATISMWITHOUT REGARD TO SIGN (OBSERVERS A, B & C)ALIEN JEWISH BOYS

Diagram 75.

between - .375 D. and - .75 D.; for had it exceeded - .75 D. numerically they would have been reckoned as myopic astigmatics. Such a table while it tells us for each range of astigmatism what proportion of each refraction class to expect, does not tell us for each refraction class what proportions fall in each astigmatic range. In order to ascertain this we must work our percentages vertically instead of horizontally. We find:

Table CLXXXIII b. *Percentages of Astigmatism for each Refraction Class.*

Central Values	Group Range	Hypermetropic Astigmatism	Hypermetropia	Emmetropia	Myopia	Myopic Astigmatism	Mixed Astigmatism
+ 1.8125 D.	Above + 1.125 D.	5.9 %	0.0 %	0.0 %	0.0 %	3.8 %	3.8 %
+ .75 D.	+ 1.125 D. to + .375 D.	8.8	5.9	4.6	9.7	16.2	0.0
.0 D.	+ .375 D. to - .375 D.	0.0	88.2	86.6	77.8	0.0	0.0
- .75 D.	- .375 D. to - 1.125 D.	20.6	5.9	8.8	12.5	31.2	23.1
- 1.9135 D.	- 1.125 D. to - 2.625 D.	44.9	0.0	0.0	0.0	41.3	50.0
- 3.6923 D.	Below - 2.625 D.	19.8	0.0	0.0	0.0	7.5	23.1
General Population		7.6	7.6	63.0	16.0	4.4	1.4

The meaning of this table is as follows: Consider 44.9 % in the column of Hypermetropic Astigmatism. In the General Population there are 7.6 % of boys with hypermetropic astigmatism; of this group of boys 44.9 % have General Astigmatism with the rule lying between  $-1.125$  D. and  $-2.625$  D.

If we wish to obtain a general measure of the association of Refraction Class with General Astigmatism we can turn to the correlation ratios of General Astigmatism on Refraction Class. We have:

Regarding sign:

$$\eta'^2_{GA.RC} = .471,068, \quad \bar{\eta}^2_{GA.RC} = .005,556 \pm .002,363;$$

$\eta'^2_{GA.RC}$  is thus substantially significant, and

$$\eta'_{GA.RC} = .6863.$$

Disregarding sign:

$$\eta'^2_{GA.RC} = .735,907, \quad \bar{\eta}^2_{GA.RC} = .005,556 \pm .002,363;$$

$\eta'^2_{GA.RC}$  is thus still more substantially significant, and

$$\eta'_{GA.RC} = .8578.$$

We thus obtain the higher relationship by paying no attention to the arbitrary sign of the astigmatism. A general idea of the relation of Refraction Class to General Astigmatism may be obtained from the accompanying radiogram, Diagram 75 (p. 167), where the areas (or angles) represent the percentages in the given refraction classes and the rays the astigmatism disregarding sign.

(v) *Refraction Class and Corneal Refraction.* Our data are given in the following table:

Table CLXXXIV. *Refraction Class and Corneal Refraction.*

Corneal Refraction (Central Values) in Dioptres

Refraction Class	38-125	38-625	39-125	39-625	40-125	40-625	41-125	41-625	42-125	42-625	43-125	43-625	44-125	44-625	45-125	45-625	46-125	46-625	47-125	47-625	Totals
Emmetropia ... ..	—	—	—	3	5	12	22	27	45	61	79	82	74	50	37	32	26.5	4.5	1	2	563
Hypermetropia ... ..	—	—	—	1	—	—	3	1	4	9	10	10	10	6	2	4	2	2	1	—	65
Hypermetropic Astigmatism	—	—	—	1	1	5	1	5	4	11	7	6	5	7	1	1	3	—	—	—	65
Mixed Astigmatism...	—	—	—	1	1	—	1	2	—	1	—	2	—	1	1	—	1	2	—	—	13
Myopic Astigmatism ... ..	—	—	—	—	—	—	3	4	1	7	6	3	5	2	5.5	2	0.5	—	—	—	39
Myopia ... ..	1	—	2	—	5	4	8	4	14	4	9	15	27	14	7	9	10	4	—	—	137
Totals ... ..	1	—	3	6	16	17	42	42	75	89	111	118	121	80	53.5	48	43	12.5	2	2	882

The Mean Corneal Refractions for the various Refraction Classes are as follows:

Emmetropia...	...	...	...	...	43.5455 D.
Hypermetropia	...	...	...	...	43.6712 D.
Hypermetropic Astigmatism	...	...	...	...	43.7635 D.
Mixed Astigmatism	...	...	...	...	44.3713 D.
Myopic Astigmatism	...	...	...	...	44.3942 D.
Myopia	...	...	...	...	44.5630 D.
General Population*	...	...	...	...	43.4896 D.

The relations here are nothing like as close as in the case of Refraction Class and General Refraction, but they are in the same directions in each case. The accompanying radiogram exhibits them graphically.

If we desire a single measure of the relationship we have:

$$\eta'^2_{CR.RC} = .019,084, \quad \bar{\eta}^2_{CR.RC} = .005,669 \pm .002,411.$$

Having regard to  $\bar{\eta}^2_{CR.RC}$ ,  $\eta'^2_{CR.RC}$  is definitely significant, and we have:  $\eta'_{CR.RC} = .1381$ , less than

\* Standard Deviation: 1.5349 D.

a quarter of the relation we have found for General Refraction and Refraction Class. To measure to some extent the degree in which this rather low association has been influenced by combining *A*, *B* and *C* together, a table was worked out for *A* and *B* only. This is given below:

Table CLXXXV. *Refraction Class and Corneal Refraction (A and B only).*

Corneal Refraction (Central Values) in Dioptres

Refraction Class	38-125	38-625	39-125	39-625	40-125	40-625	41-125	41-625	42-125	42-625	43-125	43-625	44-125	44-625	45-125	45-625	46-125	46-625	47-125	47-625	Totals
Emmetropia ...	—	—	—	—	1	2	2	6	10	15	23	29	31	23	14	13	13.5	1.5	1	2	187
Hypermetropia ...	—	—	—	—	—	—	2	—	1	7	7	10	8	6	2	4	2	2	1	—	52
Hypermetropic Astigmatism ...	—	—	—	—	3	—	3	1	3	5	2	3	4	3	—	—	3	—	—	—	30
Mixed Astigmatism ...	—	—	—	—	1	—	—	—	—	2	—	2	—	—	1	—	1	2	—	—	9
Myopic Astigmatism ...	—	—	—	—	—	—	—	1	—	2	2	4	1	4	1	—	—	—	—	—	15
Myopia ...	1	—	—	—	3	—	—	1	6	2	2	4	11	9	5	5	8	4	—	—	61
Totals ...	1	—	—	—	8	2	7	9	20	31	36	50	58	42	26	23	27.5	9.5	2	2	354

**REFRACTION CLASS & CORNEAL REFRACTION**

(OBSERVERS A, B &amp; C) ALIEN JEWISH BOYS

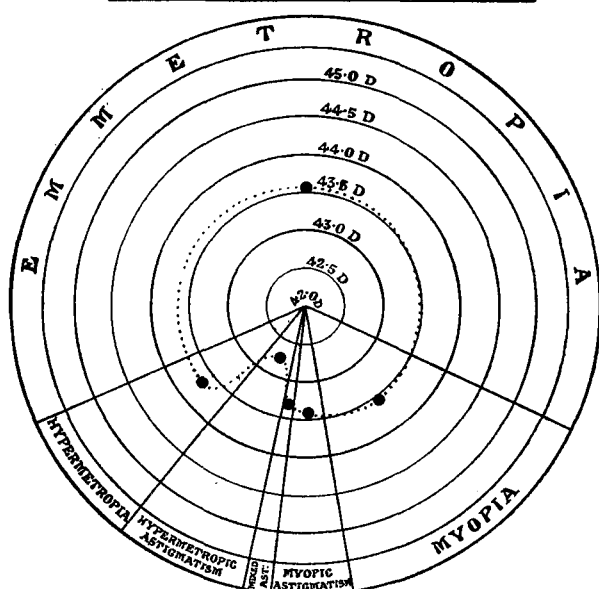


Diagram 76.

**REFRACTION CLASS & CORNEAL REFRACTION**

(OBSERVERS A &amp; B ONLY) ALIEN JEWISH BOYS

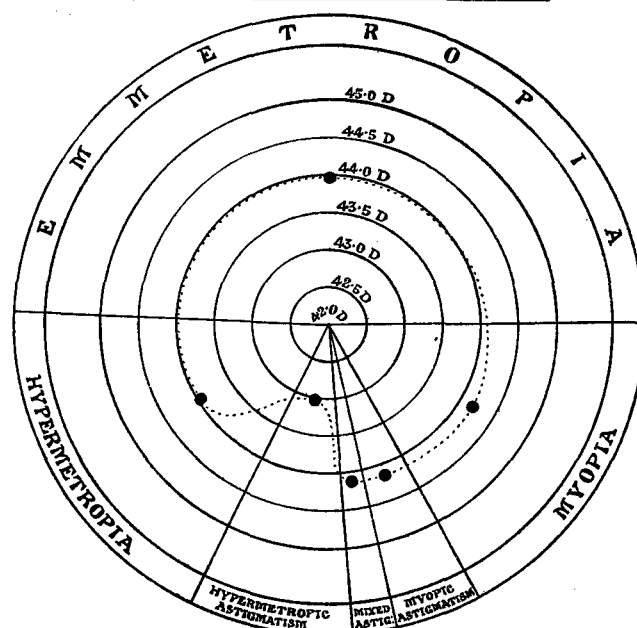


Diagram 77.

The array-means are as follows:

Emmetropia ...	...	...	...	...	...	43.9846 D.
Hypermetropia ...	...	...	...	...	...	43.9808 D.
Hypermetropic Astigmatism ...	...	...	...	...	...	43.0250 D.
Mixed Astigmatism ...	...	...	...	...	...	44.1250 D.
Myopic Astigmatism ...	...	...	...	...	...	44.1583 D.
Myopia ...	...	...	...	...	...	44.1824 D.
General Population*	...	...	...	...	...	43.9477 D.

We find:

$$\eta'^2_{CR,RC} = .038,293,$$

$$\bar{\eta}^2_{CR,RC} = .014,124 \pm .005,983,$$

$\eta'^2_{CR,RC}$  is accordingly significant and we have:

$$\eta'_{CR,RC} = .1957.$$

\* Standard Deviation: 1.4916 D.

This value is somewhat higher than we have found when  $C$  is combined with  $A$  and  $B$ , and is therefore a slight argument for treating them apart. The radiogram in Diagram 76 may be compared with the present one, Diagram 77 (see p. 169).

(vi) *Refraction Class and Corneal Astigmatism.* We can hardly expect the relation between Refraction Class and one factor of Astigmatism to be as great as between Refraction Class and the total Astigmatism. Still Corneal Astigmatism is on the whole capable of more accurate measurement than General Astigmatism (without a mydriatic) and we have confined ourselves to the observations of  $A$  and  $B$ . Table CLXXXVI gives their results.

Table CLXXXVI. *Refraction Class and Corneal Astigmatism.*

Refraction Class							
Corneal Astigmatism in Dioptries	Hypermetropic Astigmatism	Hypermetropia	Emmetropia	Myopia	Myopic Astigmatism	Mixed Astigmatism	Totals
- 1.50	2	—	1	2	—	—	5
- 0.75	—	3	8	2	—	—	13
0.00	2	17	55	21	3	—	98
+ 0.75	6	27	110	32	5	—	180
+ 1.50	5	4	9	3	3	3	27
+ 2.25	4	—	4	1	4	2	15
+ 3.00	4	1	—	—	—	1	6
+ 3.75	3	—	—	—	1	2	6
+ 4.50	2	—	—	—	—	—	2
+ 5.25	2	—	—	—	—	—	2
Totals	30	52	187	61	16	8	354

The array-means are as follows:

Refraction Class	Mean Corneal Astigmatism	
	Disregarding Sign	Regarding Sign
Hypermetropic Astigmatism	2.2250 D. $\pm$ .1052	2.0250 D. $\pm$ .1123
Hypermetropia ... ..	.6058 D. $\pm$ .0799	.5192 D. $\pm$ .0853
Emmetropia... ..	.6016 D. $\pm$ .0422	.5214 D. $\pm$ .0450
Myopia ... ..	.5779 D. $\pm$ .0738	.4303 D. $\pm$ .0788
Myopic Astigmatism ...	1.3125 D. $\pm$ .1441	1.3125 D. $\pm$ .1538
Mixed Astigmatism ...	2.4375 D. $\pm$ .2038	2.4375 D. $\pm$ .2175
General Population* ...	.8093 D. $\pm$ .0306	.7119 D. $\pm$ .0327

The array-means clearly differ significantly from the mean of the General Population. Their values are shown graphically in the radiogram, Diagram 78 (p. 171).

Turning to the correlation ratios we have:

Disregarding sign:  $\eta'^2_{CA.RC} = .382,300$ ,  $\bar{\eta}^2_{CA.RC} = .014,124 \pm .005,975$ .

Regarding sign:  $\eta'^2_{CA.RC} = .322,134$ ,  $\bar{\eta}^2_{CA.RC} = .014,124 \pm .005,975$ .

Clearly both  $\eta'^2_{CA.RC}$ 's are significant relative to the  $\bar{\eta}^2_{CA.RC}$ 's. Hence we find:

$$\begin{aligned}\eta'_{CA.RC} &= .6183, \text{ paying no attention to sign,} \\ &= .5676, \text{ paying attention to sign.}\end{aligned}$$

The corresponding values for General Astigmatism and Refraction Class are†:

$$\eta'_{GA.RC} = .8578 \text{ and } .6863.$$

\* Standard Deviation: Disregarding Sign, .8547 D.; Regarding Sign, .9121 D.

† If we may make some not very wild assumptions, we can deduce from these figures some appreciation of the association of Lenticular Astigmatism and Refraction Class. Let  $u$  = General Astigmatism,  $x$  = Corneal Astigmatism,  $y$  = Lenticular Astigmatism,  $v$  = Refraction Class. We need  $r_{vy}$ , knowing  $r_{ux}$ ,  $r_{uv}$  and  $r_{vx}$ . Roughly  $\delta u = -\delta x - \delta y$ .

Now  $\sigma_y = \sqrt{\sigma_u^2 + \sigma_x^2 + 2r_{xu}\sigma_u\sigma_x}$ . Hence we easily find  $\frac{\sigma_u r_{uv} + \sigma_x r_{xv}}{\sqrt{\sigma_u^2 + \sigma_x^2 + 2r_{xu}\sigma_u\sigma_x}} = -r_{vy}$ . But  $\sigma_u = .7374$ ,  $\sigma_x = .8157$ , while, if we may replace correlation coefficients by correlation ratios,  $r_{uv} = .8578$ ,  $r_{xv} = -.6183$  and  $r_{xu} = -.6947$  disregarding sign of the Astigmatism, and remembering how Corneal Astigmatism is measured. There results:  $r_{vy} = -.2098$ , or we conclude that Lenticular Astigmatism is far less influential than Corneal in determining Refraction Class.

(vii) *Refraction Class and Near Point.* Table CLXXXVII provides the data for considering this matter.

Table CLXXXVII. *Refraction Class and Near Point.*

Near Point (Central Values) in mm.

Refraction Class	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175	180	185	190	195	200	Totals
Emmetropia	1	3	8	10	27	29	57	53	70	61	39	25	24	20	15	8	12	7	4	4	4	—	1	—	—	—	1	—	—	—	—	483
Hypermetropia	—	—	—	2	8	3	7	6	3	2	3	1	—	—	—	1	3	1	1	—	—	—	—	—	—	—	—	—	—	—	—	42
Hypermetropic Astigmatism	—	—	—	—	—	—	1	1	2	2	4	6	2	3	—	1	4	1	1	—	—	—	—	—	—	—	—	—	—	—	—	28
Mixed Astigmatism	—	—	—	0.5	0.5	—	—	—	1	2	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	8
Myopic Astigmatism	—	1	1	2	3	6	4	2	3	2	—	2	1	1	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	32
Myopia	—	2	1	9	8	7.5	14.5	14	10	9	10	6	2	2	1	3	1	6	1	—	—	—	—	—	—	—	—	—	—	—	—	107
Totals	1	6	10	23.5	46.5	45.5	83.5	76	89	78	57	41	29	27	16	13	21	16	7	5	4	—	1	—	1	1	1	—	—	—	1	700

**REFRACTION CLASS & CORNEAL ASTIGMATISM**

WITHOUT REGARD TO SIGN (OBSERVERS A &amp; B ONLY)

ALIEN JEWISH BOYS

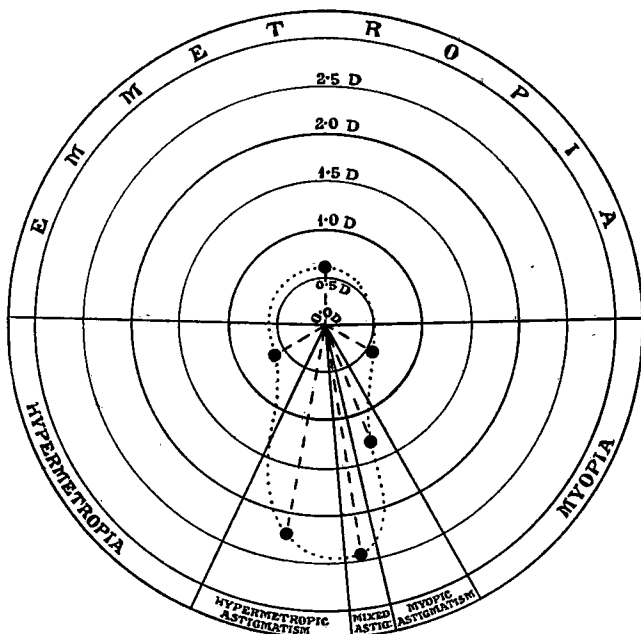


Diagram 78.

**REFRACTION CLASS & POSITION OF NEAR POINT**

ALIEN JEWISH BOYS

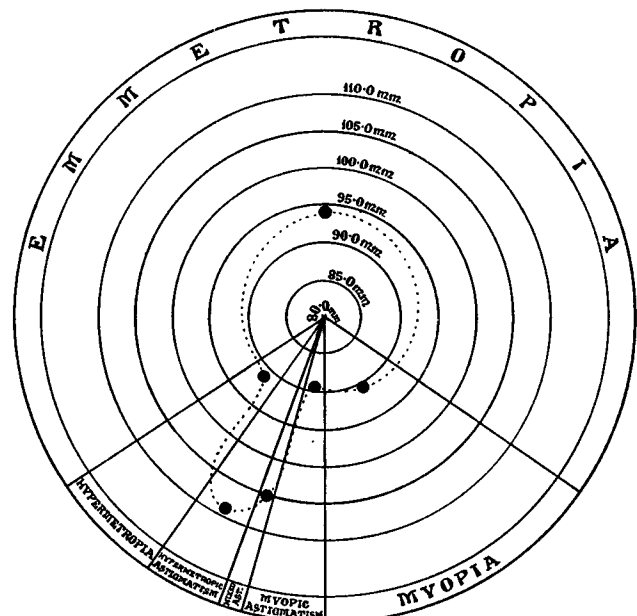


Diagram 79.

We have the following system of array-means:

Refraction Class	Mean Distance of Near Point
Hypermetropic Astigmatism	108.929
Hypermetropia ... ..	91.429
Emmetropia... ..	94.037
Myopia ... ..	90.070
Myopic Astigmatism ...	89.688
Mixed Astigmatism ...	105.313
General Population* ...	93.800

The radiogram in Diagram 79 shows the distribution of near points. For a general measure of the association we have:  $\eta'^2_{NP.RC} = .033,945$ ,  $\bar{\eta}^2_{NP.RC} = .007,143 \pm .003,035$ .

\* Standard Deviation: 20.269 mm.

$\eta'^2_{NP.RC}$  differs therefore significantly for  $\bar{\eta}^2_{NP.RC}$ , and we have:

$$\eta'_{NP.RC} = .1842,$$

corresponding to a definite but not very high degree of association.

(c) *General Refraction.* (i) We take first the General Refraction of Right and Left Eyes. Table CLXXXVIII provides the data.

Table CLXXXVIII. *General Refraction. Right Eye and Left Eye.*

General Refraction. Right Eye

General Refraction. Left Eye		+6.75	+6.00	+5.25	+4.50	+3.75	+3.00	+2.25	+1.50	+0.75	0.00	-0.75	-1.50	-2.25	-3.00	-3.75	-4.50	-5.25	-6.00	-6.75	:	:	:	-12.75	Totals
		+	+	+	+	+	+	+	+	+	—	—	—	—	—	—	—	—	—	—	—	:	:	:	—
+	6.75	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	:	:	:	—	1
+	6.00	—	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	:	:	:	—	3
+	5.25	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	:	:	:	—	1
+	4.50	—	—	—	0.5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	:	:	:	—	1.5
+	3.75	—	—	1	1	2	1.5	1	—	—	—	—	—	—	—	—	—	—	—	—	:	:	:	—	6.5
+	3.00	—	—	—	—	3	2	2	—	—	—	—	—	—	—	—	—	—	—	—	:	:	:	—	7
+	2.25	—	—	—	—	—	—	7.5	4	3.5	0.5	—	—	—	—	—	—	—	—	—	:	:	:	—	15.5
+	1.50	—	—	—	—	1	—	4	3	4	2	—	—	—	—	—	—	—	—	—	:	:	:	—	14
+	0.75	—	—	—	—	1	—	1.5	4	75.5	8.5	1.5	1	—	—	—	—	—	1	—	:	:	:	—	94
+	0.00	—	—	—	—	—	—	1	4	9	197.5	13	—	—	—	—	—	—	—	—	:	:	:	—	220.5
-	0.75	—	—	—	—	—	—	—	—	1	8	24.5	5	0.5	—	—	—	—	—	—	:	:	:	—	39
-	1.50	—	—	—	—	—	—	—	—	—	2	5	7	2	—	—	—	—	—	—	:	:	:	—	16
-	2.25	—	—	—	—	—	—	—	—	—	3.5	1	5.5	4	1	1	—	—	—	—	:	:	:	—	17
-	3.00	—	—	—	—	—	—	—	—	—	—	—	—	1	2	2	—	—	—	—	:	:	:	—	3
-	3.75	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	:	:	:	—	4
-	4.50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	:	:	:	—	—
-	5.25	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	2	—	:	:	:	—	4
-	6.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	:	:	:	—	—
-	6.75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	:	:	:	—	2
-	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
-	15.75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	:	:	:	1	1
Totals		—	1	3	1.5	10	3.5	17	11	93	222	45	19.5	8.5	4	4	—	3	1	2	..	..	..	1	450

The constants of this table are as follows:

Mean, Right Eye: ... + .0975; Left Eye: + .0950.

Standard Deviation, Right Eye: 1.5172; Left Eye: 1.6195.

Product Moment Correlation Coefficient:  $r = .9229 \pm .0047$ .

The association is thus extremely close. The regression lines are shown in Diagram 80, and have for their equations,  $x_R$  and  $y_L$ , being General Refraction of Right and Left Eyes respectively:

$$\tilde{y}_L = .9851x_R - .0010,$$

$$\tilde{x}_R = .8646y_L + .0154.$$

The array-means are as follows:

Grade of General Refraction, R. Eye in Dioptres	Mean General Refraction, L. Eye in Dioptres	Grade of General Refraction, L. Eye in Dioptres	Mean General Refraction, R. Eye in Dioptres
+ 4.26	4.0645	+ 4.702	4.1538
+ 2.38	2.2134	+ 2.48	2.1000
+ 1.50	1.5000	+ 1.50	1.4464
+ 0.75	.7500	+ 0.75	.6503
0.00	— .0287	0.00	— .0034
— 0.75	— .6000	— 0.75	— .6731
— 1.50	— 1.5962	— 1.50	— 1.1719
— 2.25	— 2.2500	— 2.25	— 1.8529
— 3.375	— 3.3750	— 4.291	— 4.3929
— 6.86	— 5.8929		
General Population:	+ .0950	General Population:	+ .0975

(ii) *General Refraction and Acuity of Vision.* This has already been dealt with: see our pp. 153 and 154.

(iii) *General Refraction and Refraction Class.* The same remark applies: see our pp. 165–167.

(iv) *General Refraction and General Astigmatism.* Here as previously General Refraction is taken to be that in the principal meridian nearer to the horizontal, while General Astigmatism is taken

### GENERAL REFRACTION, LEFT EYE & RIGHT EYE

#### ALIEN JEWISH BOYS

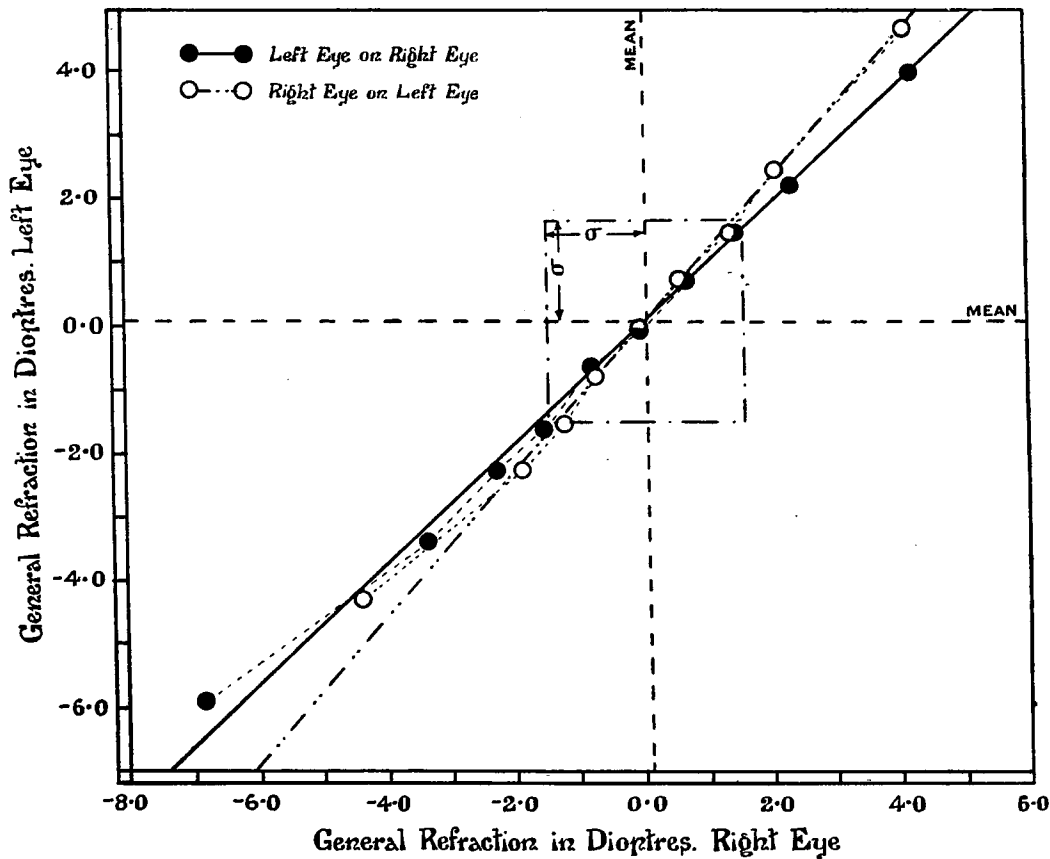


Diagram 80.

as Refraction in the principal meridian nearer to the vertical less Refraction in the principal meridian nearer to the horizontal. The following table gives our data:

Table CLXXXIX. *General Refraction and General Astigmatism.*

General Refraction in Dioptres (Central Values)

General Astigmatism in Dioptres	(Central Values)	+6.75	+6.00	+5.25	+4.50	+3.75	+3.00	+2.25	+1.50	+0.75	0.00	-0.75	-1.50	-2.25	-3.00	-3.75	-4.50	-5.25	-6.00	-6.75	∴ ∴ ∴	-12.75	∴ ∴ ∴	-15.75	Totals
	+ 1.50	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	—	—	∴ ∴ ∴	—	—	—
+ 0.75	—	—	—	—	—	1	—	1	1	6	22	15	3	4	1	—	—	—	—	—	∴ ∴ ∴	—	—	—	54
0.00	—	—	1	2	—	5	4	4	13	146	380	56	24	11	6	5	—	5	—	1	∴ ∴ ∴	—	—	—	663
- 0.75	—	—	1	—	1	1	1	7	4	30	34	9	7	6	—	2	—	—	—	1	∴ ∴ ∴	—	—	—	104
- 1.50	—	—	—	—	—	3	1	7	4	3	3	1	—	1	—	1	—	—	—	—	∴ ∴ ∴	—	—	1	25
- 2.25	—	—	—	—	3	—	3	12	2	1	2	1	—	3	—	—	—	—	1	2	∴ ∴ ∴	1	—	—	31
- 3.00	—	—	—	1	—	3	2	—	—	—	—	—	—	—	—	—	—	—	—	—	∴ ∴ ∴	—	—	—	7
- 3.75	—	—	—	—	—	4	—	—	1	1	—	1	—	—	—	—	—	2	—	—	∴ ∴ ∴	—	—	—	9
- 4.50	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	∴ ∴ ∴	—	—	—	3
- 5.25	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	∴ ∴ ∴	—	—	—	1
Totals		1	4	4	4	17	11	31	25	188	443	83	35	25	7	8	—	7	1	4	∴ ∴ ∴	1	∴ ∴ ∴	1	900

The constants of this table are:

General Refraction, Mean:	+ .1050 D.;	Standard Deviation:	1.5759 D.
General Astigmatism, Mean:	- .2375 D.;	Standard Deviation:	.7407 D.
Product Moment Correlation Coefficient: $r = - .2490 \pm .0085$ .			

We have the following system of array-means:

Grade of Refraction	Mean Astigmatism	Grade of Astigmatism	Mean Refraction
+ 5.365 D.	- 2.3077 D.	+ 0.789 D.	- .3026 D.
+ 3.75 D.	- 1.6765 D.	0.00 D.	+ .0192 D.
+ 2.335 D.	- 1.3571 D.	- 0.75 D.	+ .1587 D.
+ 1.50 D.	- .6600 D.	- 1.50 D.	+ .6300 D.
+ 0.75 D.	- .1676 D.	- 2.386 D.	+ .9671 D.
0.00 D.	- .0339 D.	- 4.038 D.	+ 2.3077 D.
- 0.75 D.	- .0361 D.		
- 1.50 D.	- .0429 D.		
- 2.25 D.	- .3900 D.		
- 5.146 D.	- .7241 D.		
General Population:	- .2375 D.	General Population:	+ .1050 D.

Correlation Ratios:  $\eta'^2_{GR.GA} = .107,073$ ,

$\bar{\eta}^2_{GR.GA} = .010,000 \pm .003,162$ ,

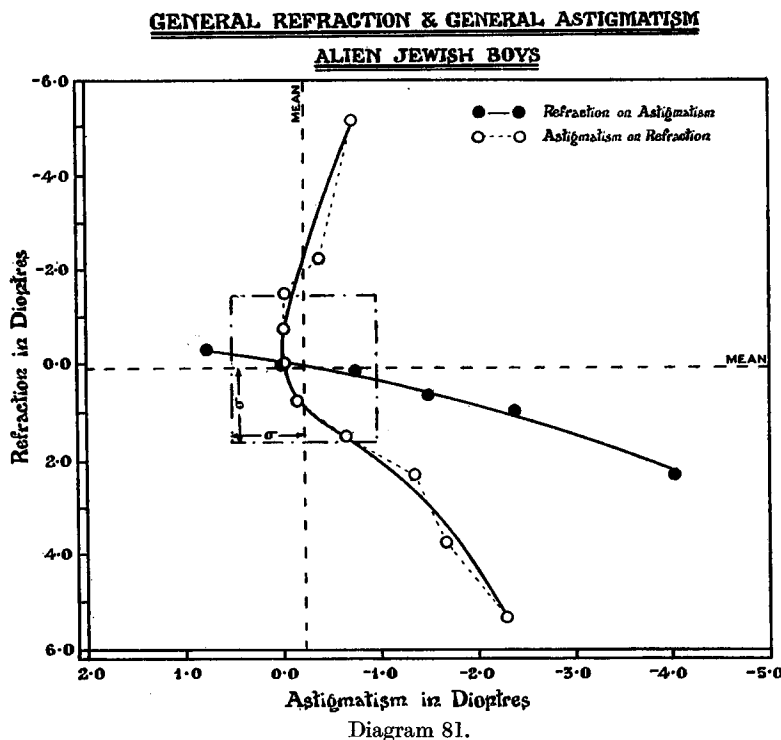
$\eta'^2_{GA.GR} = .403,995$ ,

$\bar{\eta}^2_{GA.GR} = .021,111 \pm .004,568$ .

Thus both  $\eta'^2$ 's are significantly different from their  $\bar{\eta}^2$ 's, and we have:

$\eta'_{GR.GA} = .3272$ ,

$\eta'_{GA.GR} = .6356$ .



It is clear therefore, comparing with the value of  $r$ , that the regressions are not closely linear. An examination of Diagram 81 indicates that for practical purposes it would not be possible to treat the regression of astigmatism on refraction as linear but that linearity of regression is not very far from the truth in the case of the regression of refraction on astigmatism. We have the following regression line in dioptries for the latter case:

Probable General Refraction =  $-.5297 \times$  (given General Astigmatism)  $-.0208D$ .

(iv) *bis*. We can, however, obtain a good deal more information from such a table as the present, remembering that we are here paying attention to the sign of the General Astigmatism. We first note, that if  $A$  represents the General

Astigmatism, then

$$A = R_2 - R_1,$$

where  $R_1$  and  $R_2$  are the refractions in the principal meridians,  $R_1$  corresponding to that in the meridian nearer to the horizontal, i.e. what we have above termed the General Refraction. We can therefore deduce indirectly the Mean and Standard Deviation of  $R_2$  and its correlations with  $A$  and  $R_1$ . We have:

Mean  $R_2 = \text{Mean } A + \text{Mean } R_1 = -\cdot1325 \text{ D.}$

$\sigma_{R_2} = \text{Standard Deviation of } R_2 = \sqrt{\sigma_A^2 + \sigma_{R_1}^2 + 2\sigma_A\sigma_{R_1}r_{AR_1}} = 1\cdot5655 \text{ D.}$

$r_{R_1R_2} = (\sigma_A r_{AR_1} + \sigma_{R_1})/\sigma_{R_2} = \cdot8889,$

$r_{AR_2} = (\sigma_A + \sigma_{R_1}r_{AR_1})/\sigma_{R_2} = +\cdot2225.$

We see that the variability of  $R_2$  is very nearly equal to that of  $R_1$  (1·5759 D.), and so also are their correlation coefficients with  $A$ , but these are naturally of opposite sign. As we have found that  $r_{AR_1}$  is no measure of the relationship of General Astigmatism to General Refraction we may expect that  $r_{AR_2}$  will not be so either. On the other hand, we expect the regression of  $R_2$  on  $A$  will be nearly linear in which case:

Probable Value of  $R_2 = \cdot4703$  (given General Astigmatism)  $-\cdot0208 \text{ D.}$

Comparing with the equation for the probable value of  $R_1$  on p. 174, we deduce, as we ought to:

Probable Value of  $R_2 - R_1 = \text{given General Astigmatism.}$

We have tested the general accuracy of the above results by forming a correlation table for the refractions in the two principal meridians, i.e. for  $R_1$  and  $R_2$ .

Table CXC. *Refractions in the Principal Meridians of the Eye.*

$R_1$  the Refraction in the Principal Meridian nearer to the Horizontal in Dioptres

$R_2$ the Refraction in the Principal Meridian nearest to Vertical in Dioptres	$R_1$ the Refraction in the Principal Meridian nearer to the Horizontal in Dioptres																				Totals
	+6·75	+6·00	+5·25	+4·50	+3·75	+3·00	+2·25	+1·50	+0·75	+0·00	-0·75	-1·50	-2·25	-3·00	-3·75	-4·50	-5·25	-6·00	-6·75	:	
+	6·00	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
+	5·25	—	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
+	4·50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
+	3·75	—	—	—	1	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
+	3·00	—	—	—	1	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	15
+	2·25	1	—	1	3	3	1	4	2	—	—	—	—	—	—	—	—	—	—	—	27
+	1·50	—	1	—	—	1	4	13	5	3	—	—	—	—	—	—	—	—	—	—	189
+	0·75	—	1	1	—	3	3	10	8	144	19	—	—	—	—	—	—	—	—	—	444
+	0·00	—	—	—	—	4	2	12	1	30	383	12	—	—	—	—	—	—	—	—	102
—	0·75	—	—	—	—	—	—	2	4	33	59	3	1	—	—	—	—	—	—	—	43
—	1·50	—	—	—	—	—	—	—	1	3	9	27	3	—	—	—	—	—	—	—	20
—	2·25	—	—	—	—	—	—	—	—	2	1	5	10	1	—	—	—	—	—	—	16
—	3·00	—	—	—	—	—	—	—	—	2	—	—	7	6	—	—	—	—	—	—	10
—	3·75	—	—	—	—	—	—	—	—	—	—	—	3	—	7	—	—	—	—	—	1
—	4·50	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	7
—	5·25	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	5	—	—	—	—
—	6·00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—	6·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
—	7·50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
—	8·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	3
—	9·00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	1	—	1
—	9·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—
—	10·50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	11·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	12·00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	12·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	13·50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	14·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	15·00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	15·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	16·50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	17·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	4	4	4	17	11	31	25	188	443	83	35	25	7	8	—	7	1	4	..	900

The constants of this table are:

$R_1$ , Mean:  $+\cdot1050 \text{ D.} \pm \cdot0354;$

Standard Deviation:  $1\cdot5759 \text{ D.} \pm \cdot0251.$

$R_2$ , Mean:  $-\cdot1342 \text{ D.} \pm \cdot0356;$

Standard Deviation:  $1\cdot5822 \text{ D.} \pm \cdot0252.$

There is thus a very distinct difference in the mean refractions in the horizontal and vertical principal planes, the mean value of  $R_2$  being negative, or the curvature is greater in the vertical

principal meridian than in the horizontal principal meridian. There is no significant difference between the variabilities of  $R_1$  and  $R_2$ . The values of the mean and standard deviation of  $R_2$  are in fairly close agreement with those obtained indirectly\*.

Further, we have:

Product Moment Correlation Coefficient:  $r = .8965 \pm .0044$ .

Correlation Ratios:  $\eta'^2_{R_2.R_1} = .864,867$ ,  $\bar{\eta}^2_{R_2.R_1} = .021,111 \pm .004,568$ ,

$\eta'^2_{R_1.R_2} = .810,097$ ,  $\bar{\eta}^2_{R_1.R_2} = .023,33 \pm .004,797$ .

Thus both  $\eta'^2$ 's are significant when compared with their  $\bar{\eta}^2$ 's and give the high values:

$\eta'_{R_2.R_1} = .9300$ ,  $\eta'_{R_1.R_2} = .9001$ .

Diagram 82 represents the two regression curves. It will be seen that the differences between the correlation coefficient and the two correlation ratios are marked, for neither curve is truly linear. The cubic graduations are:

$$\tilde{R}_2 = -.09451 + .88292R_1 - .05904R_1^2 - .00281R_1^3,$$

and 
$$\tilde{R}_1 = .18442 + .90357R_2 + .02465R_2^2 + .00140R_2^3,$$

giving the probable values of  $\tilde{R}_2$  and  $\tilde{R}_1$  for given values of  $R_1$  and  $R_2$  respectively.

The array-means are given below, all in dioptries.

#### GENERAL REFRACTION $R_1$ & $R_2$ IN PRINCIPAL MERIDIANS

##### ALIEN JEWISH BOYS

In Dioptries	
Grade of $R_1$	Mean of $R_2$
+ 4.45	2.5250
+ 3.00	1.6364
+ 2.25	.8226
+ 1.50	.8700
+ 0.75	.5825
0.00	-.0339
- 0.75	-.8225
- 1.50	- 1.5429
- 2.25	- 2.5800
- 3.40	- 3.4500
- 5.344	- 6.5625
- 9.250	- 10.8750

General Population: — .1342

In Dioptries	
Grade of $R_2$	Mean of $R_1$
+ 4.015	3.9706
+ 2.25	3.3500
+ 1.50	1.5278
+ 0.75	.9206
0.00	-.1419
- 0.75	-.4412
- 1.50	- 1.2384
- 2.25	- 1.6125
- 3.00	- 2.0625
- 3.75	- 3.3000
- 5.156	- 4.1250
- 10.167	- 8.0000

General Population: + .1050

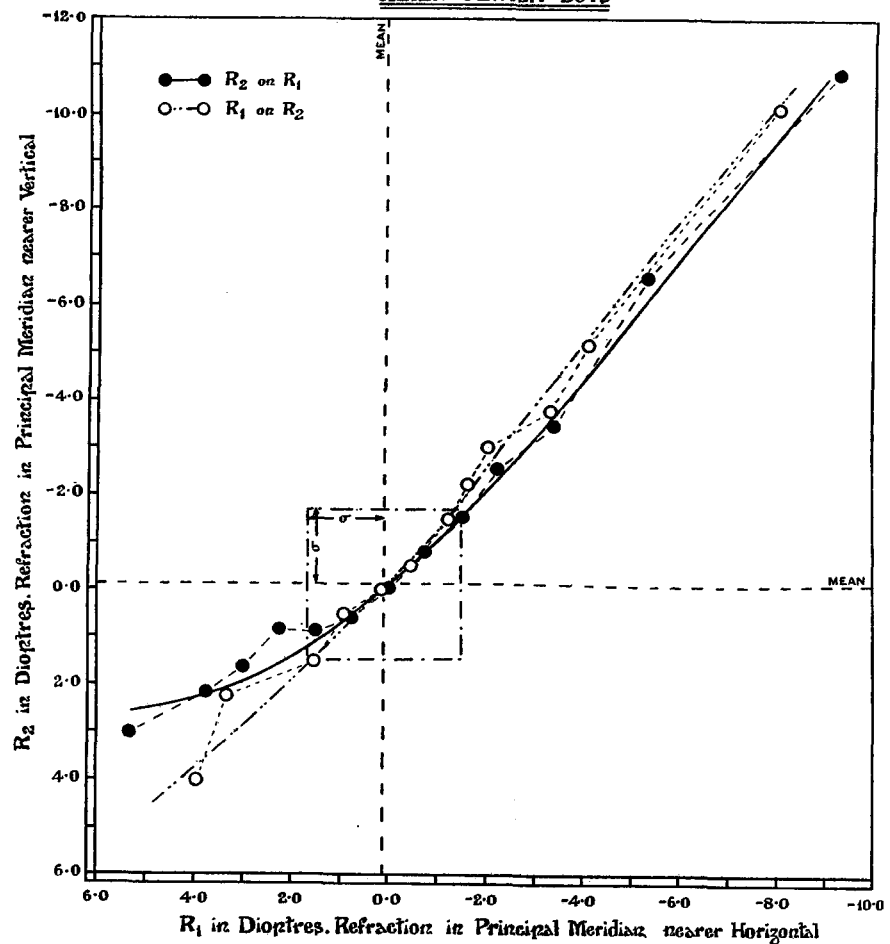


Diagram 82.

\* Theoretically they should agree exactly, but the use of Sheppard's corrections upsets this absolute agreement.

It will be noticed that for positive grades of  $R_1$  the mean  $R_2$  is less than the grade of  $R_1$ , but that for negative grades of  $R_1$ ,  $R_2$  is numerically greater, usually considerably greater, than  $R_1$ . On the other hand, the mean value of  $R_1$  is usually larger than the corresponding value of  $R_2$  for positive grades of the latter, but is numerically less, generally considerably less, for negative grades of  $R_2$ .

(v) *General Refraction and Corneal Refraction.* We shall work this out for  $A$ ,  $B$  and  $C$  observers combined and then for  $A$  and  $B$  alone. For the former we have the table:

Table CXCI. *General Refraction and Corneal Refraction. A, B and C.*

General Refraction in Dioptres (Central Values)															
Corneal Refraction in Dioptres (Central Values)															
+ 6.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
+ 6.00	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
+ 5.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
+ 4.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
+ 3.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
+ 3.00	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
+ 2.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
+ 1.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
+ 0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.00	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
- 0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
- 1.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
- 2.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
- 3.00	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
- 3.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
- 4.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
- 5.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
- 6.00	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
- 6.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Totals	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
47.625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
47.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
46.625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
46.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
45.625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
45.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
44.625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
44.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
43.625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
43.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
42.625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
42.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
41.625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
41.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40.625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
39.625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
39.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
38.625	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
38.125	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Totals	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

The constants of this table are as follows:

General Refraction, Mean: + .1314 D.;

Standard Deviation: 1.4047 D.

Corneal Refraction, Mean: 43.4898 D.;

Standard Deviation: 1.5349 D.

Product Moment Correlation Coefficient:

$$r = - .0994 \pm .0225.$$

#### GENERAL REFRACTION & CORNEAL REFRACTION

(OBSERVERS A, B & C) ALIEN JEWISH BOYS

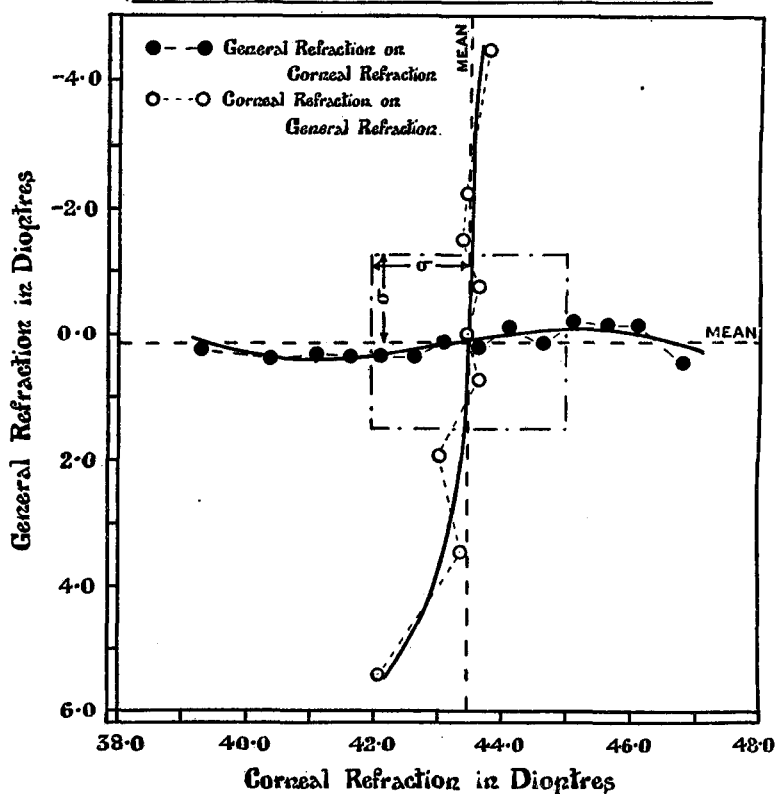


Diagram 83.

Diagram 83 indicates that only the first three and the last array-means, where in all cases we have small numbers, lie off the mean line in the case of the regression of Corneal upon General Refraction.

Indeed we have:

$$\eta'^2_{GR.CR} = \cdot 025,986,$$

$$\bar{\eta}^2_{GR.CR} = \cdot 015,873 \pm \cdot 004,013,$$

and

$$\eta'_{GR.CR} = \cdot 1612,$$

a possibly significant, but very small association.

The other way round we have:

$$\eta'^2_{CR.GR} = \cdot 044,729,$$

$$\bar{\eta}^2_{CR.GR} = \cdot 019,274 \pm \cdot 004,413,$$

thus  $\eta'^2_{CR.GR}$  is significant and  $\eta'_{CR.GR} = \cdot 2115$ .

The array-means are as follows:

Grade of Corneal Refraction	Mean of General Refraction	Grade of General Refraction	Mean of Corneal Refraction
39.325 D.	+ .2250 D.	+ 5.438 D.	42.0833 D.
40.383 D.	+ .3864 D.	+ 3.458 D.	43.3843 D.
41.125 D.	+ .3036 D.	+ 1.942 D.	43.0175 D.
41.625 D.	+ .3571 D.	+ 0.75 D.	43.6609 D.
42.125 D.	+ .3100 D.	0.00 D.	43.4815 D.
42.625 D.	+ .3497 D.	- 0.75 D.	43.6656 D.
43.125 D.	+ .1351 D.	- 1.50 D.	43.4067 D.
43.625 D.	+ .2002 D.	- 2.25 D.	43.4583 D.
44.125 D.	- .1364 D.	- 4.47 D.	43.7850 D.
44.625 D.	+ .1641 D.		
45.125 D.	- .2033 D.		
45.625 D.	- .1563 D.		
46.125 D.	- .1221 D.		
46.807 D.	+ .4318 D.		
General Population:		43.4898 D.	

The corresponding cubics are:

$$\widetilde{C.R.} = 43.5296 - \cdot 022,173 (G.R.) - \cdot 017,227 (G.R.)^2 - \cdot 004,391 (G.R.)^3,$$

$$G.R. = \cdot 11456 - \cdot 07449 (C.R. - 43.125) - \cdot 00049 (C.R. - 43.125)^2 + \cdot 002463 (C.R. - 43.125)^3,$$

both being weighted.

The surprising point is the very small, indeed almost negligible, association of these two refractions. We therefore repeated the inquiry for *A* and *B*'s records only.

Table CXCI. *General Refraction and Corneal Refraction. A and B only.*

Corneal Refraction in Dioptres (Central Values)																			
General Refraction in Dioptres (Central Values)	38.125	..	40.125	40.625	41.125	41.625	42.125	42.625	43.125	43.625	44.125	44.625	45.125	45.625	46.125	46.625	47.125	47.625	Totals
+ 6.75	—	..	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
+ 6.00	—	..	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	2
+ 5.25	—	..	—	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	3
+ 4.50	—	..	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+ 3.75	—	..	—	—	1	—	1	—	—	3	—	1	—	—	—	1	—	—	7
+ 3.00	—	..	—	—	—	—	—	—	1	—	1	3	—	—	1	1	—	—	7
+ 2.25	—	..	1	—	—	1	2	4	2	1	1	1	1	—	2	—	—	—	16
+ 1.50	—	..	—	—	—	—	—	2	1	5	2	2	1	2	1	—	—	—	16
+ 0.75	—	..	2	—	1	4	5	14.5	19.5	22	19	11	8	6	5.5	2	2	—	121.5
0.00	—	..	1	2	2	3	6	5.5	7.5	13	21	13.5	7	9	9.5	2.5	—	1.5	104
- 0.75	—	..	1	—	—	—	2	1	3	4	6	5.5	4	3	5	3	—	0.5	38
- 1.50	1	..	—	—	—	1	1	—	1	—	4	—	3	1	1.5	—	—	—	13.5
- 2.25	—	..	2	—	—	—	—	—	—	2	1	3	1	1	1	—	—	—	11
- 3.00	—	..	—	—	—	—	2	—	1	—	—	—	1	1	1	—	—	—	6
- 3.75	—	..	—	—	—	—	1	1	—	—	2	—	—	—	—	—	—	—	4
- 4.50	—	..	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
- 5.25	—	..	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
- 6.00	—	..	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
- 6.75	—	..	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	2
Totals	1	..	8	2	7	9	20	31	36	50	58	42	26	23	27.5	9.5	2	2	354

While the constants differ somewhat for this shorter series, the general result is the same. Our correlation is very slender and hardly differs from the previous correlation by more than the error of random sampling:

General Refraction, Mean:  $\cdot 2871$  D.; Standard Deviation:  $1\cdot 5709$  D.  
 Corneal Refraction, Mean:  $43\cdot 6250$  D.; Standard Deviation:  $1\cdot 4926$  D.  
 Product Moment Correlation Coefficient:  $r = -\cdot 1358 \pm \cdot 0353$ .

We have for the correlation ratio of General Refraction on Corneal:

$$\eta'^2_{GR.CR} = \cdot 066,574, \quad \bar{\eta}^2_{GR.CR} = \cdot 032,182 \pm \cdot 008,935.$$

Thus  $\eta'^2_{GR.CR}$  is significant and  $\eta'_{GR.CR} = \cdot 2580$ , but although Diagram 83 *bis* suggests some

**GENERAL REFRACTION & CORNEAL REFRACTION**  
**(OBSERVERS A & B ONLY) ALIEN JEWISH BOYS**

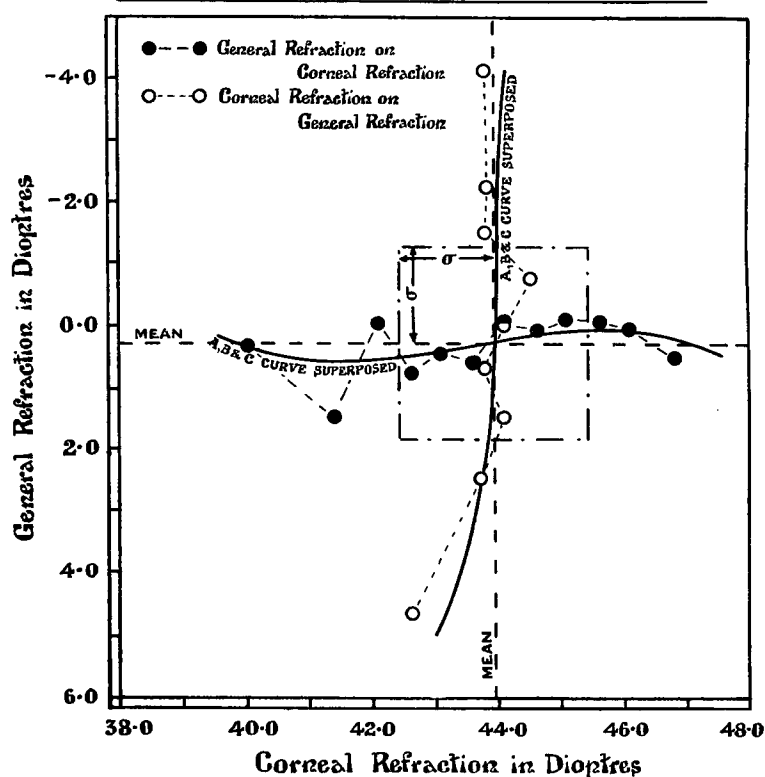


Diagram 83 *bis*.

possible compensating effect in the case of large Corneal Refractions, the data seem to us too slender to justify more than adjusting (i.e. shifting them to the new means) the cubics found for *A*, *B* and *C* to the *A*, *B* regression lines. The main point is the sufficiently startling one that the association of General Refraction with this single factor of it is low. This confirms the result we have previously obtained for the small association of Refraction Class and Corneal Refraction, whereas Corneal Astigmatism was highly associated with Refraction Class.

The suggestion accordingly is that the refraction of the lens is the chief factor in General Refraction, as it is customary to measure the latter, but that Corneal Astigmatism is much more markedly associated with General Astigmatism and so with Refraction Class. We can examine the first part of this suggestion as we have done before. Suppose Total Refraction,  $R'$ , to consist of two parts  $C$  = Corneal Refraction, and  $L$  = Lenticular Refraction of which we at present are ignorant. Then

$R' = C + L^*$  will give us the equation  $\bar{R}' = \bar{C} + \bar{L}$  for the means, but it will not be very helpful

\* This relation is not an accurate one for we do not get the combined refraction by adding  $C$  to  $L$ . If  $C$  and  $L$  be read in dioptries, a more satisfactory formula would be

$$R' = C + L - \frac{a}{100} CL,$$

where  $a$  is the distance in centimetres from the front of the cornea along the axis to the crystalline lens =  $\cdot 38$  cm. about. The mean value of  $C$  is for *A* and *B* only  $43\cdot 625$  D.; for *A*, *B* and *C*  $43\cdot 490$  D. According to Donders the focal length of the lens is  $43\cdot 707$  mm., or its refractive power  $22\cdot 880$  D. Thus the value of the last term is about  $3\cdot 78$  to  $3\cdot 79$  D. and is small as compared to  $C + L$ , which is about  $66\cdot 5$  D. This indicates the amount neglected above. We find  $R'$  accordingly to be about  $62\cdot 73$  D., or the focal length of the total combination  $1\cdot 594$  cms. =  $\cdot 628$  inch, not a bad approximation to the axial length of the vitreous humour. It is convenient to term  $L$  the pseudo-lenticular refraction.

because  $R'$  the total refraction and  $R$  the ophthalmological "General Refraction," while measured in the same units, have different origins. The equation:  $\delta R = -\delta R'$ , or

$$R' - \bar{R}' = -(R - \bar{R}) = C - \bar{C} + L - \bar{L},$$

if we may adopt it as approximate, will however be serviceable; it gives us:

$$\sigma_L = \sqrt{\sigma_R^2 + \sigma_C^2 + 2r_{RC}\sigma_C\sigma_L} = 1.9749 \text{ D.},$$

$$r_{RL} = -\frac{\sigma_R + \sigma_C r_{CR}}{\sigma_L} = -.6340, \quad r_{CL} = -\frac{\sigma_R r_{CR} + \sigma_C}{\sigma_L} = -.7065.$$

These results are, we think, of considerable interest; they indicate that the Lenticular Refraction is nearly six times as closely associated with General Refraction as Corneal Refraction with General Refraction. Thus, while a good deal may be learnt of General Astigmatism from Corneal Astigmatism, relatively little of General Refraction may be ascertained from Corneal Refraction, and this is true, although the correlation of Corneal and Lenticular Refractions is very considerable. We note further that variation in Lenticular Refraction is somewhat higher than in Corneal Refraction.

The equation giving the probable value of the Lenticular Refraction in dioptres for a given General Refraction is:

$$\tilde{L} - \bar{L} = .22883 - .79705R.$$

This will always give the probable deviation of  $L$  from its mean value (*circa* 22.880 D.) when  $R$  is known.

(vi) *General Refraction and Corneal Astigmatism.* The following table gives the records of  $A$  and  $B$  only.

Table CXCHII. *General Refraction and Corneal Astigmatism. A and B only.*

General Refraction in Dioptres (Central Values)																					
Corneal Astigmatism in Dioptres (Central Values)		+ 6.75	+ 6.00	+ 5.25	+ 4.50	+ 3.75	+ 3.00	+ 2.25	+ 1.50	+ 0.75	0.00	- 0.75	- 1.50	- 2.25	- 3.00	- 3.75	- 4.50	- 5.25	- 6.00	- 6.75	Totals
	- 1.50	—	—	—	—	—	—	—	—	1	2	1	—	1	—	—	—	—	—	—	5
	- 0.75	—	—	—	—	—	—	—	—	6	4.5	2.5	—	—	—	—	—	—	—	—	13
	0.00	—	—	1	—	—	—	5	7	26	34	12.5	5.5	4	3	—	—	—	—	—	98
	+ 0.75	—	—	1	—	3	4	3	4	74.5	55	15.5	8	4	3	3	—	1	—	1	180
	+ 1.50	—	—	—	—	—	1	5	2	7	5.5	2.5	—	2	—	1	—	—	—	1	27
	+ 2.25	—	—	—	—	—	—	2	2	5	1	4	—	—	—	—	—	—	—	—	15
	+ 3.00	—	—	—	—	1	2	1	—	1	1	—	—	—	—	—	—	—	—	—	6
	+ 3.75	—	—	1	—	2	—	—	1	1	1	—	—	—	—	—	—	—	—	—	6
	+ 4.50	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
	+ 5.25	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
	Totals	1	2	3	—	7	7	16	16	121.5	104	38	13.5	11	6	4	—	1	1	2	354

We have:

General Refraction, Mean: .2871 D. Standard Deviation: 1.5709 D.

Corneal Astigmatism\*, Mean: .7119 D.; Standard Deviation: .9121 D.

Product Moment Correlation Coefficient:  $r = +.3381 \pm .0318$ .

For the array-means we find:

Grade of Corneal Astigmatism	Mean Refraction	Grade of Refraction	Mean Corneal Astigmatism
- 0.9583 D.	+ .0208 D.	+ 3.79 D.	2.3625 D.
0.00 D.	+ .1110 D.	+ 2.25 D.	1.0781 D.
+ 0.75 D.	+ .1792 D.	+ 1.50 D.	.8906 D.
+ 1.50 D.	+ .2083 D.	+ 0.75 D.	.6451 D.
+ 2.46 D.	+ .7143 D.	0.00 D.	.5012 D.
+ 4.20 D.	+ 3.7500 D.	- 0.75 D.	.5526 D.
		- 1.84 D.	.4286 D.
		- 4.125 D.	.8036 D.
General Population:	+ .2871 D.	General Population:	.7119 D.

\* The above values are regarding sign. Disregarding sign we have: Mean: .8093 D.; Standard Deviation: .8547 D.

Diagram 84 provides graphs of both regression lines. It will be seen that the observations are fairly smooth. The graduating cubics are:

$$\begin{aligned}\tilde{R} &= .09328 + .01100A_c + .04463A_c^2 + .03470A_c^3, \\ \tilde{A}_c &= .52424 + .13010R + .05653R^2 + .00252R^3.\end{aligned}$$

**GENERAL REFRACTION & CORNEAL ASTIGMATISM**  
(OBSERVERS A & B ONLY) ALIEN JEWISH BOYS

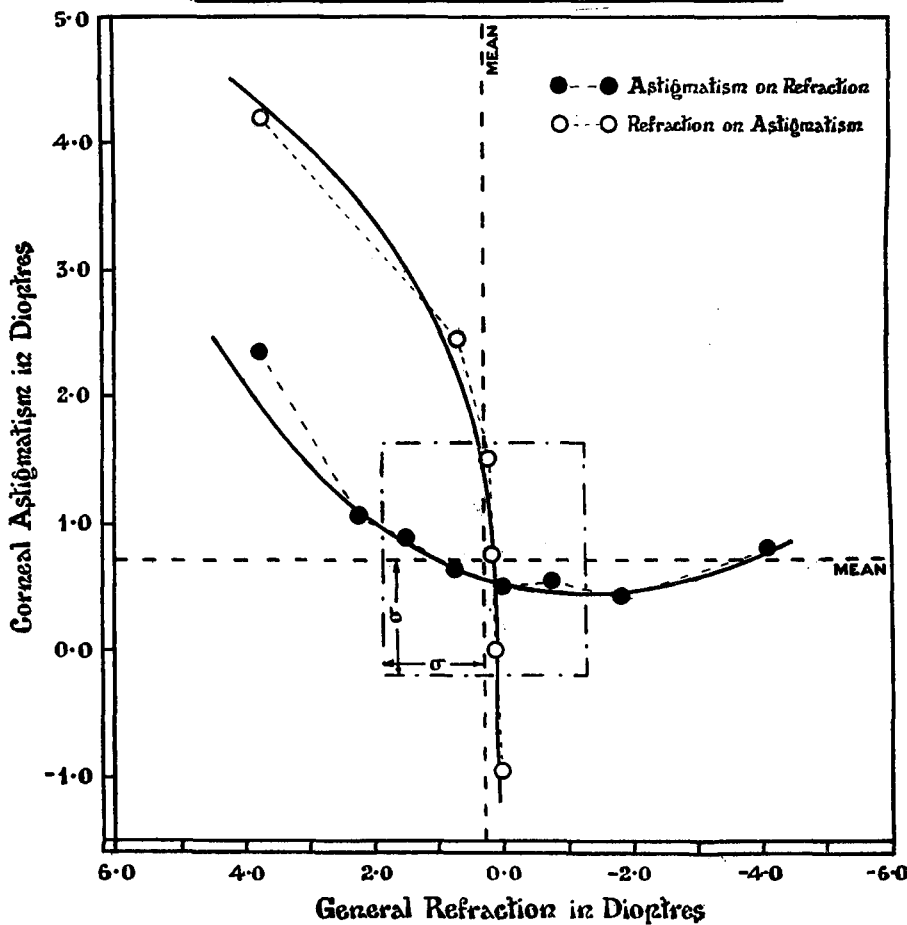


Diagram 84.

We have further for the two Correlation Ratios:

$$\begin{aligned}\eta'_{GR.CA} &= .4445, \\ \eta'_{CA.GR} &= .5756.\end{aligned}$$

Comparing these values with those found (p. 174) for General Astigmatism and General Refraction, i.e.

$$\eta'_{GR.GA} = .3272$$

$$\text{and } \eta'_{GA.GR} = .6356,$$

we conclude that the association of General Refraction with Corneal Astigmatism is as close as with General Astigmatism.

We now turn to the data for *A*, *B* and *C* which are given in Table CXCIV.

The constants of this table are:

General Refraction, Mean:  
+ .1403 D.;

Standard Deviation:  
1.4094 D.

Corneal Astigmatism, Mean:  
+ .5757 D.;

Standard Deviation:  
.8081 D.

Table CXCIV. *General Refraction and Corneal Astigmatism (A, B and C).*

		General Refraction in Dioptres																				
Corneal Astigmatism in Dioptres		+ 6.75	+ 6.00	+ 5.25	+ 4.50	+ 3.75	+ 3.00	+ 2.25	+ 1.50	+ 0.75	0.00	- 0.75	- 1.50	- 2.25	- 3.00	- 3.75	- 4.50	- 5.25	- 6.00	- 6.75	Totals	
	- 1.50	—	—	—	—	—	—	—	—	1	3	1	—	1	—	—	—	—	—	—	—	6
	- 0.75	—	1	—	—	—	—	—	—	6	5	2	—	—	—	—	—	—	—	—	—	14
	0.00	—	—	1	1	1	—	6	7	51	256	34	20	11	4	1	—	2	—	2	—	397
	+ 0.75	—	—	1	—	3	4	4	8	104	154	32	15	7	3	4	—	2	—	—	1	342
	+ 1.50	—	1	—	1	2	4	8	2	12	19	4	—	4	—	2	—	—	—	—	1	60
	+ 2.25	—	—	—	—	2	—	8	2	6	4	6	—	2	—	—	—	2	1	—	—	33
	+ 3.00	—	—	1	1	5	3	3	1	1	1	—	—	—	—	—	—	—	—	—	—	16
	+ 3.75	—	—	1	—	3	—	1	2	1	1	—	—	—	—	—	—	—	—	—	—	9
	+ 4.50	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
	+ 5.25	—	2	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
	Totals		1	4	4	4	17	11	30	22	182	443	79	35	25	7	7	—	6	1	4	882

$$r = +.3664 \pm .0197.$$
$$\begin{aligned}\eta'^2_{GR.CA} &= .202,018, \\ \eta'^2_{CA.GR} &= .327,634,\end{aligned}$$

$$\begin{aligned}\bar{\eta}^2_{GR,CA} &= .010,204 \pm .003,228, \\ \bar{\eta}^2_{CA,GR} &= .019,274 \pm .004,413.\end{aligned}$$

$$\eta'_{GR.CA} = .4495,$$

$$\eta'_{CA.GR} = .5724.$$

Grade of Corneal Astigmatism	Mean General Refraction	Grade of General Refraction	Mean Corneal Astigmatism
- 0.80 D.	+ .3375 D.	+ 5.37 D.	+ 2.4808 D.
0.00 D.	-.1115 D.	+ 3.75 D.	+ 2.4265 D.
+ 0.75 D.	-.0702 D.	+ 3.00 D.	+ 1.6364 D.
+ 1.50 D.	+.5625 D.	+ 2.25 D.	+ 1.5250 D.
+ 2.25 D.	+.2273 D.	+ 1.50 D.	+ 1.0909 D.
+ 3.00 D. { 3.55 D.	+ 2.9063 D. { 3.2000 D.	+ 0.75 D.	+ .6058 D.
+ 4.18 D. { 3.55 D.	+ 3.5357 D. { 3.2000 D.	0.00 D.	+ .3420 D.
		- 0.75 D.	+ 5127 D.
		- 1.50 D.	+ .3214 D.
		- 2.25 D.	+ .5700 D.
		- 4.47 D.	+ .7500 D.
General Population:	+ .1403 D.	General Population:	+ .5757 D.

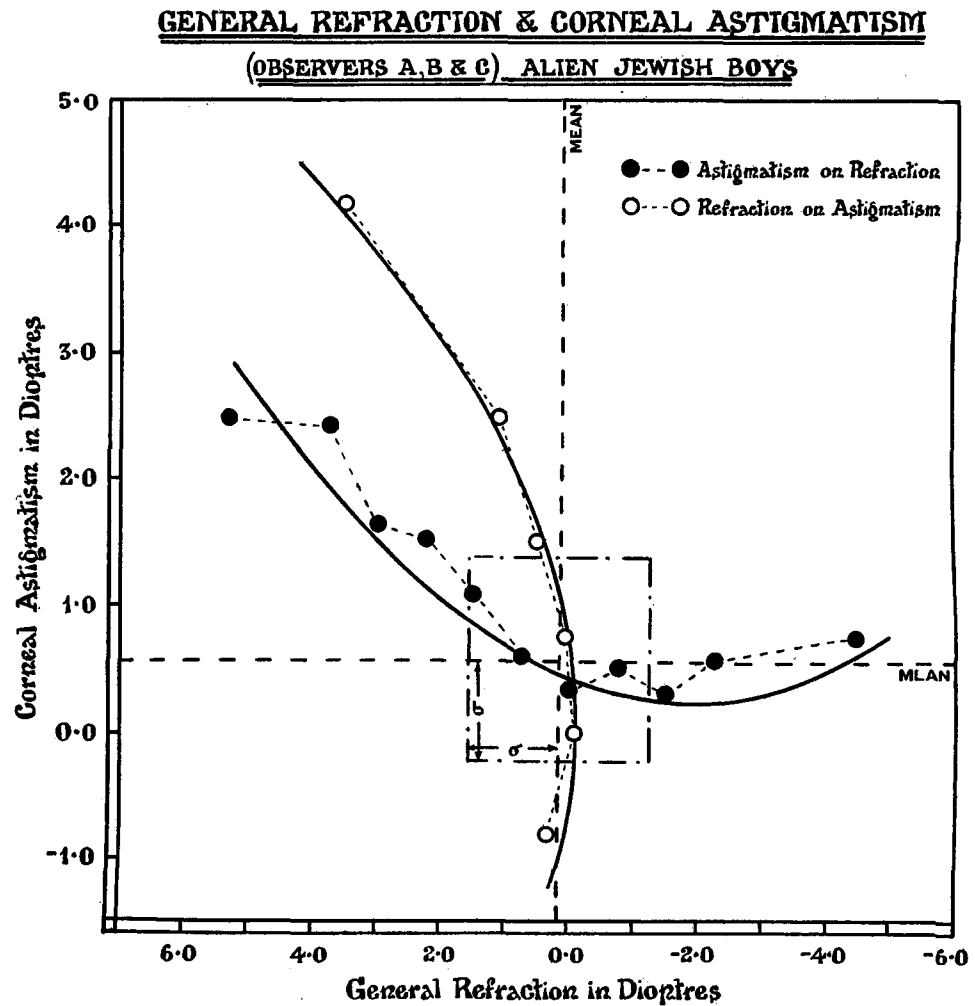


Diagram 85.

$$\begin{aligned} * \tilde{R} &= -.08857 + .00133A_c + .23057A_c^2 - .00366A_c^3, \\ \tilde{A}_c &= .44145 + .19938R + .05177R^2 - .00005R^3. \end{aligned}$$

the other hand, there is hardly any change of General Refraction with Corneal Astigmatism, until the latter is our 2 D., when the General Refraction falls rapidly.

The results for  $A$ ,  $B$  and  $C$ , if we disregard the shift of means, are not widely divergent from those for  $A$  and  $B$  only.

(vii) *General Refraction and Position of the Near Point.* We give our data in Table CXCV

Table CXCV. *Near Point and General Refraction.*

Distance of Near Point (Central Values) in mm.

General Refraction in Dioptres (Central Values)	Distance of Near Point (Central Values) in mm.																				Totals												
	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175	180	185	190	195	200	Totals	
+ 6.75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
+ 6.0	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
+ 5.25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
+ 4.5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
+ 3.75	—	—	—	—	—	1	—	—	—	—	2	1	2	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9
+ 3.0	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
+ 2.25	—	—	—	—	0.5	—	—	—	—	—	2	2	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	17
+ 1.5	—	—	—	—	2	1	1	1	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	16
+ 0.75	—	—	1	1	11.5	5	21	13	14.5	10	6	7	4	6	6	1	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	115
0.00	1	3	7	8	21.5	27	42	44	61.5	52.5	34.5	22	21	15	9	8	9	6	4	4.5	4	—	—	—	—	—	1	—	—	—	—	—	406.5
— 0.75	—	—	—	—	2.5	7.5	7.5	8	6	9.5	4.5	7	2	2	1	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	63.5
— 1.5	—	—	—	—	3.5	4	1	5	1	1	2	—	—	1	—	—	—	2.5	—	0.5	—	—	—	—	—	—	—	—	—	—	—	—	25.5
— 2.25	—	—	—	—	3	1	3	1	2	1	2	—	—	—	—	—	—	2.5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	18.5
— 3.0	—	—	—	—	1	—	—	1	1	—	—	—	—	—	—	1	—	2.5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
— 3.75	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
— 4.5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
— 5.25	—	1	1	2	—	1	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7
— 6.0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
— 6.75	—	—	1	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
Totals	1	6	10	23.5	46.5	45.5	83.5	76	89	78	57	41	29	27	16	13	21	16	7	5	4	—	1	—	1	1	1	—	—	—	1	700	

and there are some interesting reflections which follow upon an examination of this table. The constants are as follows:

General Refraction, Mean:  $-.0107$  D.; Standard Deviation:  $1.2491$  D.  
 Distance of Near Point, Mean:  $93.800$  mm.; Standard Deviation:  $20.269$  mm.  
 Product Moment Coefficient of Correlation:  $r = -.1828 \pm .0246$ .

The array-means are as follows:

Grade of Refraction	Mean Near Point Distance	Grade of Near Point Distance	Mean Refraction
+ 3.97 D.	118.53 mm.	57.6 mm.	$-1.448$ D.
+ 2.25 D.	100.74 "	65 "	$-.8298$ D.
+ 1.50 D.	89.69 "	70 "	$+.0161$ D.
+ 0.75 D.	92.30 "	75 "	$-.1566$ D.
0.00 D.	94.26 "	80 "	$-.1482$ D.
— 0.75 D.	90.67 "	85 "	$-.0987$ D.
— 1.50 D.	90.88 "	90 "	$+.0295$ D.
— 2.25 D.	96.62 "	95 "	$+.0529$ D.
— 3.54 D.	86.00 "	100 "	$+.0724$ D.
— 5.70 D.	75.50 "	105 "	$+.2561$ D.
		110 "	$+.3103$ D.
		115 "	$+.2778$ D.
		122.2 "	$+.2069$ D.
		132.2 "	$+.3953$ D.
		151.9 "	$+.1786$ D.
General Population:		General Population:	

These lead to:  $\eta'^2_{NP,GR} = .058,314$ ,

$\eta'^2_{GR,NP} = .063,671$ ,

Accordingly both  $\eta'^2$ 's are significant having regard to their  $\bar{\eta}^2$ 's, and we have:

$\eta'_{NP,GR} = .2415$ ,

$\eta'_{GR,NP} = .2523$ .

We see that  $\eta'_{NP,GR}$  is substantially larger than the coefficient of correlation  $r$ , and Diagram 86 indicates the reason for this: when the General Refraction lies between  $\pm 2.5$  D. there is hardly any sensible differences in the distance of the Near Point, but as soon as the General Refraction exceeds these limits we find great changes in the Near Point Distance; the hypermetropic may approach 120 mm. and the myopic 75 mm. in the value of this character\*. We have endeavoured to express this relation by graduating with the cubics:

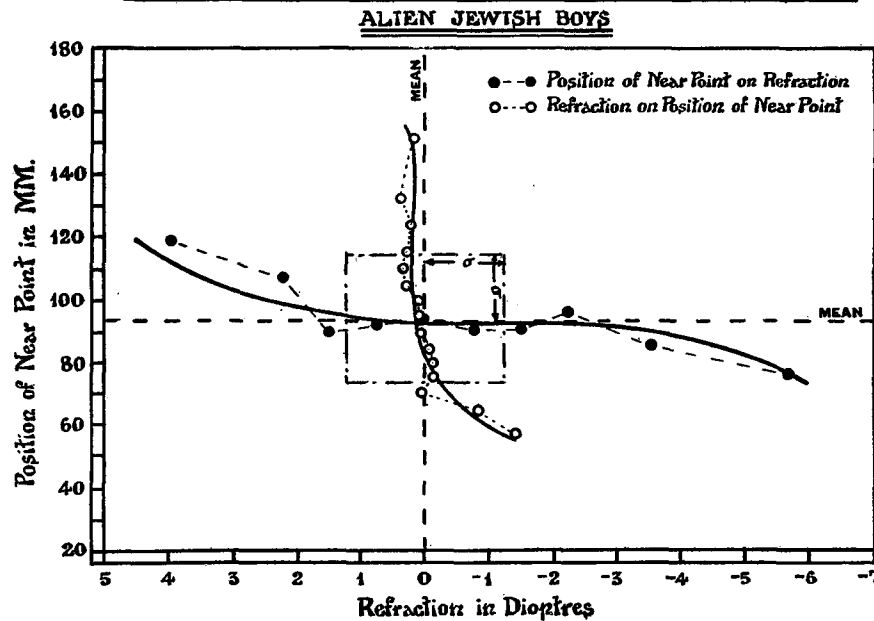
$$\tilde{D}_{NP} = 93.36530 + .44847R + .44684R^2 + .16047R^3,$$

$$\tilde{R} = .08667 + .01309(D_{NP} - 90) - .000,5154(D_{NP} - 90)^2 + .0000,0558(D_{NP} - 90)^3,$$

the units being dioptries and millimetres.

It will be seen by the diagram that the graduations are reasonably successful.

#### POSITION OF NEAR POINT & GENERAL REFRACTION



(d) *General Astigmatism.* (i) *General Astigmatism of Right and Left Eyes.* Our data are presented in Table CXCVI.

Table CXCVI. *General Astigmatism. Right Eye with Left Eye.*

General Astigmatism in Dioptries (Central Values). Right Eye

General Astigmatism in Dioptries (Central Values). Left Eye	+ 2.25	+ 1.50	+ 0.75	0.00	- 0.75	- 1.50	- 2.25	- 3.00	- 3.75	- 4.50	- 5.25	Totals
+ 3.00	0.5	—	—	—	—	—	—	—	—	—	—	0.5
+ 2.25	0.5	—	—	—	—	—	—	—	—	—	—	0.5
+ 1.50	—	1	1	—	—	—	—	—	—	—	—	2
+ 0.75	—	—	13	7.5	2	—	—	—	—	—	—	22.5
0.00	—	0.5	17	296	19	1.5	1	—	—	—	—	335
- 0.75	—	—	3	20.5	23	5	1	—	1	—	—	53.5
- 1.50	—	0.5	—	1	2	3.5	3	1	—	—	—	11
- 2.25	—	—	—	3	2	2	7.5	1	—	—	—	15.5
- 3.00	—	—	—	—	—	—	0.5	2	—	—	—	2.5
- 3.75	—	—	—	—	—	1	1	—	3	—	—	5
- 4.50	—	—	—	—	—	—	—	—	—	1	1	2
Totals	1	2	34	328	48	13	14	4	4	1	1	450

\* Donders (*Anomalies of Accommodation and Refraction of the Eye*, New Sydenham Society, 1864, p. 90) gives for myopes at age 16, 61 mm. and for hypermetropes at age 20, 122 mm.

The constants of this table are:

Right Eye, Mean:  $-.2067 \text{ D.} \pm .0240$ ;

Standard Deviation:  $.7536 \text{ D.} \pm .0169$ .

Left Eye, Mean:  $-.2317 \text{ D.} \pm .0235$ ;

Standard Deviation:  $.7387 \text{ D.} \pm .0166$ .

It is thus not possible on the basis of the above data to assert any significant difference in General Astigmatism, either as to mean or variability between right and left eyes.

Product Moment Correlation Coefficient:  $r = .8593 \pm .0083$ .

The regression straight lines are, the units being dioptries:

$$\tilde{A}_R = .8766A_L - .0036,$$

$$\tilde{A}_L = .8464A_R - .0567.$$

Here  $\tilde{A}_R$  and  $\tilde{A}_L$  are the probable values of the Astigmatism of right and left eyes respectively for given values  $A_L$  and  $A_R$  of the Astigmatism of left and right eyes respectively. Diagram 87 gives the regression lines.

The actual array-means graduated by these lines are:

Grade of General Astigmatism		Mean Astigmatism
Right Eye		Left Eye
+	0.83 D.	+ .3345 D.
-	0.00 D.	- .0549 D.
-	0.75 D.	- .4844 D.
-	1.50 D.	- 1.3269 D.
-	2.25 D.	- 1.9554 D.
-	3.00 D.	- 3.0750 D.
General Population:		- .2317 D.

Grade of General Astigmatism		Mean Astigmatism
Left Eye		Right Eye
+	0.88 D.	+ .5000 D.
-	0.00 D.	- .0157 D.
-	0.75 D.	- .5045 D.
-	1.50 D.	- 1.4318 D.
-	2.25 D.	- 1.5726 D.
-	3.71 D.	- 3.3553 D.
General Population:		- .2067 D.

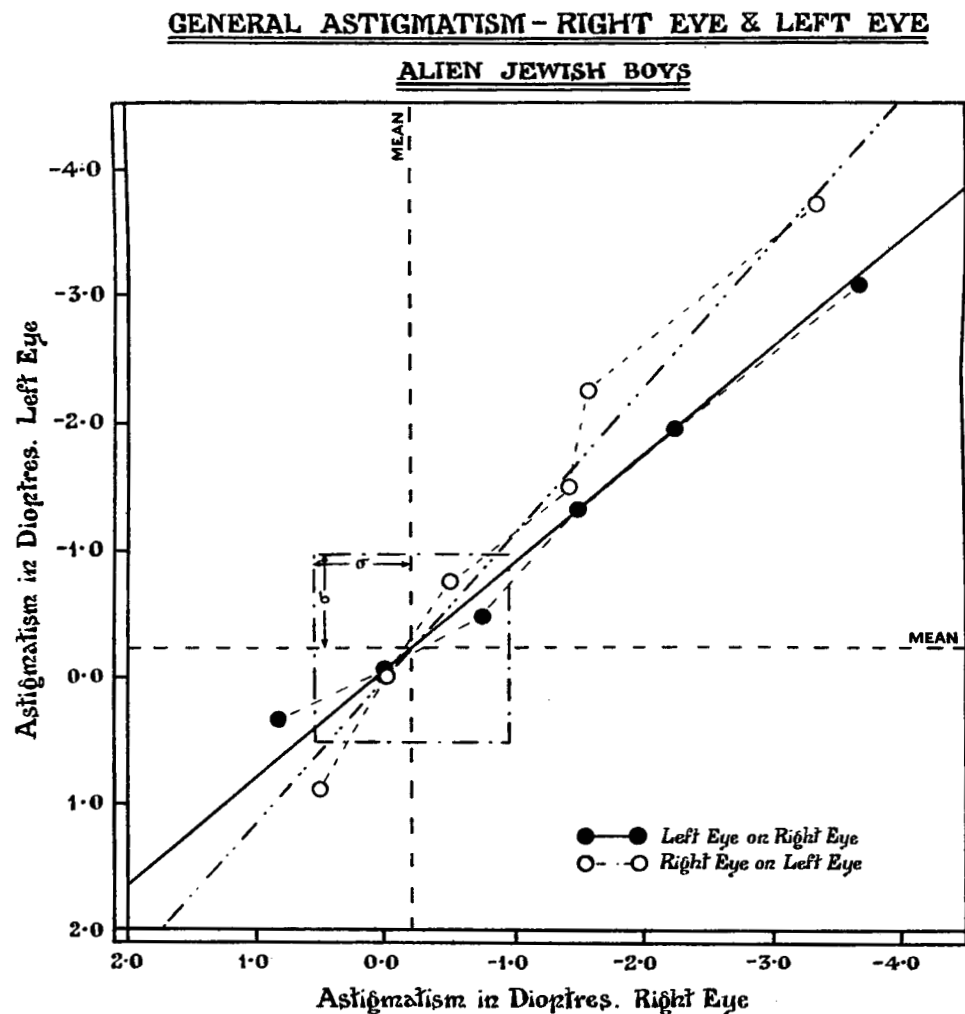


Diagram 87.

We have not worked out the correlation ratios as with such a high value of the correlation coefficient we do not think that very much better graduations would be obtained by the use of cubics.

(ii) *General Astigmatism and Visual Acuity.* This has already been discussed: see pp. 157 and 158.

(iii) *General Astigmatism and Refraction Class.* Already discussed: see pp. 166 and 167.

(iv) *General Astigmatism and General Refraction.* Already discussed: see pp. 173 and 174.

(v) *General Astigmatism and Corneal Refraction.* Our data which have been tabled first for *A* and *B*'s observations only are given in Table CXCVII.

Table CXCVII. *General Astigmatism and Corneal Refraction (A and B only).*

Corneal Refraction in Dioptres (Central Values)

General Astigmatism in Dioptres (Central Values)	38.125	...	40.125	40.625	41.125	41.625	42.125	42.625	43.125	43.625	44.125	44.625	45.125	45.625	46.125	46.625	47.125	47.625	Totals
+ 1.50	—	...	—	—	—	—	—	—	—	—	3	—	—	—	0.5	—	—	—	3.5
+ 0.75	—	...	2	—	1	1	1	2	3	7	4.5	2.5	4	3	6.5	1	0.5	1	40
0.00	1	...	3	2	4	6	16	20	29	34	39	34	17	16	14.5	4.5	1	1	242
- 0.75	—	...	—	—	—	1	1	3	2	5	6.5	2.5	4	4	3.5	3	0.5	—	36
- 1.50	—	...	—	—	—	—	—	2	—	—	3	1	1	—	2.5	1	—	—	10.5
- 2.25	—	...	1	—	—	1	1	2	1	1	2	—	—	—	—	—	—	—	9
- 3.00	—	...	1	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	3
- 3.75	—	...	—	—	1	—	1	—	1	3	—	—	—	—	—	—	—	—	6
- 4.50	—	...	1	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	3
- 5.25	—	...	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	1	...	8	2	7	9	20	31	36	50	58	42	26	23	27.5	9.5	2	2	354

The constants of this table are as follows:

General Astigmatism, Mean:  $-.2203$  D.; Standard Deviation:  $.8984$  D.

Corneal Refraction, Mean:  $43.9477$  D.; Standard Deviation:  $1.4916$  D.

Product Moment Correlation Coefficient:  $r = +.0726 \pm .0357$ .

The array-means are as follows:

Grade of General Astigmatism	Mean Corneal Refraction	Grade of Corneal Refraction	Mean General Astigmatism
+ 0.81 D.	44.269 D.	40.4583 D.	-.8750 D.
0.00 D.	43.897 D.	41.9698 D.	-.2845 D.
- 0.75 D.	44.507 D.	42.625 D.	-.5806 D.
- 1.85 D.	43.766 D.	43.125 D.	-.1488 D.
- 3.87 D.	42.548 D.	43.625 D.	-.2400 D.
		44.125 D.	-.1034 D.
		44.625 D.	-.1786 D.
		45.3596 D.	-.0459 D.
		46.125 D.	-.0273 D.
		46.8472 D.	-.1667 D.
General Population:	43.948 D.	General Population:	-.2203 D.

We find:  $\eta'^2_{CR.GA} = .053,944$ ,  $\bar{\eta}^2_{CR.GA} = .011,299 \pm .003,864$ ,  
 $\eta'^2_{GA.CR} = .054,185$ ,  $\bar{\eta}^2_{GA.CR} = .025,424 \pm .007,981$ .

Both  $\eta'^2$ 's are significant having regard to their  $\bar{\eta}^2$ 's, and we have:

$$\eta'_{CR.GA} = .2323, \quad \eta'_{GA.CR} = .2328.$$

These values, the array-means and the value of  $r$  indicate that the association, if significant, is small and of little prognostic value.

Diagram 88 shows the array-means both ways. General Astigmatism makes little sensible difference to the Corneal Refraction until it exceeds  $-2$  D., and it is not till the Corneal Refraction falls below 43 D. that its influence on General Astigmatism becomes really apparent. We have contented ourselves with fitting the regression lines with the cubics which graduate *A*, *B* and *C*'s observations shifting them to *A* and *B*'s mean values.

Here as elsewhere we are desirous of increasing the number of our records, but can only do this by adding in *C*'s observations, which show definite deviations from those of *A* and *B*; these deviations may be due either to the relative personal equation of the observers or to *C*'s boys

### GENERAL ASTIGMATISM & CORNEAL REFRACTION

(OBSERVERS A & B ONLY) ALIEN JEWISH BOYS

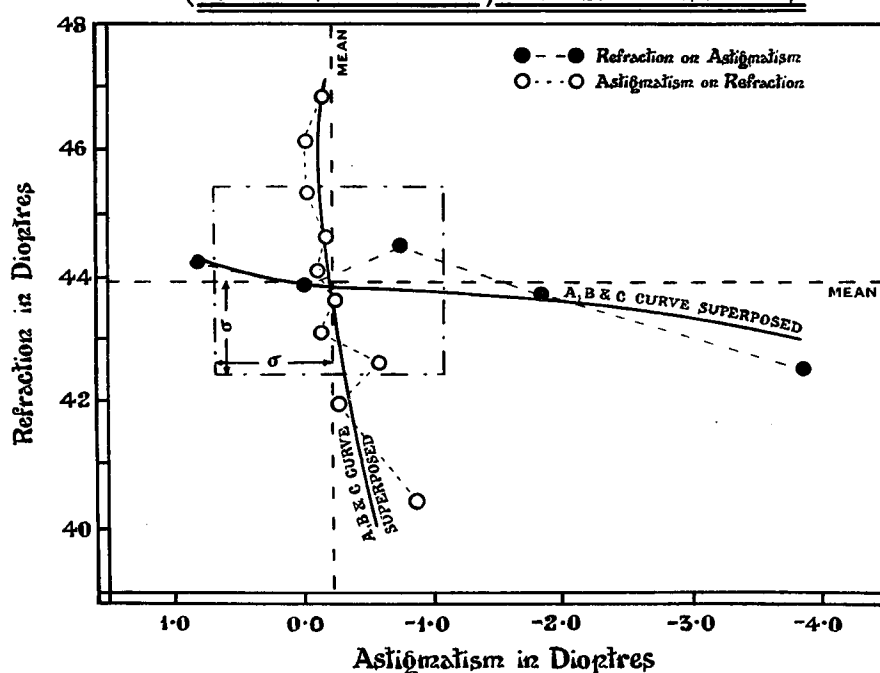


Diagram 88.

being on the whole a different series to those of *A* and *B*. Adding in *C*'s data we have the following table:

Table CXCVIII. *General Astigmatism and Corneal Refraction (A, B and C).*

Corneal Refraction in Dioptres (Central Values)

General Astigmatism in Dioptres (Central Values)	38-125	38-625	39-125	39-625	40-125	40-625	41-125	41-625	42-125	42-625	43-125	43-625	44-125	44-625	45-125	45-625	46-125	46-625	47-125	47-625	Totals
+ 1.50	—	—	—	—	—	—	1	2	1	4	7	12	3	—	—	—	—	—	—	—	3
+ 0.75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	54
0.00	1	—	—	2	3	7	16	27	31	59	66	89	88	92	63	37	36	26	7	1	652
- 0.75	—	—	1	2	2	—	9	2	6	9	9	11	14	8	10	8	6	3	1	—	101
- 1.50	—	—	—	1	1	1	1	—	1	4	2	—	3	3	1	—	3	1	—	—	22
- 2.25	—	—	—	—	1	1	2	6	5	4	3	3	2	2	1	1	—	—	—	—	30
- 3.00	—	—	—	—	2	—	—	—	2	—	—	—	1	2	—	—	—	—	—	—	7
- 3.75	—	—	—	—	—	—	—	1	1	1	—	1	3	1	1	—	—	—	—	—	9
- 4.50	—	—	—	—	1	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	3
- 5.25	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	1	—	3	6	16	17	42	42	75	89	111	118	121	80	54	48	43	12	2	2	882

The constants of this table are as follows:

Corneal Refraction, Mean: 43.4889 D.; Standard Deviation: 1.5335 D.

General Astigmatism, Mean: -0.2321 D.; Standard Deviation: 0.7410 D.

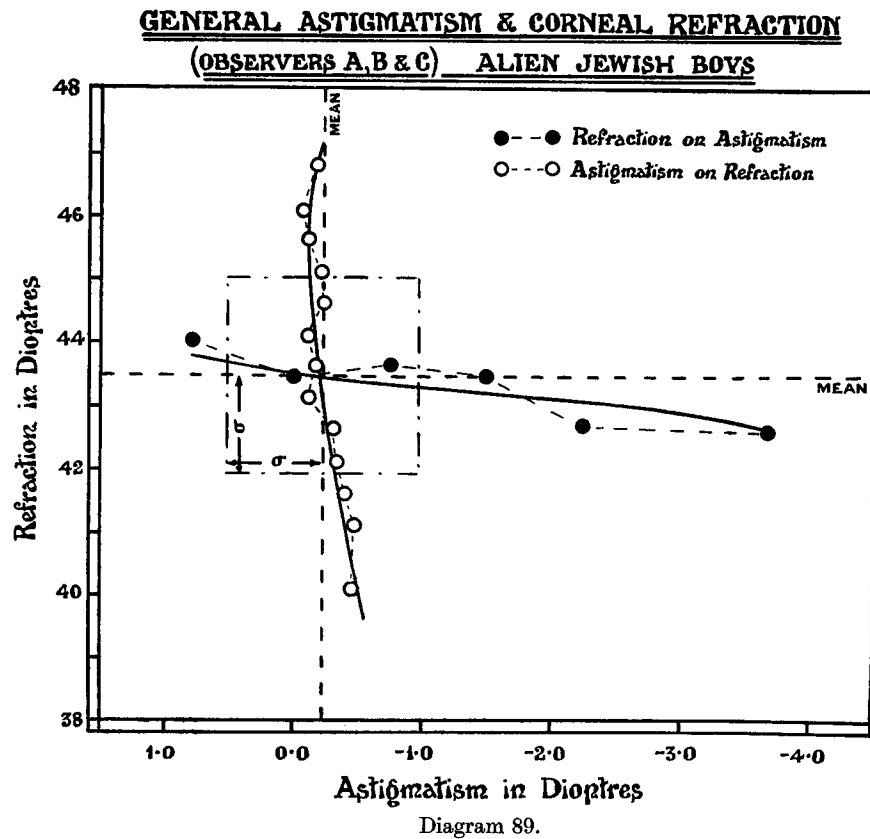
Product Moment Correlation Coefficient:  $r = +0.1293 \pm 0.0223$ .

Correlation Ratios:  $\eta^2_{CR,GA} = 0.029,520$ ,  $\bar{\eta}^2_{CR,GA} = 0.010,204 \pm 0.003,210$ ,

$\eta^2_{GA,CR} = 0.039,260$ ,  $\bar{\eta}^2_{GA,CR} = 0.020,408 \pm 0.004,492$ .

$$\eta'_{CR.GA} = .1718 \text{ and } \eta'_{GA.CR} = .1981.$$

Grade of Corneal Refraction	Mean General Astigmatism	Grade of General Astigmatism	Mean Corneal Refraction
39-325 D.	—	+ 0.79 D.	44-0285 D.
40-383 D.	—	— 0.00 D.	43-4755 D.
41-125 D.	—	— 0.75 D.	43-6745 D.
41-625 D.	—	— 1.107 D.	43-4886 D.
42-125 D.	—	— 2.25 D.	42-7250 D.
42-625 D.	—	— 3.675 D.	42-6000 D.
43-125 D.	—		
43-625 D.	—		
44-125 D.	—		
44-625 D.	—		
45-125 D.	—		
45-625 D.	—		
46-125 D.	—		
46-813 D.	—		
General Population:	— .3321 D.	General Population:	43-4889 D.


$$\begin{aligned}\widehat{G.A.} &= -\cdot 19552 + \cdot 06915 (C.R. - 43\cdot 625) - \cdot 01383 (C.R. - 43\cdot 625)^2 - \cdot 00202 (C.R. - 43\cdot 625)^3, \\ \widehat{C.R.} &= 43\cdot 5346 + \cdot 26422 (G.A.) + \cdot 07196 (G.A.)^2 + \cdot 01818 (G.A.)^3.\end{aligned}$$

**Product Moment Correlation Coefficient:**  $r = -.7578 \pm .0153$ .

\* The reader must bear in mind that the components of Corneal Astigmatism are determined from measurement of the curvatures, but the components of General Astigmatism are measures of the correcting lenses. Hence positive Corneal Astigmatism corresponds to negative General Astigmatism

Table CXCIX. *General Astigmatism and Corneal Astigmatism (A and B only).*

Corneal Astigmatism in Dioptres (Central Values)

General Astigmatism in Dioptres (Central Values)		1.50	0.75	0.00	0.75	1.50	2.25	3.00	3.75	4.50	5.25	6.00	Totals
		-	-	-	+	+	+	+	+	+	+	+	
+ 1.50	2	—	1	—	—	—	—	—	—	—	—	—	3
+ 0.75	—	2.5	16.5	16.5	1.5	2	1	—	—	—	—	—	40
0.00	3	9	75	139	12	3	1	—	—	—	—	—	242
- 0.75	—	1.5	6.5	20.5	4.5	3	—	—	—	—	—	—	36
- 1.5	—	—	—	3	7	1	—	—	—	—	—	—	11
- 2.25	—	—	—	—	2	5	1	1	—	—	—	—	9
- 3.0	—	—	—	—	—	—	2	1	—	—	—	—	3
- 3.75	—	—	—	—	—	1	1	3	1	—	—	—	6
- 4.5	—	—	—	—	—	—	—	1	1	1	—	—	3
- 5.25	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	5	13	99	179	27	15	6	6	2	2	—	—	354

Thus there is a very high relation between the two measures of Astigmatism, far higher than between General Refraction and Corneal Refraction. Assuming that we may neglect the small corrective terms\* we may write  $A_G = -(A_C + A_L)$ , where  $A_G$  is the General Astigmatism and  $A_C$ ,  $A_L$  the Corneal and pseudo-Lenticular components.

Hence we deduce  $\sigma_{A_L} = \sqrt{\sigma_{A_G}^2 + \sigma_{A_C}^2 + 2\sigma_{A_G}\sigma_{A_C}r_{A_G A_C}} = .6301 \text{ D.}$ ,

$$r_{A_G A_L} = -(\sigma_{A_G} + \sigma_{A_C}r_{A_G A_C})/\sigma_{A_L} = -.3263,$$

$$r_{A_C A_L} = -(\sigma_{A_G}r_{A_G A_C} + \sigma_{A_C})/\sigma_{A_L} = -.3695.$$

These correlations are quite appreciable and suggest that, although the correlation of Corneal Astigmatism with General Astigmatism is considerably greater than that of Lenticular Astigmatism with General Astigmatism, the lens does contribute to the General Astigmatism. Further the negative value of  $r_{A_G A_L}$  indicates that there are physical factors at work which cause the astigmatism of the lens to compensate for that of the cornea or *vice versa*.

If we consider the array-means we find:

Grade of Corneal Astigmatism	Mean General Astigmatism	Grade of General Astigmatism	Mean Corneal Astigmatism
- 0.958 D.	+ .2083 D.	+ 0.8023 D.	+ .4012 D.
0.00 D.	+ .0909 D.	0.00 D.	+ .4990 D.
+ 0.75 D.	- .0419 D.	- 0.75 D.	+ .7708 D.
+ 1.50 D.	- .6389 D.	- 1.50 D.	+ 1.3636 D.
+ 2.464 D.	- 1.3571 D.	- 2.438 D.	+ 2.5625 D.
+ 4.200 D.	- 3.9000 D.	- 4.125 D.	+ 3.9750 D.
General Population:	- 22.46 D.	General Population:	+ .7097 D.

Passing to the correlation ratios we have:

$$\eta'^2_{GA.CA} = .651,931, \quad \bar{\eta}^2_{GA.CA} = .014,124 \pm .005,974,$$

$$\eta'^2_{CA.GA} = .567,726, \quad \bar{\eta}^2_{CA.GA} = .014,124 \pm .005,974.$$

Thus both  $\eta'^2$ 's are significant as compared with the  $\bar{\eta}^2$ 's, and

$$\eta'_{GA.CA} = .8074, \quad \eta'_{CA.GA} = .7535\dagger.$$

The regression lines as shown in Diagram 90 differ very considerably from straight lines and have accordingly been fitted with the unweighted cubics:

$$\widetilde{G.A.} = .26773 - .37393 (C.A.) - .20153 (C.A.)^2 + .01666 (C.A.)^3,$$

$$\widetilde{C.A.} = .48939 - .35200 (G.A.) + .23498 (G.A.)^2 + .02513 (G.A.)^3.$$

\* See remarks, p. 179.

† This would be raised above  $r$ , if corrected for class-index correlation.

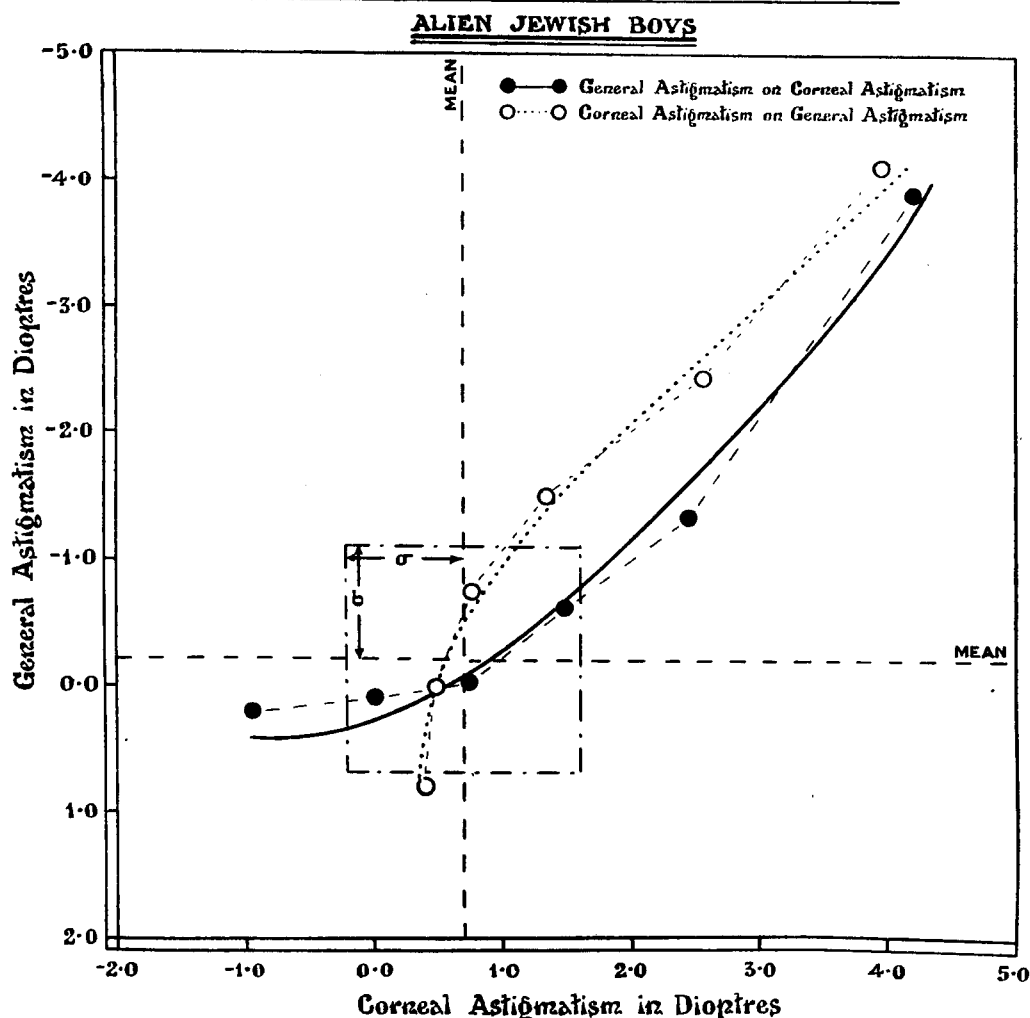
The high relations we have found between General and Corneal Astigmatism are so interesting, and to some extent so unexpected, that it seems important to check them on the larger numbers available if we include *C*'s data. We propose to work out the regression curve of General Astigmatism on Corneal Astigmatism for the combined data. These are provided in Table CC below. The constants of this table are as follows:

General Astigmatism, Mean:  $- \cdot 2111$  D.; Standard Deviation:  $\cdot 7374$  D.

Corneal Astigmatism, Mean:  $+ \cdot 5853$  D.; Standard Deviation:  $\cdot 8157$  D.

Product Moment Correlation Coefficient:  $r = - \cdot 6947 \pm \cdot 0116$ .

**GENERAL ASTIGMATISM**  
**& CORNEAL ASTIGMATISM (OBSERVERS A & B ONLY)**



Correlation Ratio: General upon Corneal Astigmatism:

$$\eta'^2_{GA.CA} = \cdot 543,977, \quad \bar{\eta}^2_{GA.CA} = \cdot 007,726 \pm \cdot 002,774,$$

$\eta'^2_{GA.CA}$  is accordingly significant and  $\eta'_{GA.CA} = \cdot 7375$ . The association is therefore not quite as great as when we take *A* and *B*'s records only, but is still very considerable. Applying the same method as we used for the data of *A* and *B* only we deduce:

$$\sigma_{A_L} = \cdot 6111 \text{ D.}, \quad r_{A_G A_L} = - \cdot 2794, \quad r_{A_Q A_L} = - \cdot 4965.$$

These results confirm in general the previous ones, they still show the lens contributing to the General Astigmatism and also the compensatory effects of the Corneal and Lenticular Astigmatisms.

Table CC. *General Astigmatism and Corneal Astigmatism (A, B and C).*

Corneal Astigmatism in Dioptres (Central Values)												
General Astigmatism in Dioptres (Central Values)		— 1.50	— 0.75	0.00	+ 0.75	+ 1.50	+ 2.25	+ 3.00	+ 3.75	+ 4.50	+ 5.25	Totals
	+ 2.25	—	—	—	—	.5	—	.5	—	.5	—	1.5
	+ 1.50	2	—	1	—	1	—	—	—	—	—	4
	+ 0.75	—	3.75	20.5	25.75	4.5	3	1	—	—	—	58.5
	0.00	4	11	357	264	27	7	2	—	—	—	672
	— 0.75	—	.75	22.5	55.75	12.5	9	—	—	—	—	100.5
	— 1.5	—	—	—	5	12	3	1	—	—	—	21
	— 2.25	—	—	1	1	6.5	11	5.5	3	.5	—	28.5
	— 3.0	—	—	—	—	—	—	6	1	—	—	7
	— 3.75	—	—	—	—	—	3	1	3	1	1	9
	— 4.5	—	—	—	—	—	—	—	1	1	1	3
	— 5.25	—	—	—	—	—	—	—	—	—	1	1
	Totals	6	15.5	402	351.5	64	36	17	8	3	3	906

than Corneal Astigmatism is, then it cannot be true as is often stated that "the cause of regular Astigmatism in the great majority of cases is a congenital irregularity of the curvature of the Cornea." This result might easily be reached because Corneal Astigmatism is such a close measure of Lenticular that it may be taken as a measure of the whole Astigmatism.

We have the following system of array-means:

Grade of Corneal Astigmatism	Mean of General Astigmatism	Grade of General Astigmatism	Mean of Corneal Astigmatism
- 1.50 D.	+ .5000 D.	+ 2.25 D.	+ 3.0000 D.*
- 0.75 D.	+ .1452 D.	+ 1.50 D.	- .3750 D.
0.00 D.	- .0056 D.	+ 0.75 D.	+ .5641 D.
+ 0.75 D.	- .0917 D.	0.00 D.	+ .3661 D.
+ 1.50 D.	- .5625 D.	- 0.75 D.	+ .7985 D.
+ 2.25 D.	- 1.2500 D.	- 1.50 D.	+ 1.5000 D.
+ 3.00 D.	- 1.9853 D.	- 2.25 D.	+ 2.2895 D.
+ 4.23 D.	- 3.3750 D.	- 3.00 D.	+ 3.1071 D.
		- 3.75 D.	+ 3.4167 D.
		- 4.50 D.	+ 4.5000 D.
		- 5.25 D.	+ 5.2500 D.
General Population:	- .2111 D.	General Population:	+ .5853 D.

These means are shown in Diagram 91 and the regression line of General on Corneal Astigmatism is there graduated with the unweighted cubic†:

$$\widetilde{G.A.} = + .01326 - .34624 (C.A.) - .03673 (C.A.)^2 - .018098 (C.A.)^3,$$

C.A. being read in dioptres and the answer given in the same.

We note that the means for General Astigmatism of A, B and of A, B and C are not widely different, i.e. - .2246 D. and - .2111 D. respectively. On the other hand, A, B and A, B and C differ considerably in their Mean Corneal Astigmatism, i.e. .7097 D. and .5853 D. respectively. Thus

\* No stress whatever must be laid on this value; it arises from the tabulator having divided into positive and negative Corneal Refractions the three Astigmatics with principal axis at 45°.

† With such small frequencies as occur in many of the arrays compared with the very large frequencies in two of them, weighted fitting is not satisfactory as a graduation. We give, however, the corresponding weighted cubic:

$$\widetilde{G.A.} = .05849 - .16619 (C.A.) - .18513 (C.A.)^2 + .00887 (C.A.)^3.$$

See Diagram 91.

we have for  $C$ 's mean  $\cdot 5055$  D. and his standard deviation  $\cdot 5405$  D. This suggests that the boys examined by  $C$  were less astigmatic and more concentrated round the zero of astigmatism than those examined by  $A$  and  $B$ . For the combined data of  $A$ ,  $B$  and  $C$  we find:

$$\eta'^2_{CA.GA} = \cdot 557,349, \quad \bar{\eta}^2_{CA.GA} = \cdot 011,111 \pm \cdot 003,331,$$

leading to the high and significant value  $\eta'_{CA.GA} = \cdot 7466$ .

**GENERAL ASTIGMATISM  
& CORNEAL ASTIGMATISM. (OBSERVERS A, B & C)**

**ALIEN JEWISH BOYS**

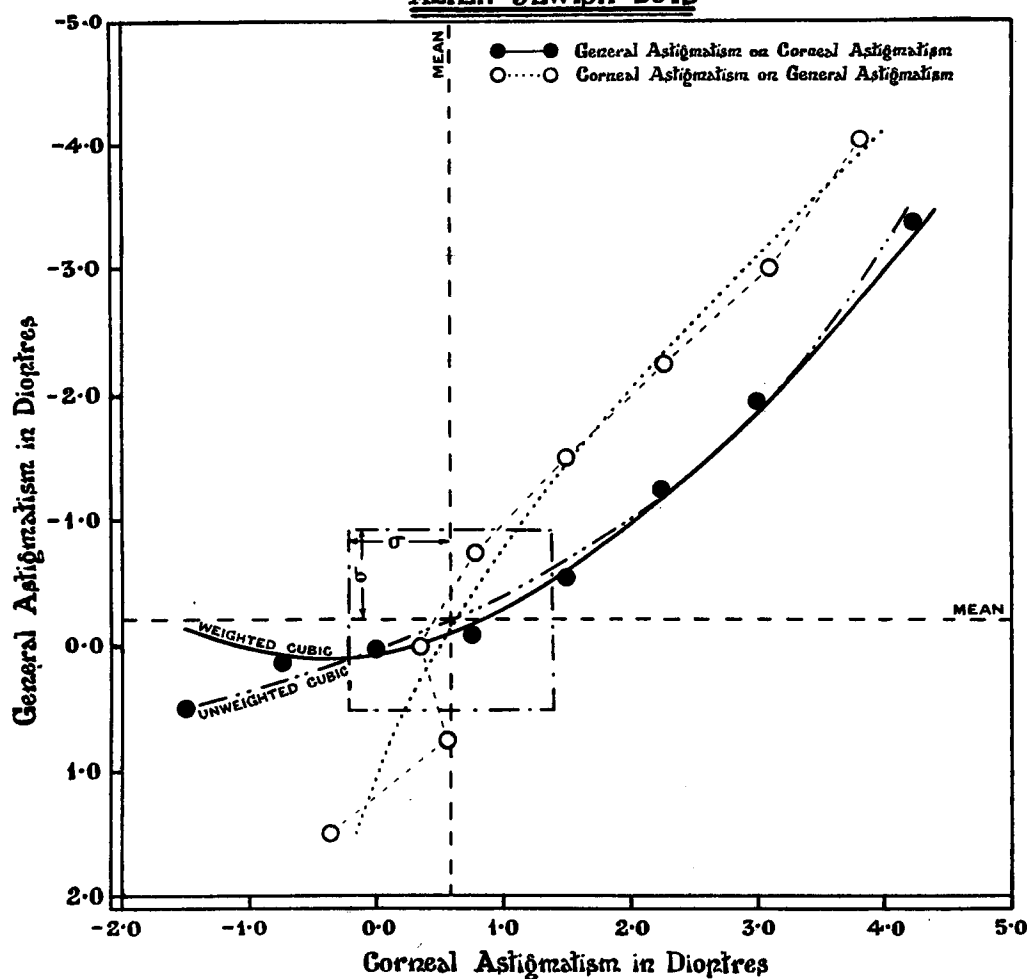


Diagram 91.

This is only slightly less than that found for  $A$  and  $B$  only. An examination of the means of the arrays (p. 191) shows that the Corneal Astigmatism for General Astigmatism *with the rule* is numerically almost exactly equal to the General Astigmatism itself, although, of course, the reverse is not true.

It will be seen from the means on p. 189 that this is very nearly true also for the data of  $A$  and  $B$  only.

The cubic regression line for Corneal on General Astigmatism for  $A$ ,  $B$  and  $C$ 's data is

$$\widehat{C.A.} = \cdot 50411 - \cdot 59097 (G.A.) + \cdot 08407 (G.A.)^2 + \cdot 00486 (G.A.)^3,$$

the points being unweighted.

(vii) *General Astigmatism and the Distance of Near Point.* Our data are given in Table CCI below.

Table CCI. *General Astigmatism and Distance of Near Point.*

Distance of Near Point in mm.

General Astigmatism in Dioptries (Central Values)	Distance of Near Point in mm.																				Totals
	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
	1.50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
	0.75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
	0.00	1	4	7	20	37	65	58	68	64	45	29	20	18	15	9	15	13	5	3	
	-0.75	—	0.5	1	1.5	6	4	10	10	12	5.5	8.5	7	3	5	1	3	2	1	1	
	-1.50	—	—	—	0.5	0.5	—	1	—	2	—	—	3	1	2	—	—	—	—	—	
	-2.25	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
	-3.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
	-3.75	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
Totals	1	6	10	23.5	46.5	45.5	83.5	76	89	78	57	41	29	27	16	13	21	16	7	5	700

The constants of this table are as follows:

Distance of Near Point, Mean: 93.800 mm.;

Standard Deviation: 19.9816 mm.

General Astigmatism, Mean: -0.1671 D.;

Standard Deviation: 0.5863 D.

Product Moment Coefficient of Correlation:  $r = -0.1417 \pm 0.0250$ .

The array-means are:

Grade of General Astigmatism	Mean Distance of Near Point
+ 0.759 D.	88.875 mm.
0.00 D.	93.145 "
- 0.75 D.	95.740 "
- 1.50 D.	100.476 "
- 2.625 D.	105.962 "

Grade of Near Point Distance	Mean General Astigmatism
61.92 mm.	-0.2727
72.50 "	-0.1427
82.38 "	-0.0494
92.34 "	-0.0117
102.09 "	-0.2219
112.41 "	-0.2143
122.24 "	-0.1810
132.16 "	-0.4257
151.90 "	-0.5000

General Population: 93.800 "

General Population: -0.1671

For the regression curve of General Astigmatism on Near Point it is needful to use a cubic; we see that for either a close or a distant Near Point the effect is the same, i.e. an increase of General Astigmatism with the rule. The equation to the cubic is:

$$\begin{aligned} \widetilde{G.A.} = & -0.10216 - 0.001,616 (N.P. - 90) \\ & - 0.000,23124 (N.P. - 90)^2 \\ & + 0.00000,11936 (N.P. - 90)^3, \end{aligned}$$

the distance of the Near Point being measured in millimetres and the General Astigmatism in dioptries.

As usual  $\widetilde{G.A.}$  denotes the probable or mean value of the astigmatism in an array of boys of given Near Point Distance  $N.P.$

Diagram 92 indicates that the regression of Near Point Distance

### GENERAL ASTIGMATISM & POSITION OF NEAR POINT

#### ALIEN JEWISH BOYS

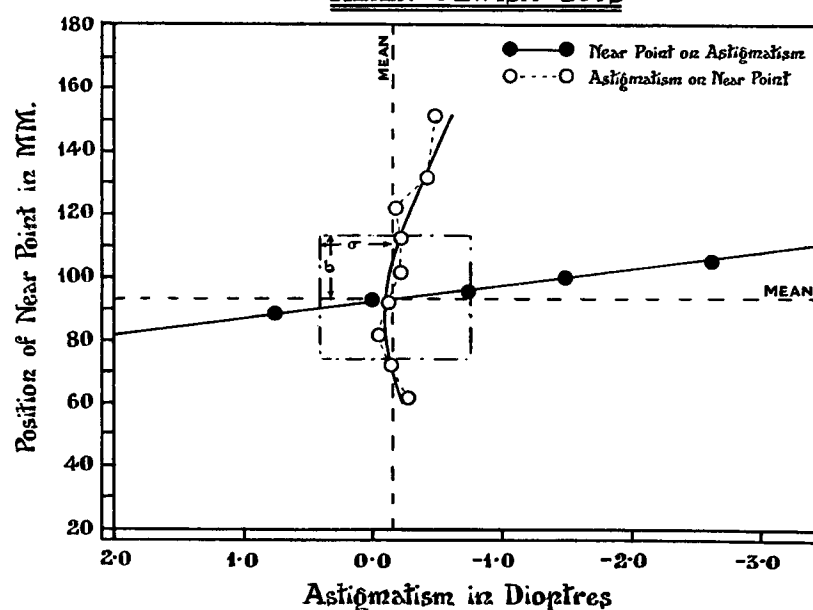


Diagram 92.

on General Astigmatism is almost exactly linear, and if  $\tilde{D}_{NP}$  = probable Near Point Distance for given General Astigmatism  $G.A.$ :  $\tilde{D}_{NP} = 92.993 - 4.829 G.A.$  gives a suitable graduation.

We see that as Astigmatism with the rule increases the Near Point gets farther and farther off. The relation, as the diagram indicates, is quite a substantial one, much more so than the association of Visual Acuity, Refraction Class or General Refraction with Near Point Distance: see Diagram 92.

(viii) *General Astigmatism and Direction of Axis\**. The accompanying table provides the data.

Table CCII. *General Astigmatism and Direction of Axis.*

Angle Axis of Principal Meridian makes with Horizontal (Central Values)

General Astigmatism (Central Values)	-45°	-40°	-35°	-30°	-25°	-20°	-15°	-10°	-5°	0°	+5°	+10°	+15°	+20°	+25°	+30°	+35°	+40°	+45°	Totals
+ 1.50	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	3
+ 0.75	—	1	—	2	—	—	1	1	—	41	—	1	1	1	—	5	—	—	—	54
0.00	4	—	—	6	—	1	2	3	—	631	—	1	1	—	—	9	—	—	5	663
- 0.75	2	—	—	3	—	—	2	—	—	83	—	1	3	1	1	5	—	—	3	104
- 1.50	—	—	—	1	—	1	—	—	—	17	1	—	—	—	1	2	—	—	2	25
- 2.25	3	—	—	2	—	—	—	3	—	20	—	1	—	2	—	—	—	—	—	31
- 3.00	1	—	—	—	—	—	—	1	—	3	—	—	2	—	—	—	—	—	—	7
- 3.75	—	—	—	—	—	—	—	2	—	6	1	—	—	—	—	—	—	—	—	9
- 4.50	—	—	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	3
- 5.25	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	10	1	—	15	—	2	5	10	—	804	2	5	8	4	2	22	—	—	10	900

The constants of this table, which shows considerable preferences for angles like 10°, 15°, 30° and 45°, are as follows:

General Astigmatism, Mean: — .2375 D.;                      Standard Deviation: .7407 D.  
 Direction of Axis, Mean: 0°·2944;                      Standard Deviation: 9°·5203.

We have not for the above results removed all the eyes, 590 in number, with no astigmatism. When this is done we have:

General Astigmatism, Mean: — .6895 D.;                      Standard Deviation: 1.1706 D.  
 Direction of Axis†, Mean: 0°·8548;                      Standard Deviation: 16°·3285.

The array-means are given below:

Given Direction of Axis	Mean General Astigmatism	Grade of General Astigmatism	Direction of Axis (disregarding sign)
— 44°·545	— .9545 D.	+ 0.790 D.	6°·667
— 30°·0	— .3500 D.	0.000 D.	13°·151
— 12°·65	— 1.1029 D.	— 0.750 D.	5°·721
0°	— .6624 D.	— 1.500 D.	9°·200
+ 13°·68	— 1.5000 D.	— 2.250 D.	8°·871
+ 29°·58	— .1562 D.	— 3.675 D.	6°·750
+ 45°·0	— .5250 D.		
General Population:	— .6895	General Population:	8°·3065

The association is clearly of a skew character and the correlation coefficient would be useless. We find:

$$\eta'^2_{GA.DA} = .059,188, \quad \bar{\eta}^2_{GA.DA} = .019,355 \pm .007,455,$$

$$\eta'^2_{DA.GA} = .042,483, \quad \bar{\eta}^2_{DA.GA} = .016,129 \pm .006,816,$$

the latter disregarding sign of angle. Both the  $\eta'^2$ 's are significant having regard to the values of the corresponding  $\bar{\eta}^2$ 's and their probable errors. We therefore conclude that association exists between the intensity of General Astigmatism and the Direction of the Axis.

\* By the "direction of axis" we mean throughout our work the direction of the principal meridian which makes the least angle with the horizontal, and the direction of axis is measured by this angle.

† Disregarding sign of angle: Mean = 8°·3065, Standard Deviation = 14°·1576.

We have  $\eta'_{DA.GA} = .2061$  and  $\eta'_{GA.DA} = .2433$ . As in the case of Corneal Astigmatism and the direction of axis (see later, p. 207) we find a maximum of astigmatism with the axis directed at about  $\pm 12^\circ$  to  $13^\circ$ , but we also find another rise in the astigmatism as the angle approaches its maximum at about  $\pm 45^\circ$ . Whether the astigmatism really takes a secondary minimum when the direction of the axis is zero, we are not prepared to assert, because the value depends so largely on what is treated as zero astigmatism, the classification being originally only to  $\frac{1}{4}$  D. In the same way the maximum angle of the axis being reached at the array with central value 0 D. is really under some doubt. For the angle could only be determined for the individuals having astigmatism of  $\pm \frac{1}{4}$  D., and in these cases, it is undoubtedly difficult to determine the direction of the axis. There may be some physiological reason why these cases with small amounts of astigmatism should be the chief contributors to the axis at  $\pm 45^\circ$ , but until that is discovered, we are inclined to think that there may have been some bias, when the direction of the axis was difficult of ascertainment, to put it half-way between horizontal and vertical.

The graphs of the array-means are shown in Diagrams 93 and 94.

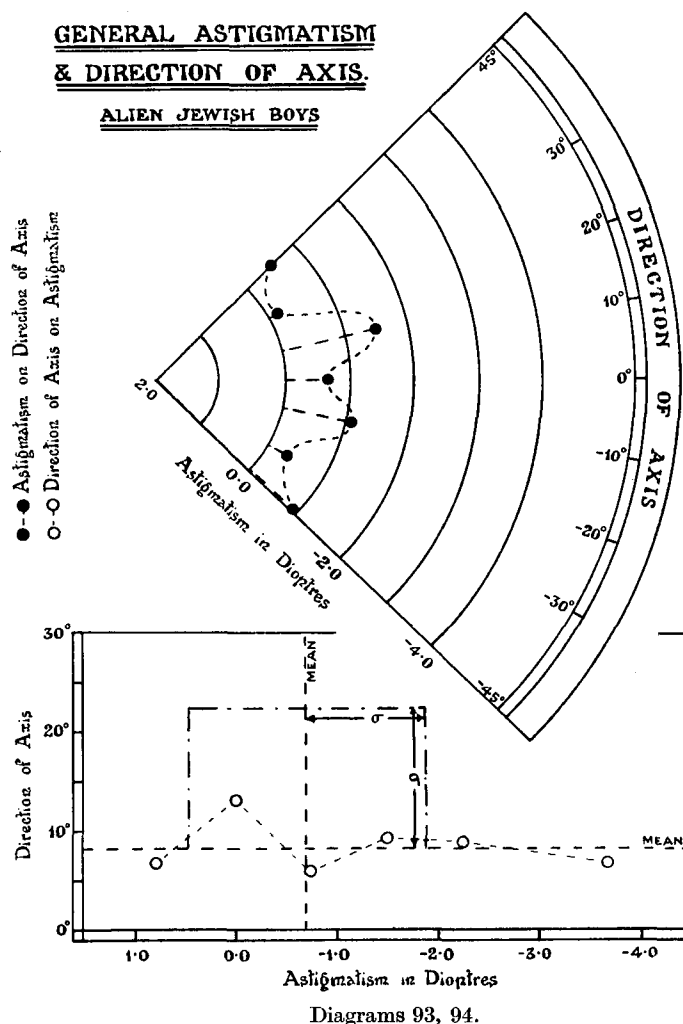


Table CCIII. *Corneal Refraction in Right and Left Eyes. Boys (A, B and C).*

Corneal Refraction in Dioptres. Right Eye. (Central Values)

Corneal Refraction in Dioptres. Left Eye. (Central Values)																					
	38-125	38-625	39-125	39-625	40-125	40-625	41-125	41-625	42-125	42-625	43-125	43-625	44-125	44-625	45-125	45-625	46-125	46-625	47-125	47-625	Totals
39-125	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
39-625	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
40-125	—	—	—	—	4	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	7
40-625	—	—	—	3	1	4	2	1	—	—	—	—	—	—	—	—	—	—	—	—	11
41-125	—	—	—	1	5	3	11	1	3	—	1	1	—	—	—	—	—	—	—	—	26
41-625	—	—	—	—	1	2	7	7	9	1	—	—	—	—	—	—	—	—	—	—	27
42-125	—	—	—	—	1	—	3	11	17	16	2	1	1	—	—	—	—	—	—	—	52
42-625	—	—	—	—	—	—	—	—	6	20	11	1	—	—	—	—	—	—	—	—	38
43-125	—	—	—	—	—	—	—	1	2	11	28	9	2	1	—	—	1	—	—	—	55
43-625	—	—	1	—	—	—	—	—	—	7	17	31	9	4	—	1	—	—	—	—	70
44-125	1	—	—	—	—	—	—	—	—	2	4	15	32	13	1	—	—	1	—	—	69
44-625	—	—	—	—	—	—	—	—	—	—	1	4	16	19	6	1	—	—	—	—	47
45-125	—	—	—	—	—	—	—	—	—	—	1	—	5	10	15	2	2	—	—	—	35
45-625	—	—	—	—	—	—	—	—	—	—	—	—	1	2	5.5	13	9	0.5	—	—	31
46-125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	4	10	3	1	—	21
46-625	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4	3	—	—	7
47-125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	2
47-625	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
Totals	1	—	2	5	13	10	24	22	38	57	65	62	66	49	30.5	21	28	7.5	1	1	503

(e) *Corneal Refraction.* (i) *Corneal Refraction, Right and Left Eyes.* Our data are given in Table CCIII on p. 195.

The constants of this table are as follows\*:

\* There is no significant difference for Corneal Refraction between either the means or variabilities of Right and Left Eyes. The corresponding table was worked out for *A* and *B*'s observations only and gave:

Right Eye. Corneal Refraction.

Mean:  $43.887 \pm .073$ .

Standard Deviation:  $1.5458 D. \pm .051$ .

Left Eye. Corneal Refraction.

Mean:  $43.930 \pm .074$ .

Standard Deviation:  $1.5725 D. \pm .052$ .

Product Moment Correlation Coefficient:

$r = .8706 \pm .0114$ .

The main difference lies, as we have before seen, in the mean values.

The Regression Straight Lines are:

$$\tilde{E}_L = 5.0618 + .88564E_R,$$

$$\tilde{E}_R = 5.2912 + .85582E_L,$$

which may be compared with those for the combined observations of *A*, *B* and *C*. The regression curves for *A* and *B* only tend to be more flat cubics than straight lines, and a slighter tendency to deviate from the straight line may also be traced in those for *A*, *B* and *C* fitted in Diagram 95 with straight lines.

**CORNEAL REFRACTION- LEFT EYE & RIGHT EYE**  
(OBSERVERS A & B ONLY) ALIEN JEWISH BOYS

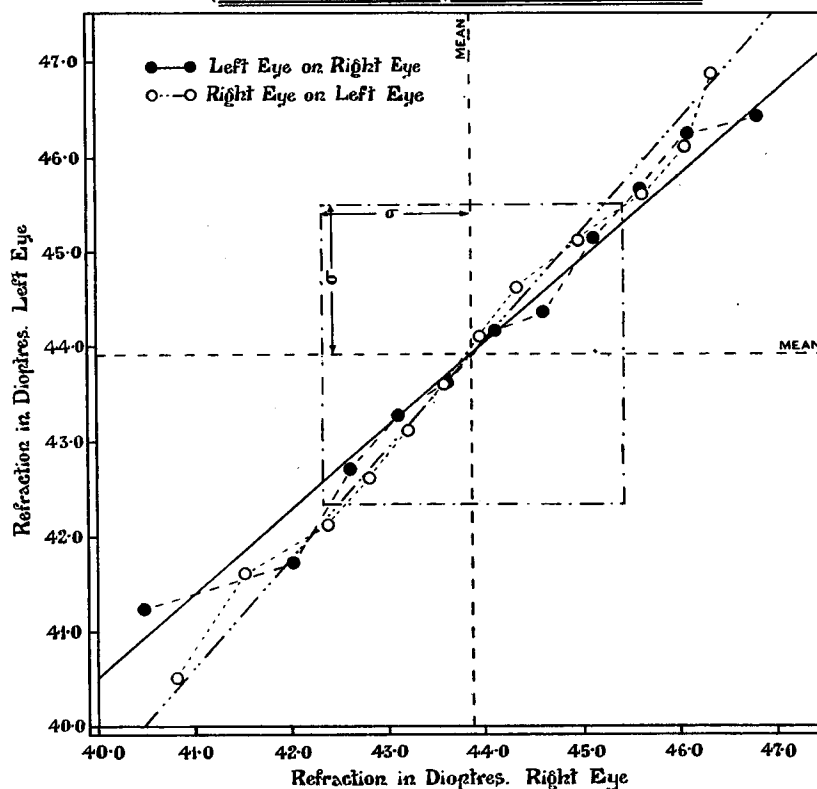


Diagram 95 bis.

Table CCIV. *Corneal Refraction of Right Eye and Left Eye. A and B only.*

Corneal Refraction of Left Eye (Central Values)

	40-125	40-625	41-125	41-625	42-125	42-625	43-125	43-625	44-125	44-625	45-125	45-625	46-125	46-625	47-125	47-625	Totals
38-125	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
38-625	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39-125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39-625	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
40-125	2	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	5
40-625	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
41-125	1	—	2	2	1	—	—	—	—	—	—	—	—	—	—	—	6
41-625	1	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	3
42-125	1	—	—	5	3	—	—	—	—	—	—	—	—	—	—	—	11
42-625	—	—	—	—	7	1	6	1	—	—	—	—	—	—	—	—	22
43-125	—	—	—	—	—	4	7	8	1	—	—	—	—	—	—	—	20
43-625	—	—	—	—	1	1	5	14	4	3	—	—	—	—	—	—	28
44-125	—	—	—	—	—	—	2	4	14	7	2	—	—	—	—	—	29
44-625	—	—	—	—	—	—	1	2	9	11	3	—	—	—	—	—	26
45-125	—	—	—	—	—	—	—	—	1	1	10	4	2	—	—	—	18
45-625	—	—	—	—	—	—	—	—	—	—	2	6	3	—	—	—	11
46-125	—	—	—	—	—	—	—	—	—	—	—	3.5	7	3	2	—	15.5
46-625	—	—	—	—	—	—	—	—	—	—	—	0.5	3	—	—	—	5.5
47-125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
47-625	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	5	2	3	10	13	13	22	29	31	22	17	14	16	5	2	1	205

Corneal Refraction, Mean, Right Eye: 43·4446 D.  $\pm$  ·0476 D., Left Eye: 43·5142 D.  $\pm$  ·0461 D.  
 Standard Deviation, Right Eye: 1·5819 D.  $\pm$  ·0336 D., Left Eye: 1·5322 D.  $\pm$  ·0326 D.

Product Moment Correlation Coefficient:  $r = \cdot91235 \pm \cdot0050$ .

The correlation is so high that we can hardly anticipate any great improvement on linear regression. The array-means run thus, giving mean Corneal Refraction for one eye for given grade of same character in the other eye:

Grade, R. E.	Mean, L. E.	Grade, R. E.	Mean, L. E.	Grade, L. E.	Mean, R. E.	Grade, L. E.	Mean, R. E.
40·498 D.	41·028 D.	44·125 D.	44·216 D.	40·239 D.	40·466 D.	44·125 D.	43·966 D.
41·125 D.	41·312 D.	44·625 D.	44·523 D.	41·125 D.	41·125 D.	44·625 D.	44·423 D.
41·625 D.	41·807 D.	45·125 D.	45·182 D.	41·625 D.	41·569 D.	45·125 D.	44·868 D.
42·125 D.	42·007 D.	45·625 D.	45·530 D.	42·125 D.	42·182 D.	45·625 D.	45·585 D.
42·625 D.	42·739 D.	46·125 D.	45·929 D.	42·625 D.	42·717 D.	46·125 D.	45·887 D.
43·125 D.	43·225 D.	46·783 D.	46·030 D.	43·125 D.	43·161 D.	46·825 D.	46·300 D.
43·625 D.	43·657 D.			43·625 D.	43·489 D.		
General Population:		43·514 D.		General Population:		43·445 D.	

Diagram 95 indicates how closely the regression curves can be considered as linear. The association between Right and Left Eyes for Corneal Refraction is almost the same as that between the eyes for General Refraction (see p. 172).

The equations to the regression straight lines are:

$$\tilde{E}_L = 5\cdot1244 + \cdot88365E_R,$$

$$\tilde{E}_R = 2\cdot4550 + \cdot94198E_L,$$

in dioptries,  $\tilde{E}_L$  and  $\tilde{E}_R$  being the probable Corneal Refractions for given Corneal Refractions  $E_R$  and  $E_L$  of the right and left eyes respectively.

(ii) *Corneal Refraction and Visual Acuity.* This has been already discussed: see pp. 154 to 156.

(iii) *Corneal Refraction and Refraction Class.* Already discussed: see pp. 168 and 169.

(iv) *Corneal Refraction and General Refraction.* Already discussed: see pp. 177 to 179.

(v) *Corneal Refraction and General Astigmatism.* Already discussed: see pp. 186 to 188.

(vi) *Corneal Refraction and Corneal Astigmatism.* Table CCV (p. 198) gives the data for *A* and *B* only. The constants of this table, having regard to the sign of the astigmatism, are as follows:

Corneal Refraction, Mean: 43·9085 D.; Standard Deviation: 1·5277 D.

Corneal Astigmatism, Mean: ·7610 D.; Standard Deviation: ·9456 D.

Product Moment Coefficient of Correlation:  $r = -\cdot2722 \pm \cdot0308$ .

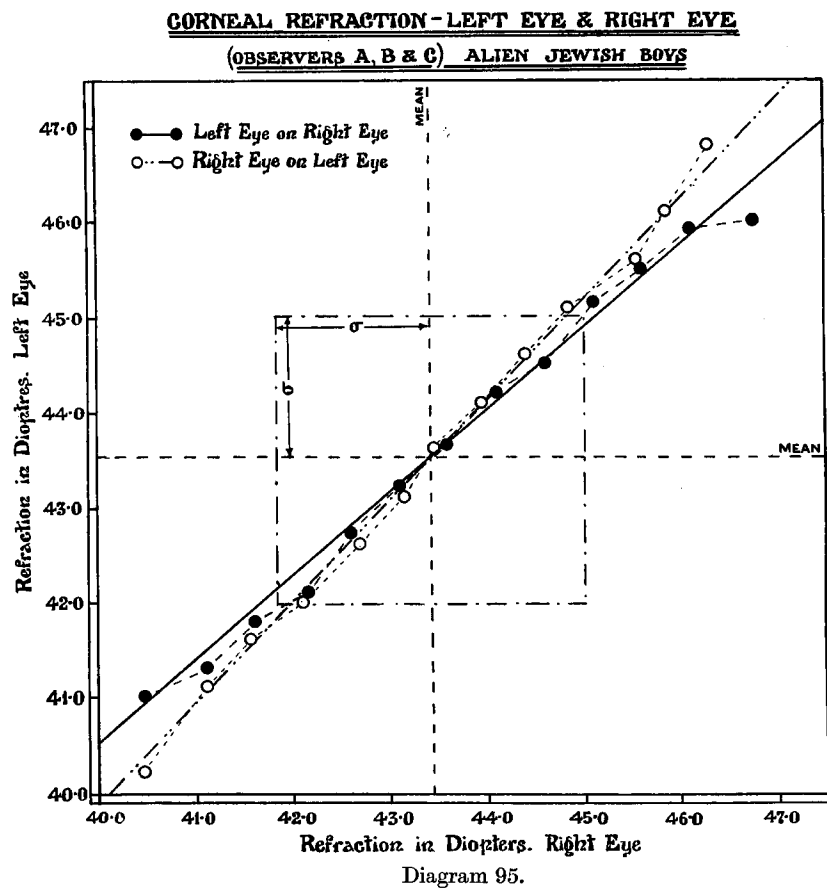


Table CCV. *Corneal Refraction and Corneal Astigmatism. A and B only.*

Corneal Refraction in Dioptres (Central Values)

Corneal Astigmatism in Dioptres (Central Values)	38.125	40.125	40.625	41.125	41.625	42.125	42.625	43.125	43.625	44.125	44.625	45.125	45.625	46.125	46.625	47.125	47.625	Totals
- 2.25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
- 1.50	—	—	—	—	—	—	—	—	—	2	1	—	—	2	—	—	—	5
- 0.75	—	—	—	—	—	—	—	—	—	3	3	—	—	2	—	—	—	16
0.00	—	—	3	1	3	7	8	10	9	15	10	11	11	8.5	1.5	—	—	109
+ 0.75	1	—	2	1	3	2	7	13	22	34	28	19	11	16	6	1	2	202
+ 1.50	—	—	—	1	—	2	4	5	3	4	3	3	2	3	1	—	—	33
+ 2.25	—	—	2	1	1	2	2	5	6	2	3	1	1	—	—	—	—	25
+ 3.00	—	—	1	—	—	1	1	—	1	3	—	2	—	—	—	—	—	9
+ 3.75	—	—	1	—	1	1	1	—	—	2	—	—	—	—	—	—	—	6
+ 4.50	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
+ 5.25	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	—	3
+ 6.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	..	10	4	9	13	24	35	42	57	60	48	35	25	31.5	10.5	3	410

Precisely as we have done previously we can determine the Corneal Refraction in the "vertical" meridian from a knowledge of the above constants. For let  $A_C$  measure the Corneal Astigmatism and  $R_{C_1}$  and  $R_{C_2}$  the two Corneal Refractions, i.e. the principal Corneal Refractions nearest to the horizontal and vertical respectively. Then  $A_C = R_{C_2} - R_{C_1}$ , and

$$\sigma_{R_{C_2}} = \sqrt{\sigma_{A_C}^2 + \sigma_{R_{C_1}}^2 + 2\sigma_{A_C}\sigma_{R_{C_1}}r_{A_C R_{C_1}}} = 1.5626 \text{ D.},$$

$$r_{R_{C_1} R_{C_2}} = (\sigma_{A_C}r_{A_C R_{C_1}} + \sigma_{R_{C_1}})/\sigma_{R_{C_2}} = +.8130,$$

$$r_{A_C R_{C_2}} = (\sigma_{A_C} + \sigma_{R_{C_1}}r_{A_C R_{C_1}})/\sigma_{R_{C_2}} = +.3391.$$

Of course Mean  $R_{C_2} = \text{Mean } A_C + \text{Mean } R_{C_1} = 44.6695 \text{ D.}$

### CORNEAL REFRACTION & CORNEAL ASTIGMATISM

(OBSERVERS A & B ONLY) ALIEN JEWISH BOYS

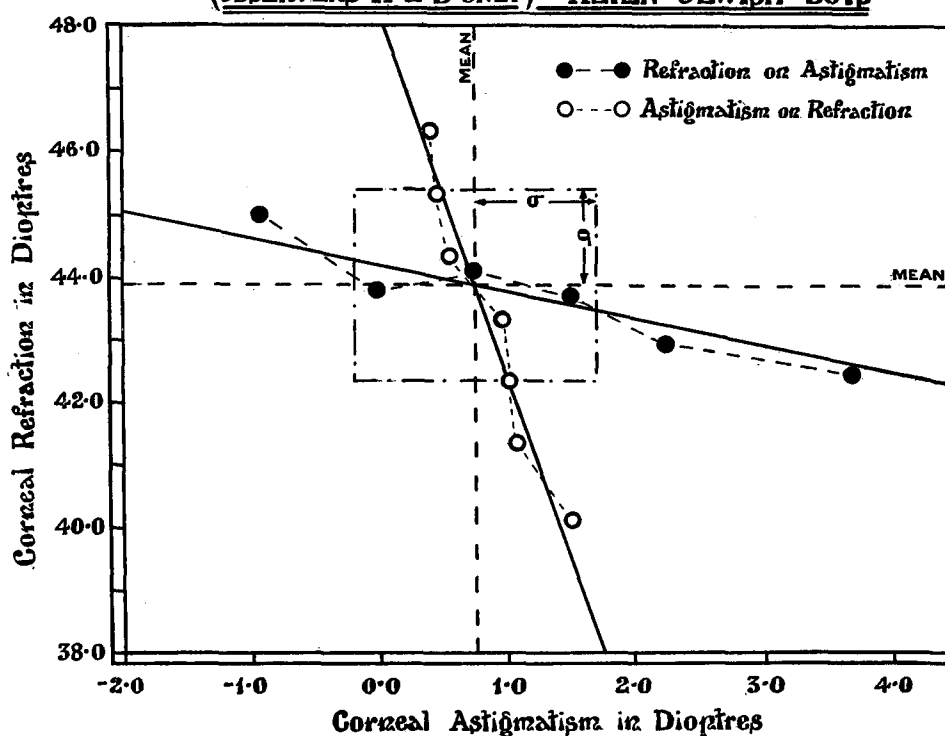


Diagram 96.

Thus we see that the refractive power in the vertical principal is larger than in the horizontal principal meridian, that this refractive power is slightly more variable and slightly more closely associated with the Corneal Astigmatism. Further, the correlation of the two principal Corneal Refractions is somewhat less than that between the two principal General Refractions.

The array-means are as follows:

Grade of Corneal Astigmatism	Mean Corneal Refraction	Grade of Corneal Refraction	Mean Corneal Astigmatism
- 0.9286 D.	45.0298 D.	40.125 D.	+ 1.5000 D.
0.00 D.	43.8108 D.	41.375 D.	+ 1.0909 D.
+ 0.75 D.	44.1374 D.	42.375 D.	+ 1.0297 D.
+ 1.50 D.	43.7007 D.	43.375 D.	+ .9545 D.
+ 2.25 D.	42.9650 D.	44.375 D.	+ .5694 D.
+ 3.7125 D.	42.4750 D.	45.375 D.	+ .4750 D.
		46.364 D.	+ .4309 D.
General Population:	43.9085 D.	General Population:	+ .7610 D.

These means and Diagram 96 (p. 198) do not suggest that other graduations than by straight lines would be of much service, and these have been used. Their equations are, in dioptries as unit:

$$\tilde{R}_C = 44.2431 - .4397A_C,$$

$$\tilde{A}_C = 8.3073 - .1685R_C,$$

where  $\tilde{R}_C$  and  $\tilde{A}_C$  are the probable values of the Corneal Refraction and Corneal Astigmatism for given values of the Corneal Astigmatism  $A_C$  and Corneal Refraction  $R_C$  respectively.

(vi) *bis* We can test directly the values indirectly found for  $R_{C_1}$  and its correlation with  $R_{C_2}$ . Table CCVI gives the direct distribution of  $R_{C_1}$  and  $R_{C_2}$ .

Table CCVI. *Correlation of Corneal Refractions in Principal Meridians (A and B only).*

$R_{C_1}$ , Corneal Refraction in Principal Meridian nearer to Horizontal in Dioptries

	38.125	38.625	39.125	39.625	40.125	40.625	41.125	41.625	42.125	42.625	43.125	43.625	44.125	44.625	45.125	45.625	46.125	46.625	47.125	47.625	Totals
$R_{C_2}$ , Corneal Refraction in Principal Meridian nearer to Vertical in Dioptries	38.625	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
39.125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39.625	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
40.125	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
40.625	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
41.125	—	—	—	—	1	2	2	—	1	—	—	—	—	—	—	—	—	—	—	—	6
41.625	—	—	—	—	—	1	1	6	—	—	—	—	—	—	—	—	—	—	—	—	7
42.125	—	—	—	—	—	1	2	2	7	—	—	—	—	—	—	—	—	—	—	—	12
42.625	—	—	—	—	2	1	1	6	6	1	—	2	—	—	—	—	—	—	—	—	19
43.125	—	—	—	—	1	—	1	2	14	7	1	—	1	—	—	—	—	—	—	—	29
43.625	—	—	—	—	1	—	—	4	5	16	9	3	—	—	—	—	—	—	—	—	38
44.125	—	—	—	—	—	—	—	—	2	3	8	21	12	4	—	—	—	—	—	—	50
44.625	—	—	—	—	1	—	—	1	4	3	14	21	7	2	2	—	2	—	—	—	55
45.125	—	—	—	—	—	—	1	—	1	2	17	20	6	2	6	2	—	—	—	—	52
45.625	—	—	—	—	—	—	1	1	1	2	4	10	18	9	18	9	3	1	—	—	51
46.125	—	—	—	—	—	—	—	—	—	1	2	5	2	6	5	6	—	1	—	—	28
46.625	—	—	—	—	—	—	—	—	—	—	2	—	—	1	8	7	1	1	—	—	20
47.125	—	—	—	—	—	—	—	—	—	—	—	—	3	2	1	11	3	—	—	—	20
47.625	—	—	—	—	—	—	—	—	2	—	2	—	1	—	—	1	3	1	—	—	10
48.125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	2
48.625	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	2	—	2	5
Totals	1	—	—	—	10	4	9	13	24	35	42	57	60	48	35	25	32	10	3	2	410

The constants of this table are as follows:

“Horizontal” Corneal Refraction,  $R_{C_1}$ , Mean: 43.9079 D.; Standard Deviation: 1.5267 D.

“Vertical” Corneal Refraction,  $R_{C_2}$ , Mean: 44.6774 D.; Standard Deviation: 1.6030 D.

Product Moment Correlation Coefficient:  $r = .8184 \pm .0110$ .

Correlation Ratios:

$$\begin{aligned}\eta'^2_{R_{C_2}, R_{C_1}} &= .673,157, & \bar{\eta}^2_{R_{C_2}, R_{C_1}} &= .002,927 \pm .002,542, \\ \eta'^2_{R_{C_1}, R_{C_2}} &= .686,939, & \bar{\eta}^2_{R_{C_1}, R_{C_2}} &= .002,927 \pm .002,542.\end{aligned}$$

The  $\eta'^2$ 's are both significant compared to the  $\bar{\eta}^2$ 's, and we have the high values

$$\eta'_{R_{C_2}, R_{C_1}} = .8205, \quad \eta'_{R_{C_1}, R_{C_2}} = .8288.$$

It will be seen, however, that these values do not exceed considerably the value of the correlation coefficient and accordingly straight lines will graduate fairly successfully. These lines are:

$$\begin{aligned}\tilde{R}_{C_2} &= 6.9473 + .8593R_{C_1}, \\ \tilde{R}_{C_1} &= 9.0841 + .7794R_{C_2},\end{aligned}$$

where  $\tilde{R}_{C_2}$ ,  $\tilde{R}_{C_1}$  are the probable values of the Corneal Refractions in the "vertical" and "horizontal" principal meridians for given values  $R_{C_1}$ ,  $R_{C_2}$  respectively of the given refractions in "horizontal"

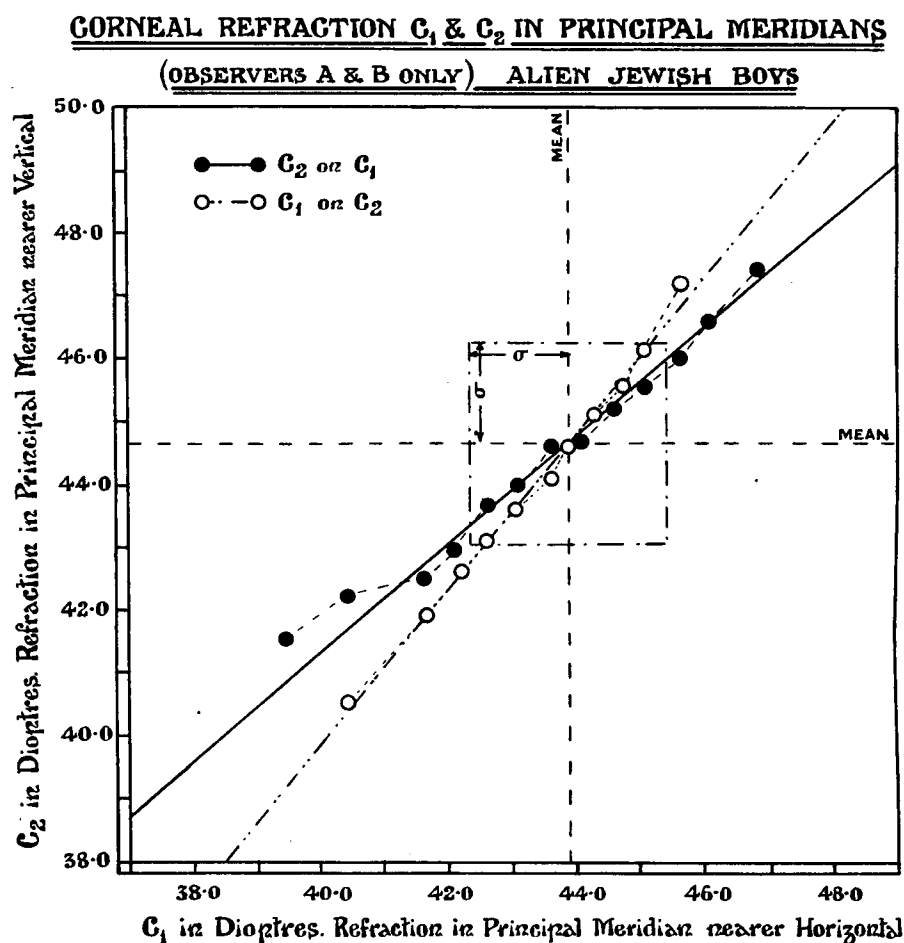


Diagram 97.

and "vertical" meridians respectively, all quantities being read in dioptries. Diagram 97 shows the observational points graduated by straight lines.

We worked out again the relation between  $R_{C_2}$  and  $R_{C_1}$ , using the data of *A*, *B* and *C*, or 1006 cases. We found:

Corneal Refraction, Principal Axis nearer horizontal, Mean: 43.4779 D.; Standard Deviation: 1.5567 D.

Corneal Refraction, Principal Axis nearer vertical, Mean: 44.0828 D.; Standard Deviation: 1.6357 D.

Product Moment Correlation Coefficient:  $r = .8552 \pm .0057$ .

Correlation Ratios:

$$\eta'^2_{R_{C_2}, R_{C_1}} = .733,164, \text{ whence}$$

$$\eta'_{R_{C_2}, R_{C_1}} = .8562,$$

$$\eta'^2_{R_{C_1}, R_{C_2}} = .766,496, \text{ whence}$$

$$\eta'_{R_{C_1}, R_{C_2}} = .8755.$$

As before the correlation ratios are so close to the correlation coefficient that we shall scarcely improve on linear regressions. The regression straight lines are (in dioptries):

$$\tilde{R}_{C_2} = 5.0136 + .8986R_{C_1},$$

$$\tilde{R}_{C_1} = 7.5991 + .8139R_{C_2}.$$

These, with the observation points, are figured in Diagram 98, and the following table gives the data:

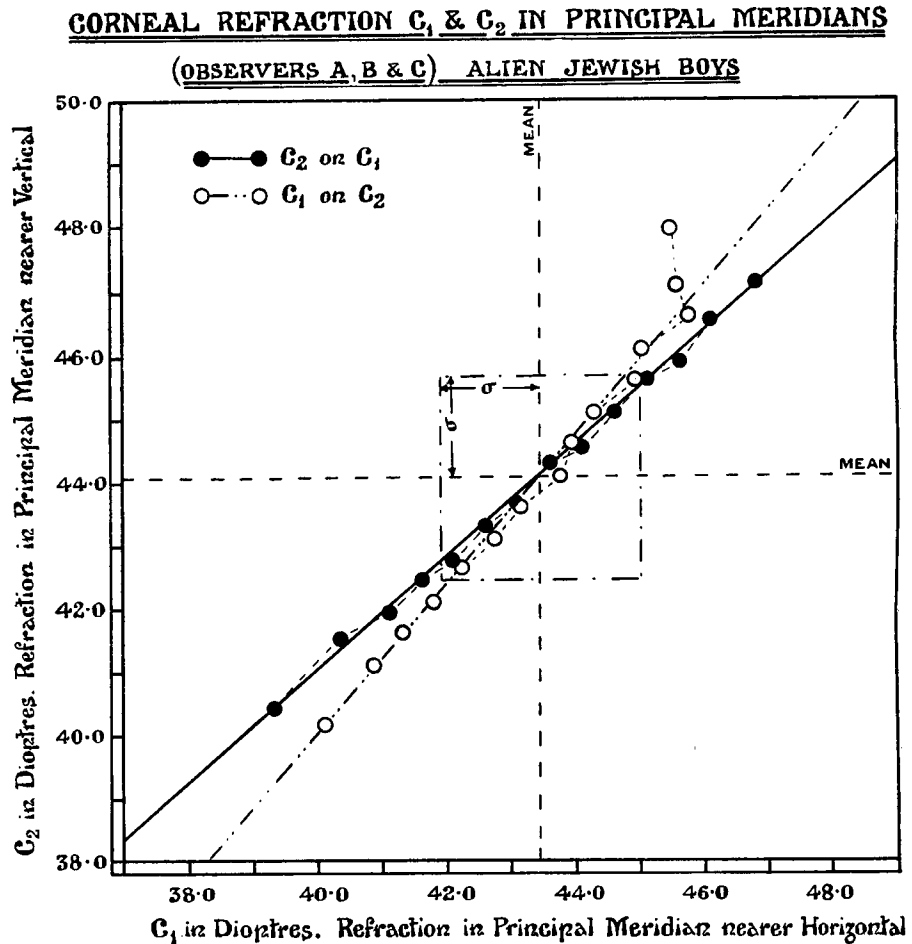


Table CCVI bis. *Corneal Refractions in Principal Meridians (A, B and C).*

$R_{C_1}$ , Corneal Refraction in Principal Meridian nearer Horizontal in Dioptries

$R_{C_2}$ , Corneal Refraction in Principal Meridian nearer Vertical in Dioptries	38.125	38.625	39.125	39.625	40.125	40.625	41.125	41.625	42.125	42.625	43.125	43.625	44.125	44.625	45.125	45.625	46.125	46.625	47.125	47.625	Totals
38.125	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38.625	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
39.125	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
39.625	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
40.125	—	—	—	—	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
40.625	—	—	—	2	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
41.125	—	—	2	—	2	5	19	—	1	—	—	—	—	—	—	—	—	—	—	—	29
41.625	—	—	—	3	—	3	5	24	—	1	—	—	—	—	—	—	—	—	—	—	36
42.125	—	—	—	1	—	2	15	8	44	—	—	—	—	—	—	—	—	—	—	—	71
42.625	—	—	—	—	3	3	5	5	17	36	1	—	—	—	—	—	—	—	—	—	73
43.125	—	—	—	—	3	—	2	4	9	25	58	2	—	1	—	—	—	—	—	—	104
43.625	—	—	—	—	1	—	—	2	6	18	27	40	4	—	—	—	—	—	—	—	98
44.125	—	—	—	—	—	—	—	—	6	5	17	41	56	4	—	—	—	—	—	—	129
44.625	—	—	—	—	1	—	1	3	2	5	7	31	33	37	2	—	2	1	—	—	125
45.125	—	—	—	—	—	—	1	—	4	3	4	4	30	28	23	2	—	—	—	—	99
45.625	—	—	—	—	—	—	1	2	1	—	4	3	1	11	25	30	3	1	—	—	82
46.125	—	—	—	—	1	—	—	—	—	—	1	4	9	7	11	10	14	—	1	—	58
46.625	—	—	—	—	—	—	—	—	—	—	—	2	—	3	2	8	13	4	1	—	33
47.125	—	—	—	—	—	—	—	1	—	—	—	1	—	3	3	2	13	3	—	—	26
47.625	—	—	—	—	—	—	—	—	—	2	1	2	—	1	—	—	2	3	1	—	12
48.125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	2
48.625	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	2	—	2	5
Totals	1	—	4	7	20	21	50	49	90	95	120	132	135	96	66	52	49	14	3	2	1006

We see that the results for  $A$ ,  $B$  and  $C$  confirm those for  $A$  and  $B$  in the fact that  $R_{C_1}$ , Corneal Refraction in the principal meridian nearer to the vertical is greater and more variable than  $R_{C_2}$ , Corneal Refraction in the principal meridian nearer to the horizontal. The chief difference between the present longer and the former shorter series is the shifting of the means roughly through half a dioptré.

(vii) *Corneal Refraction and Distance of Near Point.* Table CCVII below provides the material furnished by the three observers  $A$ ,  $B$  and  $C$ .

The following constants are derived from the table:

Corneal Refraction, Mean: 43.4413 D.; Standard Deviation: 1.3906 D.  
Distance of Near Point, Mean: 92.9642 mm.; Standard Deviation: 20.3431 mm.

Product Moment Coefficient of Correlation:  $r = -0.1156 \pm 0.0242$ .

Correlation Ratios:

$$\eta'^2_{NP.CR} = 0.039,806, \quad \bar{\eta}^2_{NP.CR} = 0.017,241 \pm 0.004,520, \\ \eta'^2_{CR.NP} = 0.053,511, \quad \bar{\eta}^2_{CR.NP} = 0.037,135 \pm 0.006,565.$$

The first correlation ratio is significant and gives  $\eta'_{NP.CR} = 0.1995$ , the second is not definitely significant, but is higher and we have  $\eta'_{CR.NP} = 0.2313$ . In the former case it is difficult to improve much on graduation by the straight line. In the latter case it seems probable that graduation by a cubic might show that the straight line gives exaggerated mean values of the Corneal Refraction at small and great values of the Near Point Distance but this is the case where the correlation ratio is not significant, and the elaboration of higher curve fitting is hardly justifiable.

Table CCVII. *Distance of Near Point and Corneal Refraction.*

Distance of Near Point (Central Values) in mm.

Corneal Refraction (Central Values) in Dioptres	Distance of Near Point (Central Values) in mm.																				Totals
	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	
39-125	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2	4
39-625	—	—	—	—	—	—	—	1	1	—	1	1	—	1	1	—	2	—	—	—	7
40-125	—	—	—	1	—	—	1	—	—	—	—	1	2	1	1	1	—	—	—	—	9
40-625	—	—	—	—	—	—	1	—	1	1	3	3	5	—	—	—	2	1	—	—	17
41-125	—	—	—	—	1	—	—	5	2	1	10	6	4	3	—	1	3	1	—	—	40
41-625	—	—	—	—	—	—	—	2	2	2	3	4	8	3	2	—	3	1	1	1	36
42-125	—	—	—	—	—	1	—	2	4	7	4	6	4	6	8	5	3	3	5	4	65
42-625	—	—	—	—	—	1	2	6	3	6	6	14	14	3	4	7	4	—	2	2	75
43-125	—	—	—	—	—	1	5	6	4	19	7	18	9	8	5	5	5	—	2	—	99
43-625	1	—	—	1	2	1	3	9	9	13	17	13	8	5	3	3	1	1	4	2	102
44-125	—	—	—	—	3	3	5	5	10	12	14	9	5	9	8	5	1	5	—	2	98
44-625	—	2	—	—	1	4	3	4	5	12	5	10	7	8	6	1	3	2	1	—	76
45-125	—	—	—	—	—	1	0.5	3.5	3	2	3	3	10	7	—	2	—	1	2	—	43.5
45-625	—	—	—	—	—	1	2	6	4	2	6	3	3	3	3	2	—	2	1	2	41
46-125	—	—	—	—	—	1	2	2	3	6	2	4	3	1	1	—	1	1	—	0.5	30.5
46-625	—	—	—	—	—	1	—	—	0.5	1.5	—	—	1	1	3	—	—	—	—	—	8
47-125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
47-625	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	2
Totals	1	—	2	2	7	15	24.5	51.5	51.5	84.5	81	98	84	62	42	32	28	18	14	19	754

Diagram 99 shows that the observations give somewhat irregular means, and that we shall hardly be likely to graduate with anything better than the straight line. We have:

$$\tilde{D}_{NP} = 166.4283 - 0.169,111R_{C_1}, \\ \tilde{R}_{C_1} = 44.1759 - 0.007,9021D_{NP}.$$

Here  $\tilde{D}_{NP}$  and  $\tilde{R}_{C_1}$  are the probable Distance of Near Point in mm. and the probable Corneal Refraction in dioptrés for given Corneal Refraction and given Distance of Near Point respectively. The correlation, however, is so low that the results to be obtained are subject to large probable errors.

These results were confirmed by considering the same pair of characters for  $A$  and  $B$  only, when the product moment correlation coefficient sank to  $r = -\cdot0242 \pm \cdot0479$ , i.e. to an insignificant value, but one not significantly different (considering its probable error) from that for  $A$ ,  $B$  and  $C$ 's combined records. We conclude that the Corneal Refraction has little influence on the Distance of the Near Point.

### POSITION OF NEAR POINT & CORNEAL REFRACTION

(OBSERVERS A, B & C) ALIEN JEWISH BOYS

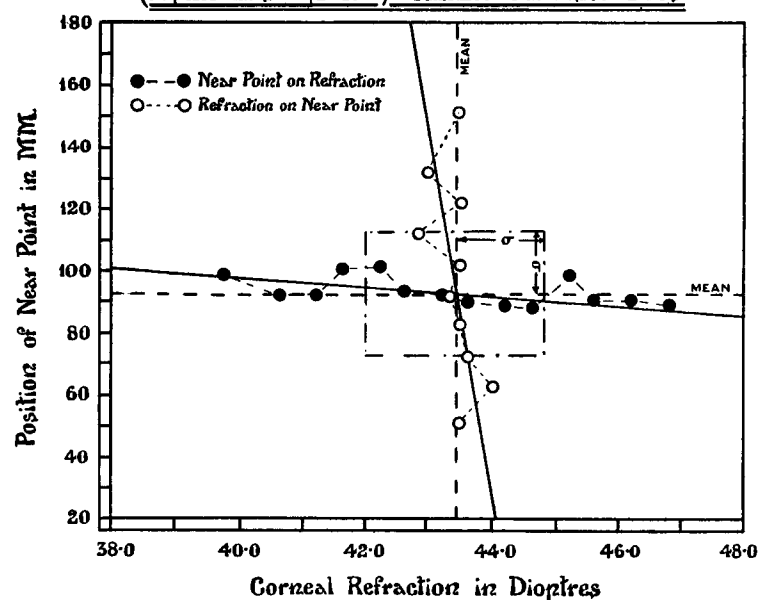


Diagram 99.

(f) *Corneal Astigmatism.* (i) *Corneal Astigmatism, Right and Left Eyes.* Table CCVIII contains our data for  $A$ ,  $B$  and  $C$ .

Table CCVIII. *Degree of Corneal Astigmatism.*

*Right Eye and Left Eye (A, B and C).*

Corneal Astigmatism in Dioptres. Right Eye

Corneal Astigmatism in Dioptres. Left Eye	Corneal Astigmatism in Dioptres. Right Eye												Totals
	- 2.25	- 1.50	- 0.75	0.00	+ 0.75	+ 1.50	+ 2.25	+ 3.00	+ 3.75	+ 4.50	+ 5.25	+ 6.00	
- 2.25	—	1	—	—	—	—	—	—	—	—	—	—	1
- 1.50	—	2	1	—	—	—	—	—	—	—	—	—	3
- 0.75	—	—	1	5	2	—	1	—	—	—	—	—	9
+ 0.00	—	1	5.5	155	51.5	2	2	—	—	—	—	—	217
+ 0.75	—	—	2	48	115	22	5	—	—	—	—	1	193
+ 1.50	—	—	—	3	13	11	6	1	—	1	1	—	36
+ 2.25	—	—	—	1	12	3	9	—	—	—	1	—	26
+ 3.00	—	—	—	1	1	1	1	6	1	—	—	—	11
+ 3.75	—	—	—	—	—	—	2	1	1	1	—	—	5
+ 4.50	—	—	—	—	—	—	—	—	1	—	—	—	1
+ 5.25	—	—	—	—	—	—	—	—	—	—	1	—	1
+ 6.00	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	—	4	9.5	213	194.5	39	26	8	3	2	3	1	503

The constants of this table are as follows:

Mean, Right Eye:

$\cdot6277 \text{ D.} \pm \cdot0266.$

Mean, Left Eye:

$\cdot6069 \text{ D.} \pm \cdot0250.$

Standard Deviation, Right Eye:  $\cdot8858 \text{ D.} \pm \cdot0186.$

Standard Deviation, Left Eye:

$\cdot8298 \text{ D.} \pm \cdot0176.$

Product Moment Correlation Coefficient:

$r = \cdot7171 \pm \cdot0146.$

The array-means are:

Grade of Corneal Astigmatism, Right Eye	Probable Corneal Astigmatism, Left Eye	Grade of Corneal Astigmatism, Left Eye	Probable Corneal Astigmatism, Right Eye
- 0.972 D.	- .444 D.	- 1.038 D.	- .173 D.
0.00 D.	+ .197 D.	0.00 D.	+ .187 D.
+ 0.75 D.	+ .690 D.	+ 0.75 D.	+ .699 D.
+ 1.50 D.	+ 1.096 D.	+ 1.50 D.	+ 1.458 D.
+ 2.25 D.	+ 1.644 D.	+ 2.25 D.	+ 1.500 D.
+ 3.882 D.	+ 2.912 D.	+ 3.417 D.	+ 2.833 D.
General Population:	+ .607 D.	General Population:	+ .628 D.

The plotted means of the arrays (see Diagram 100) do not suggest that any considerable betterment on straight line graduation will be obtained. The straight regression lines are:

$$\tilde{E}_L = \cdot 67177E_R + \cdot 1852,$$

$$\tilde{E}_R = \cdot 76549E_L + \cdot 1632,$$

where  $\tilde{E}_L$  and  $\tilde{E}_R$  are the probable values in dioptries of the Corneal Astigmatism of Left and Right Eyes for given Corneal Astigmatism  $E_R$ ,  $E_L$  of Right and Left Eyes respectively.

The relationship was recomputed for *A* and *B*'s records only (see Table CCIX below) and gave:

Corneal Astigmatism, Right Eye, Mean:  $\cdot 7463 \pm \cdot 0441$ ; Standard Deviation:  $\cdot 9369 \pm \cdot 0312$ .

Corneal Astigmatism, Left Eye, Mean:  $\cdot 7756 \pm \cdot 0449$ ; Standard Deviation:  $\cdot 9540 \pm \cdot 0318$ .

Product Moment Correlation Coefficient:  $r = \cdot 7430 \pm \cdot 0211$ .

The correlation is not materially altered, but *A* and *B* show higher Corneal Astigmatism than *C*, and more variability in its range of values.

It will be seen, having regard to the probable errors, that there is no difference, either in the combined records of *A*, *B* and *C*, or in those of *A* and *B* alone, in the Corneal Astigmatism of Right and Left Eyes, nor is there a difference in the variabilities of Right and Left Eyes.

(ii) *Corneal Astigmatism and Visual Acuity*. This association has been discussed already: see pp. 158–162 and Diagrams 70–72.

Table CCIX. *Corneal Astigmatism. Left Eye and Right Eye (A and B only).*

Corneal Astigmatism. Right Eye

Corneal Astigmatism. Left Eye	Corneal Astigmatism. Right Eye									
	-1.50	-0.75	0.00	+0.75	+1.50	+2.25	+3.00	+3.75	+4.50	+5.25
-1.50	2	1	—	—	—	—	—	—	—	—
-0.75	—	1	4	2	—	—	—	—	—	—
0.00	—	5	28	21	—	1	—	—	—	—
+0.75	—	2	19	64	11	3	—	—	—	—
+1.50	—	—	2	7	4	3	1	—	—	—
+2.25	—	—	—	8	1	3	—	—	—	1
+3.00	—	—	1	1	—	1	2	—	—	—
+3.75	—	—	—	—	—	1	1	1	—	—
+4.50	—	—	—	—	—	—	—	1	—	—
+5.25	—	—	—	—	—	—	—	—	1	—
Totals	2	9	54	103	16	12	4	2	1	2
										205

CORNEAL ASTIGMATISM - LEFT EYE & RIGHT EYE

(OBSERVERS A, B & C) ALIEN JEWISH BOYS

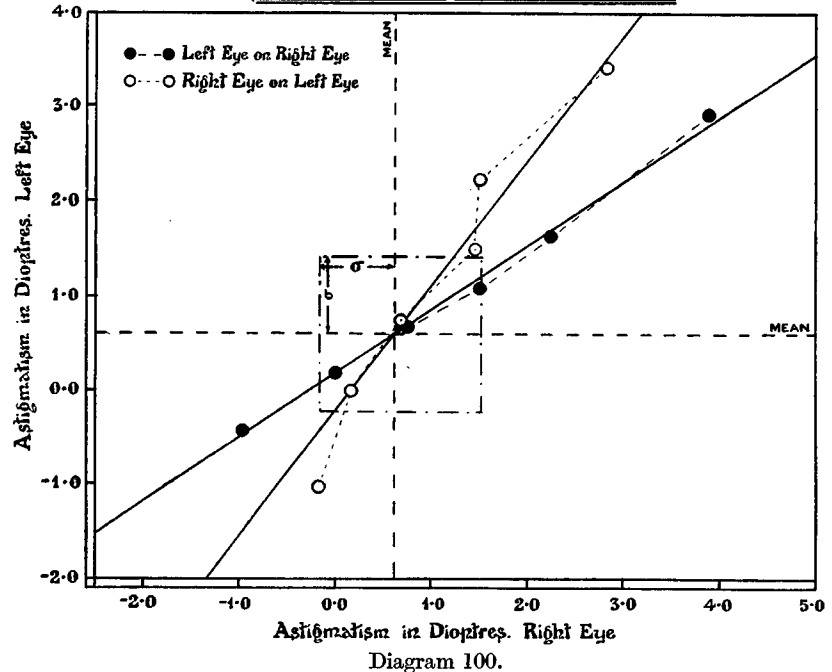


Diagram 100.

(iii) *Corneal Astigmatism and Refraction Class*. Already dealt with: see p. 170 and Diagram 78.

(iv) *Corneal Astigmatism and General Refraction*. Already dealt with: see pp. 180–182 and Diagrams 84–85.

(v) *Corneal Astigmatism and General Astigmatism*. Already dealt with: see pp. 188–192 and Diagrams 90–91.

(vi) *Corneal Astigmatism and Corneal Refraction*. Already dealt with: see pp. 197–199 and Diagram 96.

(vii) *Corneal Astigmatism and Distance of Near Point*. We give first in Table CCX (p. 206) the combined observations of *A*, *B* and *C*.

The constants of this table are as follows:

Distance of Near Point, Mean: 92.9642 mm.; Standard Deviation: 20.3431 mm.

Corneal Astigmatism, Mean: + .5212 D.; Standard Deviation: .7258 D.

Product Moment Correlation Coefficient:  $r = -.0190 \pm .0238$ .

Correlation Ratios:

$$\eta'^2_{NP.AG} = .043,327,$$

$$\bar{\eta}^2_{NP.AG} = .013,262 \pm .003,972,$$

$$\eta'^2_{AG.NP} = .052,372,$$

$$\bar{\eta}^2_{AG.NP} = .018,568 \pm .004,687.$$

Both  $\eta'^2$ 's, if small, are significant and we have  $\eta'_{NP.AG} = .2082$ , and  $\eta'_{AG.NP} = .2288$ .

These values are not large, but differ considerably from that of the correlation coefficient. If we examine, however, Diagram 101, it is difficult to detect any continuous relationship of a marked character between the two variates. With extreme Corneal Astigmatism the Distance of the Near Point increases by about 10 mm., but for values of the astigmatism from  $-1.0$  to  $+2.5$  D. there is no regular change in the distance. In the same way only in the case of very close Near Points do we find any sensible increase above the average of Corneal Astigmatism and then only to  $+1.2$  D. We think on the whole that we must conclude that our total data do not indicate any real relationship between Corneal Astigmatism and Distance of Near Point. Nevertheless, as both correlation ratios are significant, we have fitted the array-means of Near Point on Corneal Astigmatism with the parabola:

$$\widetilde{N.P.} = 92.2702 - 3.421,105A_C + 1.891,397A_C^2,$$

and those of Corneal Astigmatism on Near Point with the cubic:

$$\tilde{A}_C = .43936 - .002,038(N.P. - 90) + .000,274(N.P. - 90)^2 - .000,0025(N.P. - 90)^3.$$

As the relationship, if there be any, is of some importance we worked it out for  $A$  and  $B$ 's observations only. We found:

Distance of Near Point, Mean: 81.5909 mm.;

Standard Deviation: 15.4415 mm.

Corneal Astigmatism, Mean: .6061 D.;

Standard Deviation: .6619 D.

Product Moment Correlation Coefficient:  $r = .1106 \pm .0473$ .

Correlation Ratio:  $\eta'^2_{NP.CA} = .024,608,$

$$\bar{\eta}^2_{NP.CA} = .020,202 \pm .009,536.$$

#### POSITION OF NEAR POINT & CORNEAL ASTIGMATISM

(OBSERVERS A, B & C) ALIEN JEWISH BOYS

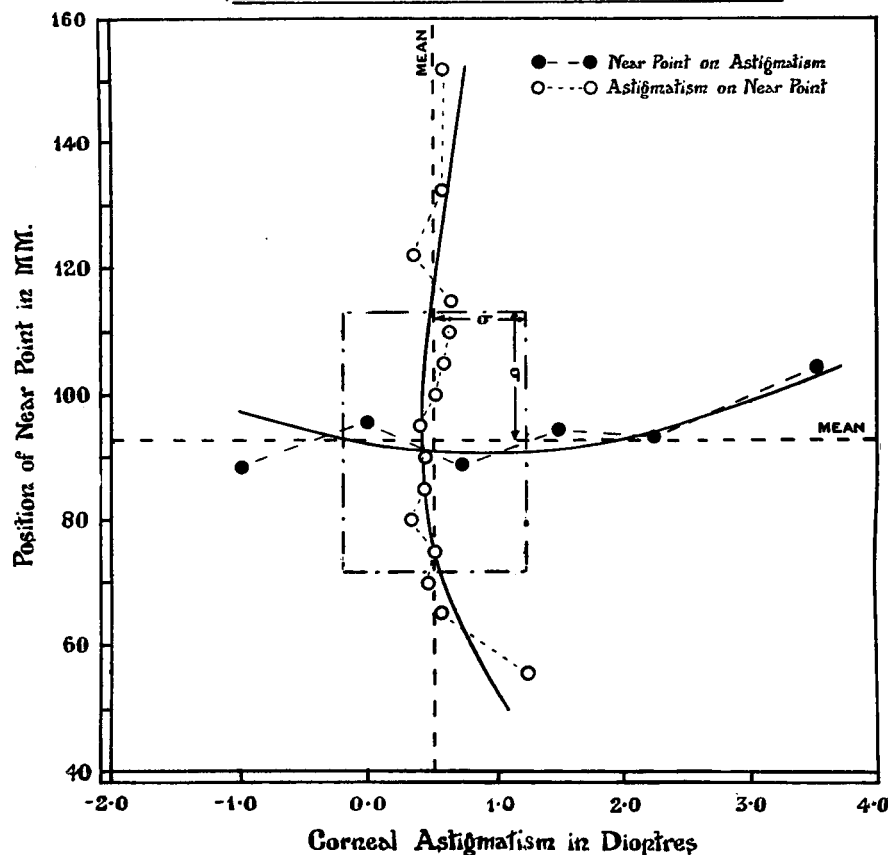


Diagram 101.

Table CCX. *Corneal Astigmatism and Distance of Near Point (A, B and C).*

Distance of Near Point in mm.

Corneal Astigmatism in Dioptres	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175	180	...	200	Totals
- 2.25	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
- 1.50	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
- 0.75	—	—	—	—	—	—	—	2	1.5	2.5	1	—	2	—	—	—	—	—	—	—	1.5	—	—	—	—	—	—	—	—	—	—	—	11.5
0.00	—	—	—	1	1	2	9	20	22	42	40	44	49	34	19	15	12	13	8	9	11	4	2	3	—	—	—	—	—	1	—	—	363
+ 0.75	1	—	—	—	3	8	13	25	19	35	32	43	26	19	13	12	9	3	4	4	2.5	2	1	1	—	—	—	—	—	—	—	—	275.5
+ 1.50	—	—	2	—	2	2	1.5	3.5	3	2	6	5	2	3	6	1	5	1	2	3	—	1	2	—	—	—	—	—	—	—	—	—	53
+ 2.25	—	—	—	—	1	2	1	1	5	2	2	4	1	5	2	2	2	1	—	2	—	—	—	—	—	—	—	1	—	—	—	—	34
+ 3.00	—	—	—	—	—	1	—	—	—	—	—	—	3	—	1	2	—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	9
+ 3.75	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
+ 4.50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+ 5.25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+ 6.00	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	—	2	2	7	15	24.5	51.5	51.5	84.5	81	98	84	62	42	32	28	18	14	19	16	7	5	4	—	1	—	1	1	1	..	1	754

It will be seen that  $\eta'^2_{NP.CA}$  is not significantly different from  $\bar{\eta}^2_{NP.CA}$ , and that further  $r$  does not differ significantly from zero, having regard to its probable error. We should not therefore be able to assert that the degree of Corneal Astigmatism does affect the Distance of the Near Point on the basis of these observations only.

Table CCXI. *Corneal Astigmatism and Distance of Near Point (A and B only).*

Distance of Near Point in mm.

Corneal Astigmatism in Dioptres (Central Values)	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	..	175	Totals
- 2.25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	..	—	—
- 1.50	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	..	—	1
- 0.75	—	—	—	2	1.5	2.5	1	—	2	—	—	1	—	—	—	—	—	—	..	—	10
0.00	—	—	6	9	2	19	6	6	3	2	—	3	—	—	—	—	—	—	..	—	56
+ 0.75	2	7	6	18	14	18	13	12	3	7	—	1	—	—	1	1	—	1	..	—	104
+ 1.50	2	—	0.5	2.5	1	1	2	2	1	1	—	—	1	—	—	1	—	—	..	—	15
+ 2.25	—	—	1	—	4	1	—	2	—	—	—	2	—	—	—	—	—	—	..	—	10
+ 3.00	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	..	1	2
Totals	4	8	13.5	31.5	22.5	42.5	22	22	9	10	1	6	1	—	1	2	—	1	..	1	198

Considering the relatively high relation of General Astigmatism and the Distance of the Near Point, it is somewhat surprising to find how little relation Corneal Astigmatism has to this distance. The inference is that it is the astigmatism of the lens, not the cornea, which influences the position of the Near Point.

(viii) *Corneal Astigmatism and Direction of Axis.* (Principal Axis nearer to horizontal.) It appears just worth while asking whether the slope of the axis is in any way related to the intensity of the astigmatism. We deal first with *A*, *B* and *C*'s records combined. Table CCXII provides the data.

From this table all those with no Corneal Astigmatism are excluded.

The following are the constants of this table:

Corneal Astigmatism, Mean: .9350 D.; Standard Deviation: .9170 D.

Direction of Axis, Mean: + 0°41'73 = 25'038; Standard Deviation: 8°8374.

Product Moment Correlation Coefficient:  $r = - .0597 \pm .0261$ .

But this is not very helpful, because the relationship may really be independent of the sign of the

Table CCXII. *Corneal Astigmatism and Direction of Axis to Horizontal.*

Direction of Axis in Degrees (Central Values)\*

Corneal Astigmatism in Dioptres (Central Values)	-45°	-40°	-35°	-30°	-25°	-20°	-15°	-10°	-5°	0	+5°	+10°	+15°	+20°	+25°	+30°	+35°	+40°	+45°	Totals
-2.25	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
-1.50	—	—	—	—	1	1	1	—	1	3	—	—	—	—	—	—	—	—	—	7
-0.75	—	1	—	—	—	—	—	—	1	12	—	—	1	—	—	2	—	1	—	18
0.00	1	—	—	1	—	—	—	2	1	76	2	1	—	1	—	—	1	2	1	89
+0.75	1	1	1	3	—	4	7	13	2	326	6	10	6	1	1	1	4	1	—	388
+1.50	—	—	—	—	—	2	4	1	1	55	2	4	1	—	3	1	1	—	—	75
+2.25	—	—	—	—	—	1	1	1	1	37	2.5	2.5	2	2	2	—	—	—	—	52
+3.00	—	—	1	—	—	—	1	2	—	9	2	2.5	1.5	—	—	—	—	—	—	19
+3.75	—	—	—	—	—	—	1	1	1	1	2	—	2	—	—	—	—	—	—	8
+4.50	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	—	3
+5.25	—	—	—	—	—	—	—	—	—	3	0.5	0.5	—	—	—	—	—	—	—	4
+6.00	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	2	2	2	4	1	8	15	20	8	525	17	21.5	13.5	5	6	4	6	4	1	665

angle. Accordingly it seemed best to find the means of the arrays of Corneal Astigmatism for given grades of angle. We find the following values:

Data for A, B and C		Data for A and B only	
Angle of Axis	Mean Corneal Astigmatism	Angle of Axis	Mean Corneal Astigmatism
-35°·00	+ ·4773 D.	-35°·00	+ ·3750 D.
-16°·74	+ 1·1087 D.	-16°·53	+ 1·2115 D.
-8°·57	+ 1·0714 D.	-8°·33	+ 1·1667 D.
0°·00	+ ·8571 D.	0°·00	+ ·7331 D.
+7°·79	+ 1·6364 D.	+7°·19	+ 1·3947 D.
+16°·35	+ 1·7838 D.	+13°·23	+ 1·4516 D.
+32°·62	+ ·7143 D.	+33°·125	+ ·4219 D.
General Population:	+ ·9350 D.	General Population:	+ ·8478 D.

It is clear from these results that there is a sort of rhythmic change of Corneal Astigmatism with the direction of the axis. We have a low value of the Astigmatism when the axis is horizontal; it rises on either side, reaches a maximum and then declines to become lower than the horizontal value when we reach 45°. Determining the measure of non-linearity we find for the correlation ratios:

For A, B and C:  $\eta'^2_{CA,DA} = \cdot071,518$ ,  $\bar{\eta}^2_{CA,DA} = \cdot009,023 \pm \cdot003,495$ .

Thus  $\eta'^2_{CA,DA}$  is significant and  $\eta'_{CA,DA} = \cdot2674$ .

For A and B only:  $\eta'^2_{CA,DA} = \cdot074,650$ ,  $\bar{\eta}^2_{CA,DA} = \cdot016,316 \pm \cdot006,288$ .

Thus  $\eta'^2_{CA,DA}$  is again significant and  $\eta'_{CA,DA} = \cdot2732$ .

Accordingly there does appear to be a relation between the intensity of the Corneal Astigmatism and its direction, this association having about the same magnitude for both series of observations.

The coefficient of correlation for A and B only is given by

$$r = -\cdot0751 \pm \cdot0350,$$

which differs insensibly from zero. This suggests that the actual distribution of Corneal Astigmatism about the horizontal is really given by a nearly symmetrical curve.

The additional constants of the A and B table are:

Corneal Astigmatism, Mean: ·8478 D.; Standard Deviation: ·9633 D.  
Direction of Axis, Mean: 1°·3247; Standard Deviation: 9°·1819.

The fact that in this case the correlation coefficient is practically zero, but the correlation ratio, nearly ·3, will serve to indicate the danger of trusting to the former only in such investigations as the present.

\* Angle positive when axis is elevated above horizontal on nasal side, negative when depressed below horizontal.

Table CCXIII. *Corneal Astigmatism and Direction of Axis (A and B only).*

Direction of Axis in Degrees (Central Values)

Corneal Astigmatism in Dioptres (Central Values)	40°	35°	30°	25°	20°	15°	10°	5°	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	Totals
- 2.25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
- 1.50	—	—	—	—	1	1	—	1	2	—	—	—	—	—	—	—	—	—	5
- 0.75	—	—	—	—	—	—	—	1	10	—	—	—	—	—	2	—	2	—	16
0.00	—	—	1	—	—	—	2	1	55	2	1	—	1	—	—	1	2	1	67
+ 0.75	1	—	—	—	2	3	12	2	158	6	8	6	—	1	1	2	—	—	202
+ 1.50	—	—	—	—	—	2	—	1	21	2	2	1	1	2	1	—	—	—	33
+ 2.25	—	—	—	—	1	1	1	1	14	2.5	0.5	1	2	1	—	—	—	—	25
+ 3.00	—	—	—	—	—	—	2	—	3	1	1.5	1.5	—	—	—	—	—	—	9
+ 3.75	—	—	—	—	—	2	—	2	—	—	—	—	—	—	—	—	—	—	6
+ 4.50	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	2
+ 5.25	—	—	—	—	—	—	—	—	2	0.5	0.5	—	—	—	—	—	—	—	3
Totals	1	—	1	—	4	9	18	9	266	15	13.5	10.5	5	4	4	3	4	1	368

The accompanying radiograms show for the two series the distribution of Corneal Astigmatism with Direction of Axis: see Diagrams 102 and 103.

If this symmetry of the distribution account for the lowness of the correlation coefficient (the sign of the axial angle not being related to the intensity of the Corneal Astigmatism) it is clear that we may combine the negative and positive angles of the same value. Our table then becomes:

Table CCXIII bis. *Corneal Astigmatism and Direction of Axis, disregarding sign.*

A, B and C's data

A and B's data only

	Corneal Astigmatism in Dioptres											Totals	Corneal Astigmatism in Dioptres											Totals
	- 2.25	- 1.50	- 0.75	0.00	+ 0.75	+ 1.50	+ 2.25	+ 3.00	+ 3.75	+ 4.50	+ 5.25		+ 6.00	- 1.50	- 0.75	0.00	+ 0.75	+ 1.50	+ 2.25	+ 3.00	+ 3.75	+ 4.50	+ 5.25	
0°	1	3	12	76	326	55	37	9	1	2	3	—	525	2	10	55	158	21	14	3	—	1	2	266
± 5°	—	1	1	3	8	3	3.5	2	3	—	0.5	—	25	1	1	3	8	3	3.5	1	3	—	0.5	24
± 10°	—	—	—	3	23	5	3.5	4.5	1	1	0.5	—	41.5	—	—	3	20	2	1.5	3.5	1	—	0.5	31.5
± 15°	—	—	—	—	13	5	3	2.5	3	—	—	—	28.5	1	1	—	9	1	2	1.5	2	—	—	19.5
± 20°	—	1	—	1	5	2	3	—	—	—	—	1	13	1	—	1	2	3	3	—	1	—	—	9
± 25°	—	1	—	—	1	3	2	—	—	—	—	—	7	—	—	—	1	2	1	—	—	—	—	4
± 30°	—	—	2	1	4	1	—	—	—	—	—	—	8	—	2	1	1	1	—	—	—	—	—	5
± 35°	—	—	—	1	5	1	—	1	—	—	—	—	8	—	—	1	2	—	—	—	—	—	—	3
± 40°	—	—	2	2	2	—	—	—	—	—	—	—	6	—	2	2	1	—	—	—	—	—	—	5
± 45°	—	—	—	2	1	—	—	—	—	—	—	—	3	—	—	1	—	—	—	—	—	—	—	1
Totals	1	7	18	89	388	75	52	19	8	3	4	1	665	5	16	67	202	33	25	9	6	2	3	368

The constants of this table are:

For A, B and C: Corneal Astigmatism, Mean: ·9350 D; Standard Deviation: ·9170 D.

For A and B only: Corneal Astigmatism, Mean: ·8478 D; Standard Deviation: ·9633 D.

For A, B and C: Direction of Axis, Mean: 3°·4549; Standard Deviation: 8°·2715.

For A and B only: Direction of Axis, Mean: 4°·0965; Standard Deviation: 8°·4477.

Means of Arrays:

Data for A, B and C

Grade of Corneal Astigmatism	Mean Angle of Axis
- 1.59 D.	8°·125
- 0.75 D.	8°·889
0.00 D.	3°·371
+ 0.75 D.	2°·603
+ 1.50 D.	4°·267
+ 2.25 D.	3°·990
+ 3.00 D.	6°·711
+ 4.41 D.	6°·719

General Population: 3°·4549

Data for A and B only

Grade of Corneal Astigmatism	Mean Angle of Axis
- 1.50 D.	8°·000
- 0.75 D.	10°·000
0.00 D.	3°·806
+ 0.75 D.	2°·871
+ 1.50 D.	5°·455
+ 2.25 D.	5°·900
+ 3.00 D.	6°·944
+ 4.30 D.	7°·500

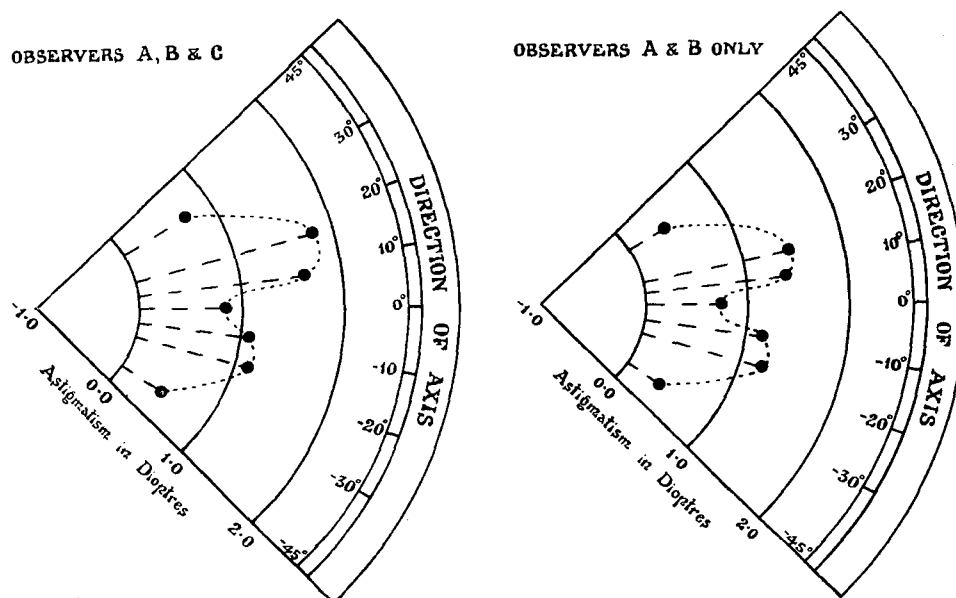
General Population: 4°·0965

Here, although the results are somewhat irregular, it is clear that the angle gets larger as the Corneal Astigmatism increases, whether with, or against, the rule. As a general measure of the association we can find  $\eta'_{DA,CA}$ . We have:

$$\begin{array}{ll} \text{For } A, B \text{ and } C: & \eta'^2_{DA,CA} = \cdot031114, & \bar{\eta}^2_{DA,CA} = \cdot010,526 \pm \cdot003,772. \\ \text{For } A \text{ and } B \text{ only:} & \eta'^2_{DA,CA} = \cdot048,941, & \bar{\eta}^2_{DA,CA} = \cdot019,036 \pm \cdot006,787. \end{array}$$

### CORNEAL ASTIGMATISM & DIRECTION OF AXIS.

#### ALIEN JEWISH BOYS



Diagrams 102, 103.

In both cases  $\eta'^2$  is significant, having regard to  $\bar{\eta}^2$  and its probable error. The distributions for  $A, B$  and  $C$  and for  $A, B$  only are very similar, and the values of  $\eta'$  are:

$$\begin{array}{ll} \text{for } A, B \text{ and } C: & \eta'_{DA,CA} = \cdot1765, \\ \text{and for } A \text{ and } B \text{ only:} & \eta'_{DA,CA} = \cdot2212. \end{array}$$

These values are less than we have found for the correlation ratios in the like series of General Astigmatism on Direction of Axis (p. 207). The plotted array-means are shown for both cases in Diagram 104. They have been graduated by aid of the spline only, as the apparent existence of two points of inflection disqualified the use of a cubic, and the subject did not seem of sufficient importance to justify the great labour involved in fitting quartics. But the graphs indicate a possible evolutionary tendency for a principal meridian to approach the vertical.

### DIRECTION OF AXIS & CORNEAL ASTIGMATISM

#### ALIEN JEWISH BOYS

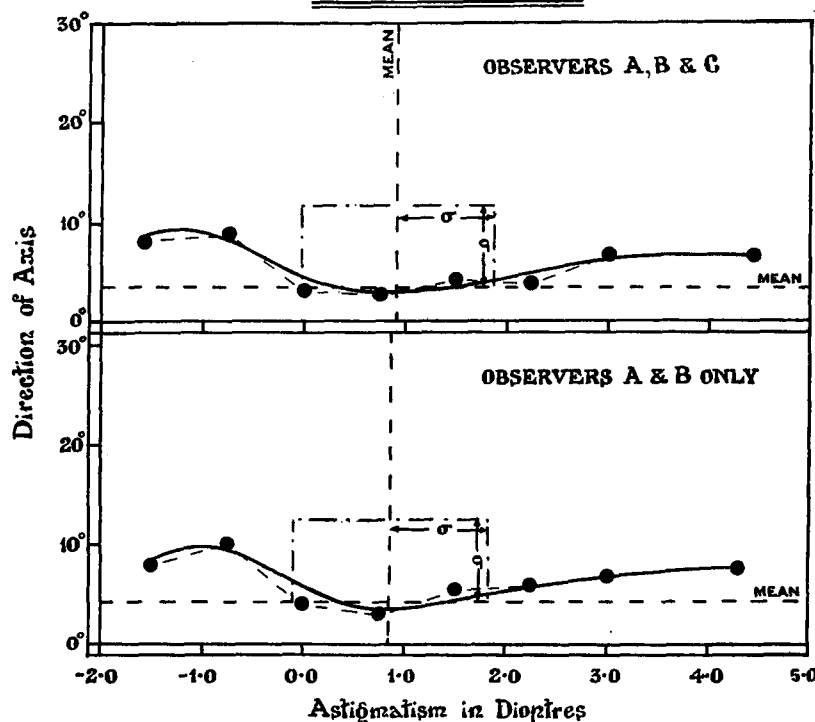


Diagram 104.

(g) *Distance of the Near Point.* (i) *Distances of the Near Points of Right and Left Eyes.* Our material is exhibited in Table CCXIV below.

Table CCXIV. *Near Point of Right Eye and Near Point of Left Eye.*

Left Eye. Distance of Near Point in mm. (Central Values)

Right Eye. Distance of Near Point in mm. (Central Values)	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175	Totals
45	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
55	—	—	—	—	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
60	—	—	—	—	1	2	2	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7
65	—	—	—	1	—	1	4	3	1	0.5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	11.5
70	—	—	—	—	1	1	3	11	4.5	2	1	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	25.5
75	—	—	—	—	—	1	3	6	8	7	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	28
80	—	—	—	—	—	1	—	1	3	21	10	8	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	46
85	1	—	—	—	—	—	1	—	—	2	14	6	4	—	2	—	—	—	1	—	—	—	—	—	—	—	—	—	—	38
90	—	—	—	—	—	—	—	1	2	7	9	18	4	4	2	1	1	—	—	1	—	—	—	—	—	—	—	—	—	50
95	—	—	—	—	—	—	—	—	—	1	4	13	15	11	3	3	—	1	—	—	—	—	—	—	—	—	—	—	—	51
100	—	—	—	—	—	—	—	—	—	1	1	4	8	9	5	1	—	—	—	—	1	—	—	—	—	—	—	—	—	31
105	—	—	—	—	—	—	—	1	—	—	—	3	2	5	6	1	1	—	—	1	—	1	—	—	—	—	—	—	—	21
110	—	—	—	—	—	—	—	—	—	—	1	—	1	4	2	3	1	1	—	2	—	—	—	—	—	—	—	—	—	16
115	—	—	—	—	—	—	—	—	—	—	1	1	—	1	3	3	3	2	—	1	2	1	—	—	—	—	—	—	—	18
120	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	3	—	—	—	—	—	—	—	—	—	—	6
125	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	1	1	1	1	—	—	—	—	—	—	—	—	5
130	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	2	—	2	2	1	—	—	—	—	—	—	—	11
135	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	1	2	—	—	—	—	—	—	—	—	6
140	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	3
145	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	3
150	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
155	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
165	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
170	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—
175	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
180	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
185	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
190	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
195	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
200	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—
Totals	1	—	1	1	4	8	13	26	23.5	42.5	43	51	35	34	22	19	10	12	9	10	10	4	2	3	—	1	—	—	1	386

The constants of this table are as follows:

Distance of Near Point, Right Eye, Mean:  $92.856 \pm .665$  mm.;

Standard Deviation:  $19.8950 \pm .4831$  mm.

Distance of Near Point, Left Eye, Mean:  $93.335 \pm .705$  mm.;

Standard Deviation:  $20.5440 \pm .4988$  mm.

No significant difference between Right and Left Eyes is indicated in either mean or variability.

Product Moment Coefficient of Correlation:  $r = .7828 \pm .0133$ .

The high correlation and an examination of Diagram 105 indicate that we shall hardly better a straight line graduation. There is, however, some indication that for great distances of the Near Point of one eye, that of the other eye scarcely keeps pace with it. The equations to the Regression straight lines are:

$$\tilde{E}_L = .80834E_R + 18.2765, \quad \tilde{E}_R = .75807E_L + 22.1013,$$

where  $\tilde{E}_L$  and  $\tilde{E}_R$  are the probable Near Point Distances of Left and Right Eyes for given Near Point Distances  $E_R$  and  $E_L$  of Right and Left Eyes respectively.

For the association of the Near Point with: (ii) *Visual Acuity*, see p. 163; (iii) *Refraction Class*, see p. 171; (iv) *General Refraction*, see p. 183; (v) *General Astigmatism*, see p. 193; (vi) *Corneal Refraction*, see p. 202; (vii) *Corneal Astigmatism*, see p. 204.

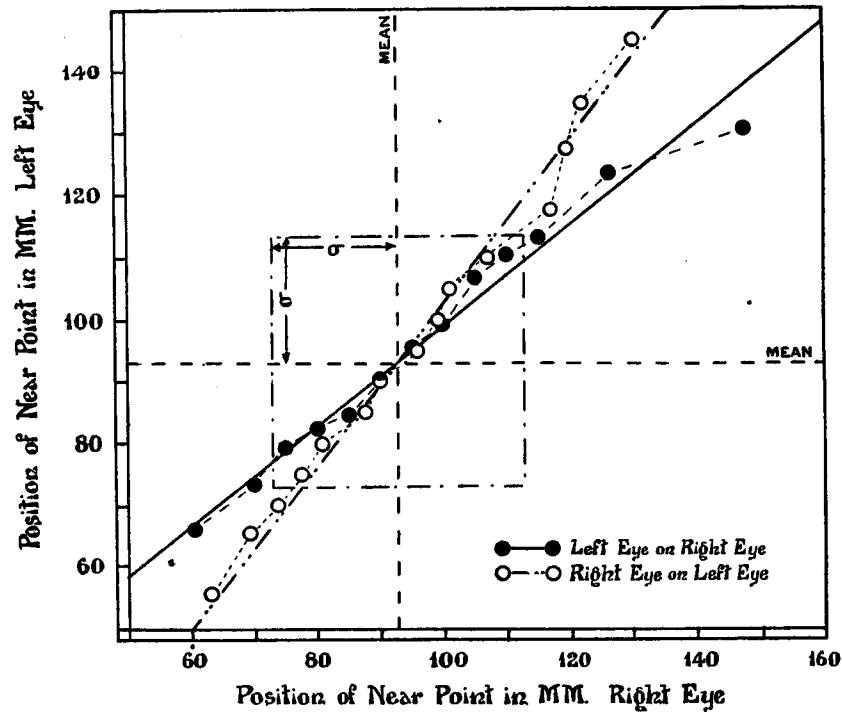
POSITION OF NEAR POINT - LEFT EYE & RIGHT EYEALIEN JEWISH BOYS

Diagram 105.

(h) *Monocular Accommodation in relation to Age and Visual Acuity.* (i) *Influence of Age on Accommodation in Children.* From the Near Point we naturally turn to the Monocular Accommodation, taking it to be  $100/d + R$ , where  $d$  is the Distance of the Near Point in centimetres and  $R$  the General Refraction for the eye.

The accompanying Table CCXV gives the correlation of Age and Accommodation.

Table CCXV. *Correlation of Accommodation and Age.*

Accommodation in Dioptres

Age in Years (Central Values)		2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	Totals
	7-7083	—	—	—	—	—	—	2	1	1	—	—	—	2	—	—	—	—	—	—	6
	8-2083	—	—	—	—	—	—	—	—	—	—	2	—	1	—	1	—	—	—	—	4
	8-7083	—	—	—	—	—	—	1	2	3	2	1	—	1	—	—	—	—	—	—	10
	9-2083	—	—	—	—	1	2	4	8	4	3	2	2	1	2	1	—	—	—	—	30
	9-7083	—	—	—	—	—	4	2	1	9	10	4	—	2	1	—	—	—	—	1	34
	10-2083	1	1	—	2	2	—	—	2	3	7	6	6	4	—	—	—	—	—	—	34
	10-7083	—	—	—	—	3	3	4	6	13	8	2	3	1	1	—	—	—	—	—	44
	11-2083	—	—	—	—	1	3	7	10	11	17	11	7	1	—	—	—	—	—	—	68
	11-7083	—	—	—	—	—	1	4	4	12	12	3	4	3	1	—	—	—	—	—	44
	12-2083	—	—	—	3	2	2	5	16	12	23	8	4	4	4	—	1	—	—	—	84
	12-7083	—	—	1	2	3	8	5	13	15	12	7	17	7	1	5	1	1	—	—	98
	13-2083	—	—	—	—	2	2	12	10	25	19	17	17	4	6	—	—	—	—	—	114
	13-7083	—	—	—	1	3	3	7	4	13	16	18	11	5	1	4	—	2	—	—	88
	14-2083	—	—	—	—	—	—	2	4	7	2	3	1	1	1	—	1	—	—	—	22
	14-7083	—	—	—	—	—	—	1	1	1	3	1	1	—	—	—	—	—	—	—	8
15-2083	—	—	—	—	1	1	—	—	1	2	1	—	1	1	—	—	—	—	—	8	
Totals	1	1	1	8	18	29	56	82	130	136	86	73	38	19	11	3	3	—	1	696	

The constants of this table are as follows:

Age, Mean: 12.0732 years; Standard Deviation, 1.5274 years.  
 Accommodation, Mean: 11.1411 D.; Standard Deviation, 2.3576 D.  
 Product Moment Correlation Coefficient:  $r = +.0792 \pm .0254$ .

Correlation Ratio of Accommodation on Age in years:

$$\eta'^2_{Acc.Y} = .028,109, \quad \cdot\bar{\eta}^2_{Acc.Y} = .021,552 \pm .005,248.$$

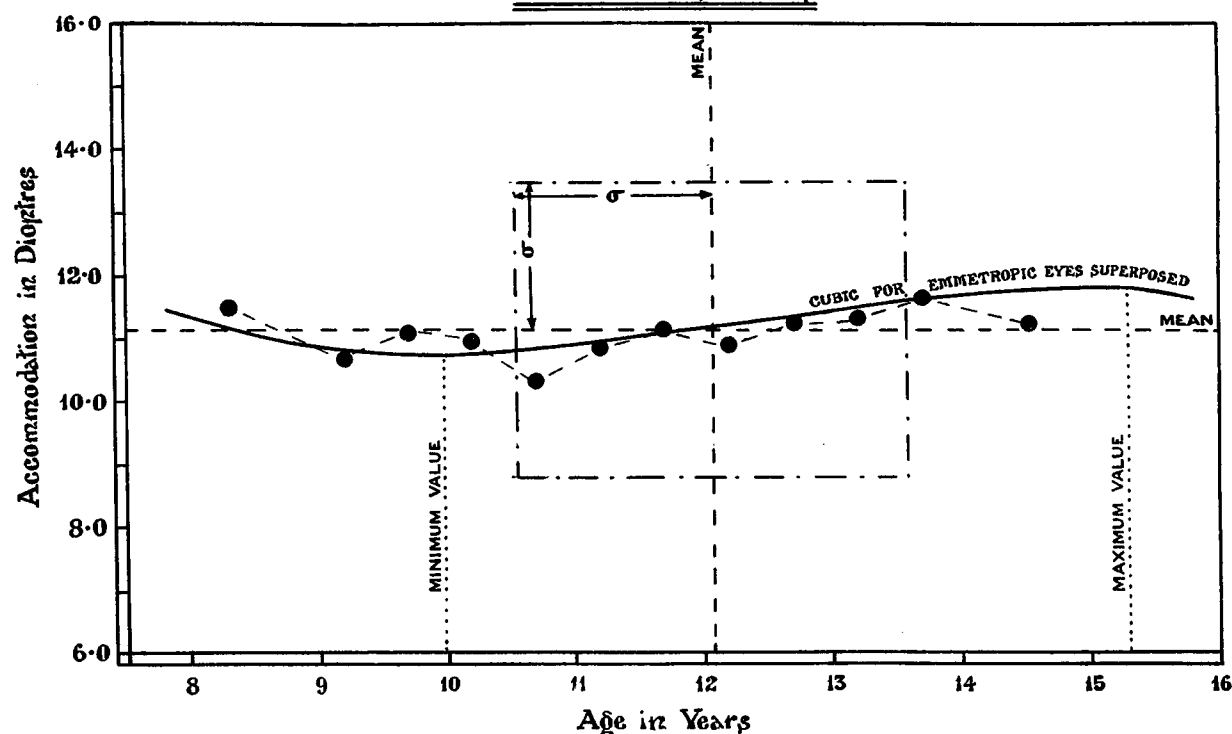
These results indicate that the correlation ratio is not significantly different from zero. On the other hand, the correlation coefficient might be considered as just significant, but its small intensity indicates an *increase* of accommodation with age. The array-means are as follows:

Grade of Age	Mean Accommodation
8.3083 years	11.500 D. $\pm$ .356
9.2083 "	10.683 D. $\pm$ .290
9.7083 "	11.156 D. $\pm$ .273
10.2083 "	10.979 D. $\pm$ .273
10.7083 "	10.336 D. $\pm$ .240
11.2083 "	10.847 D. $\pm$ .193
11.7083 "	11.167 D. $\pm$ .240
12.2083 "	10.914 D. $\pm$ .173
12.7083 "	11.266 D. $\pm$ .161
13.2083 "	11.345 D. $\pm$ .149
13.7083 "	11.654 D. $\pm$ .170
14.5241 "	11.240 D. $\pm$ .258

General Population: 11.141 D.  $\pm$  .060

Only two of these array-means can be considered as possibly significant, and the whole series shows no orderly sequence (see Diagram 106). We were accordingly driven to the conclusion that

### ACCOMMODATION & AGE (ALL EYES) ALIEN JEWISH BOYS



Age in Years

Diagram 106.

(The cubic from Diagram 107 has been superposed for comparison.)

between the ages of 8 and 15 years no definite change in accommodation was to be traced in our Jewish material. This seems so opposed to the views expressed in current textbooks that we went further into the matter. As far as we have been able to trace the statements they appear all to go back directly or indirectly to Donders' epoch-making work; there appear to be no fresh detailed measurements on age and accommodation actually provided\*. Now Donders' age curve is for emmetropic eyes only and it occurred to us that we may have confused the issue by dealing with all eyes. We accordingly extracted the Near Points and formed the accommodation for all emmetropic eyes, reckoning as such those with refraction not more than  $\pm .25$  D. We found 354 such eyes arranged as in Table CCXVI.

Table CCXVI. *Correlation of Age and Accommodation. Emmetropic Eyes.*  
Age in Years (Central Values)

Accommodation in Dioptres	Age in Years (Central Values)															Totals	
	7-7083	8-2083	8-7083	9-2083	9-7083	10-2083	10-7083	11-2083	11-7083	12-2083	12-7083	13-2083	13-7083	14-2083	14-7083		15-2083
5-6	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
6-7	—	—	—	1	—	2	2	1	—	1	—	—	1	—	—	—	8
7-8	—	—	—	2	3	—	3	2	1	2	6	—	1	—	—	—	20
8-9	2	—	1	4	1	—	2	3	3	2	3	4	1	2	1	—	29
9-10	1	—	—	8	—	1	3	6	—	9	5	4	1	1	1	—	40
10-11	1	—	2	4	7	1	9	5	7	7	10	10	7	6	—	1	77
11-12	—	—	1	3	7	6	6	9	9	12	6	5	9	2	—	1	76
12-13	—	2	1	2	4	5	1	3	2	3	3	6	7	1	—	—	40
13-14	—	—	—	1	—	4	2	1	1	2	4	6	6	1	—	—	28
14-15	2	1	1	—	1	2	—	—	1	1	3	3	2	—	—	1	18
15-16	—	—	—	2	1	—	—	—	—	4	—	2	—	—	—	—	9
16-17	—	—	—	1	—	—	—	—	—	—	1	—	2	—	—	—	4
17-18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18-19	—	—	—	—	—	—	—	—	—	—	1	—	2	—	—	—	3
19-20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20-21	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
Totals	6	3	6	28	25	22	28	30	24	43	42	40	39	13	2	3	354

The constants of this table are:

Age, Mean: 11·7139 years; Standard Deviation: 1·6588 years.  
Accommodation, Mean: 11·0715 D.; Standard Deviation: 2·2296 D.

Thus the emmetropic boys are slightly younger and more variable in age, but the accommodation remains very nearly the same. The array-means here exhibit a more orderly sequence:

Grade of Age	Mean Accommodation
8-2083 years	11·517 D. $\pm$ .388
9-4441 "	10·883 D. $\pm$ .207
10-4883 "	10·690 D. $\pm$ .213
11-4305 "	10·580 D. $\pm$ .205
12-2083 "	11·031 D. $\pm$ .229
12-7083 "	10·950 D. $\pm$ .232
13-2083 "	11·575 D. $\pm$ .238
13-7083 "	12·168 D. $\pm$ .241
14-4305 "	10·672 D. $\pm$ .354
General Population:	11·072 D. $\pm$ .080

Diagram 107 shows these means and we have graduated them with a cubic. We see that although the arrays give a fairly orderly series of points, the probable errors of the system of means would not alone justify us in stressing the differences. But we begin to see that there is not a *continuous* fall of accommodation with age; there is a fall from 8 years to 11·5 or 12 years,

\* This is for example essentially the case in Fuchs' textbook. He gives no reference beyond Donders. The smaller books extract from each other without reference, but their data are from Donders.

then a rise during the pubescent years followed by a probable fall about 15, which no doubt would be continuous throughout the remainder of life\*. Representing the association by a single constant we have:

$$\eta'^2_{Acc.Y} = .048,743, \quad \bar{\eta}^2_{Acc.Y} = .022,599 \pm .007,525.$$

$\eta'^2_{Acc.Y}$  is therefore probably significant as compared with  $\bar{\eta}^2_{Acc.Y}$  and we have an association measured by  $\eta'_{Acc.Y} = .2208$ , a small but definite relationship. This relationship would undoubtedly disappear, if we combined the prepubescent dip with the pubescent rise. Now, how

### ACCOMMODATION & AGE (EMMETROPIC EYES)

#### ALIEN JEWISH BOYS

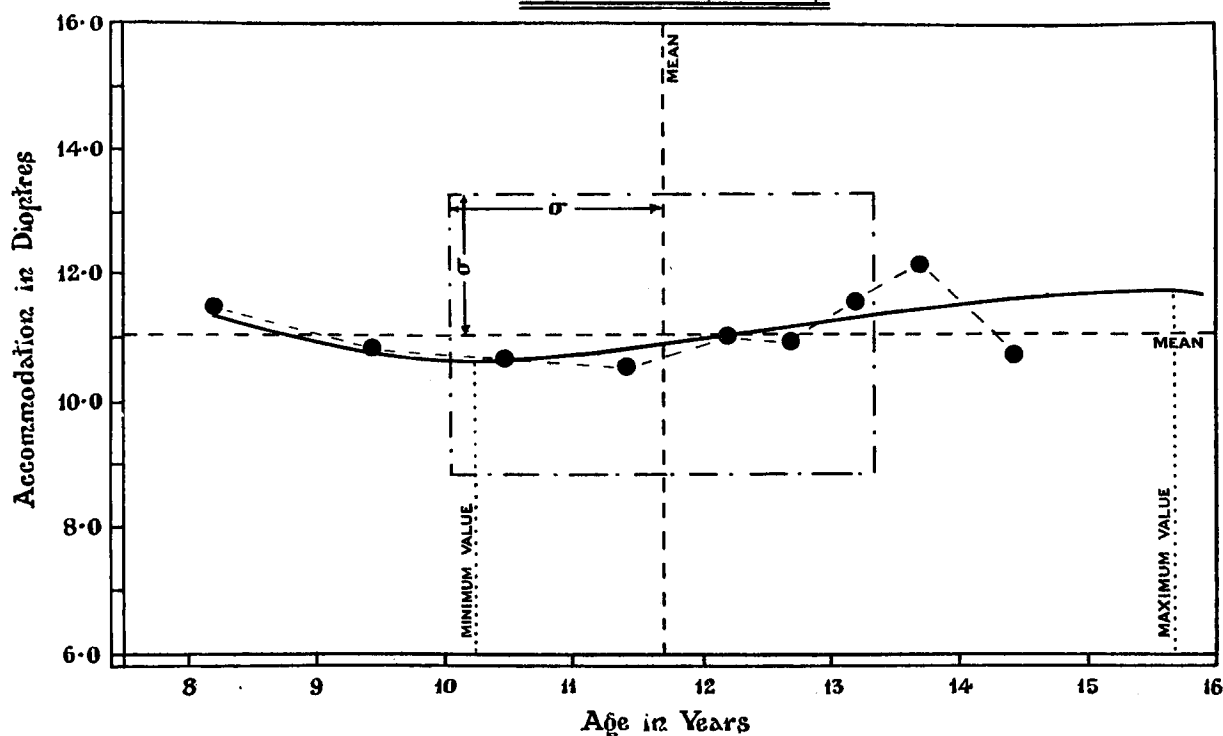


Diagram 107.

do we meet the statements of the textbooks that accommodation of emmetropic eyes continually decreases with age? We believe all these statements are based ultimately on the measurements of Donders and particularly on Fig. 104 of p. 207 of his work† (see our Diagrams 108 and 109). This diagram is the one reproduced by Fuchs (*Text-Book of Ophthalmology*, 2nd ed., 1905) with the vertical scale changed to that of dioptres, and it is from Fuchs or the original Donders that the tables in the smaller textbooks of ophthalmology have been obtained; like Donders they all begin at 10 years of age. Here we have a perfectly continuous falling curve. How did Donders obtain it? Luckily, while we have not the individual measurements on which Donders

\* Graduating by a cubic we find for its equation

$$\bar{Acc} = 10.89320 + .26936 (Y - 11.7083) + .05388 (Y - 11.7083)^2 - .01800 (Y - 11.7083)^3,$$

where  $\bar{Acc}$  is the mean or probable accommodation in dioptres for a boy of age  $Y$  in years. The minimum value of the accommodation is found from this cubic to be 10.67 at 10.24 yrs., and the maximum value just after puberty (at 15.68 yrs.) is 11.72. Thus the total average difference in accommodation for the seven years under consideration is only about one dipotre.

† *On the Anomalies of Accommodation and the Refraction of the Eye*, New Sydenham Society, 1864.

based his curve, he has given us in his Fig. 107 (p. 209; it should be labelled, as called in Donders' text, Fig. 105) one of the earliest scatter-diagrams exhibiting the correlation of accommodation and age. It thus becomes evident, that while the *general sweep* of his swarm of dots is undeniably given by his Fig. 104, there are only 16 dots for the years 10 to 15. It is quite clear that on the basis of

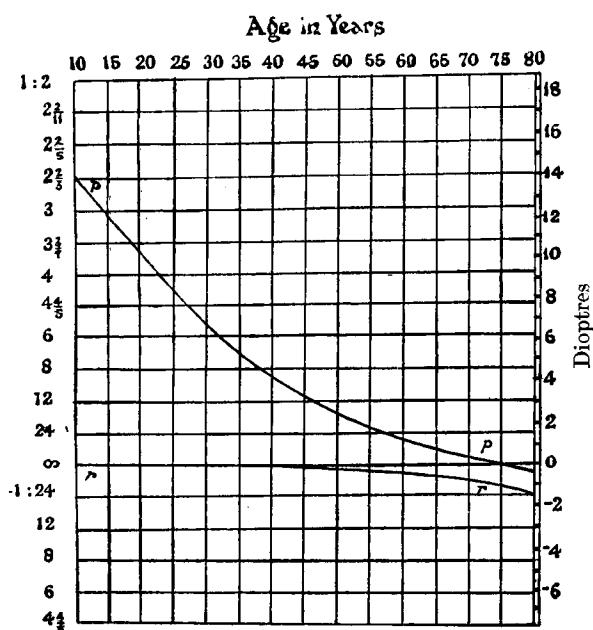


Diagram 108. Donders' Accommodation Curve.

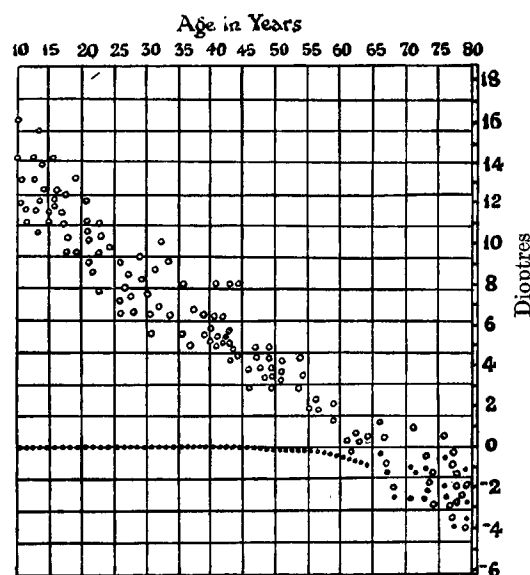


Diagram 109. Donders' Data.

these 16 dots, it would have been hard to determine the nature of his curve between 10 and 15, and accordingly Donders, as we have said, has taken the general sweep of his scatter-diagram from 10 to 80 years of age, wholly disregarding the fact that his curve does not by a considerable distance pass through the mean of his 10 to 15 years group. We give on p. 216 the 16 points of Donders for these 6 years on an enlarged scale taken as accurately as it was possible to do it from his diagram on p. 209. The mean of them for ages 10 to 15 is 12.82 D.\* The individuals were presumably of Dutch extraction and accordingly we must infer that our children of Jewish extraction, with an average accommodation of 11.07 D., were inferior to the Europeans, in so far as their Near Point was farther off. It would be of interest to know whether the accommodation of all middle eastern races is less than that of the European†.

We think that any conclusion that it has been shown once and for all by Donders that accommodation uniformly decreases from 10 to 80 years of age cannot be accepted, and that our very small correlation between accommodation and age between the years 8 and 15 is not really contradicted by Donders' observations when these are analysed. He has, indeed, overlooked the noteworthy changes which occur in most growth curves as they reach the pubescent years. When, however, we approach Donders' observations with the knowledge derived from our Jewish Boys, it is not hard to see that they not only do not refute but in a general way confirm our

\* This is the mean found from Donders' 16 observations between ages 10 and 15, not of course the value read from his smoothed curve for the entire "swarm" of his observations from age 10 to age 80, which is about 13.4 D.

† Observations made in our Laboratory give for the Accommodation of eyes of Jewish undergraduates 18-25 years of age (Mean Age: 21.27 yrs.) 8.805 D. Donders' data—probably for Europeans—show 10 D. for this age. But Donders was working with emmetropic eyes only. There were scarcely any of these Jewish students (about 4 %) who had emmetropic eyes; their mean accommodation was 8.67 D., which seems to indicate that there is not much difference on the average in this case between all eyes and emmetropic eyes.

results. In Diagram 110 below we have placed Donders' observations indicated by small circles in their approximate arrays, and then marked by a cross the means of each array of two or more points. These means exhibited as crosses were then joined by a splined curve; we thus get a regression curve of accommodation on age singularly like that of our 354 observations, namely, there is a fall in accommodation from 10 to 12, followed by a pubescent rise and then a further rapid fall. We hold therefore that the Donders 10 to 80 years' accommodation curve cannot be cited as contravening our results; indeed his isolated values are in accordance with them. There is very little change in accommodation on the whole between 8 and 15 years, the loss of the first three years being regained in the next three; thus we find the correlation coefficient extremely small, but this is not true of the correlation ratio.

(ii) *Monocular Accommodation and Visual Acuity.*

We have already indicated that as there is an association between the Distance of the Near Point and Visual Acuity (see p. 163) and between General Refraction and Visual Acuity (see p. 153), we should anticipate that our measurement of accommodation will be influenced by the degree of Visual Acuity. Our data are given in Table CCXVII.

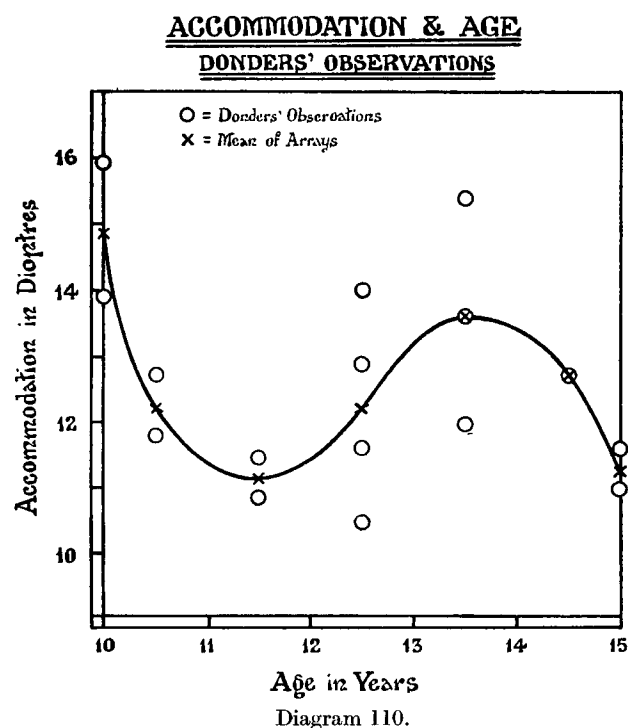


Table CCXVII. *Monocular Accommodation and Visual Acuity.*

		Accommodation in Dioptres																			
Visual Acuity (Central Values)		2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	Totals
	1-50	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	3
	1-40	—	—	—	—	—	—	—	—	—	—	—	3	4	2	2	—	—	—	—	11
	1-29	—	—	—	1	—	2	5	8	20	19	9	2	2	—	—	—	1	—	—	69
	1-11	—	—	—	—	2	7	19	25	36	36	15	13	6	6	2	—	1	—	—	169
	.91	—	—	—	—	3	3	3	13	32	35	28	29	12	4	3	1	1	—	—	167
	.75	—	—	—	—	3	5	12	14	17	15	10	6	6	2	1	—	—	—	—	91
	.58	—	—	—	—	3	6	4	9	9	17	11	6	2	—	—	1	—	—	—	68
	.37	—	—	—	2	3	2	4	6	7	6	4	6	4	1	—	1	—	—	—	46
	.25	—	—	—	—	1	—	2	2	5	1	1	6	1	—	—	—	—	—	—	19
	.14	1	1	—	2	2	—	2	1	2	5	3	3	1	—	—	—	—	—	—	23
	.08	—	—	1	2	1	2	2	1	1	—	—	—	—	—	—	—	—	—	—	10
	.04	—	—	—	1	—	1	2	1	1	1	1	—	—	—	—	—	—	—	—	8
Totals	1	1	1	8	18	28	55	80	130	135	82	75	38	16	9	3	3	—	1	684	

The constants of this table are as follows:

Visual Acuity, Mean: .8513; Standard Deviation: .3335.  
Accommodation, Mean: 11.1195 D.; Standard Deviation: 2.3366 D.  
Product Moment Coefficient of Correlation:  $r = .1890 \pm .0249$ .

Correlation Ratios:

$$\eta'^2_{VA.Acc} = .104,000, \quad \bar{\eta}^2_{VA.Acc} = .024,854 \pm .005,765,$$

$$\eta'^2_{Acc.VA} = .140,334, \quad \bar{\eta}^2_{Acc.VA} = .016,082 \pm .004,588.$$

Both  $\eta'^2$ 's are significant having regard to the probable errors of the  $\bar{\eta}^2$ 's. We have accordingly:

$$\eta'_{VA, Acc} = .3225, \quad \eta'_{Acc, VA} = .3746.$$

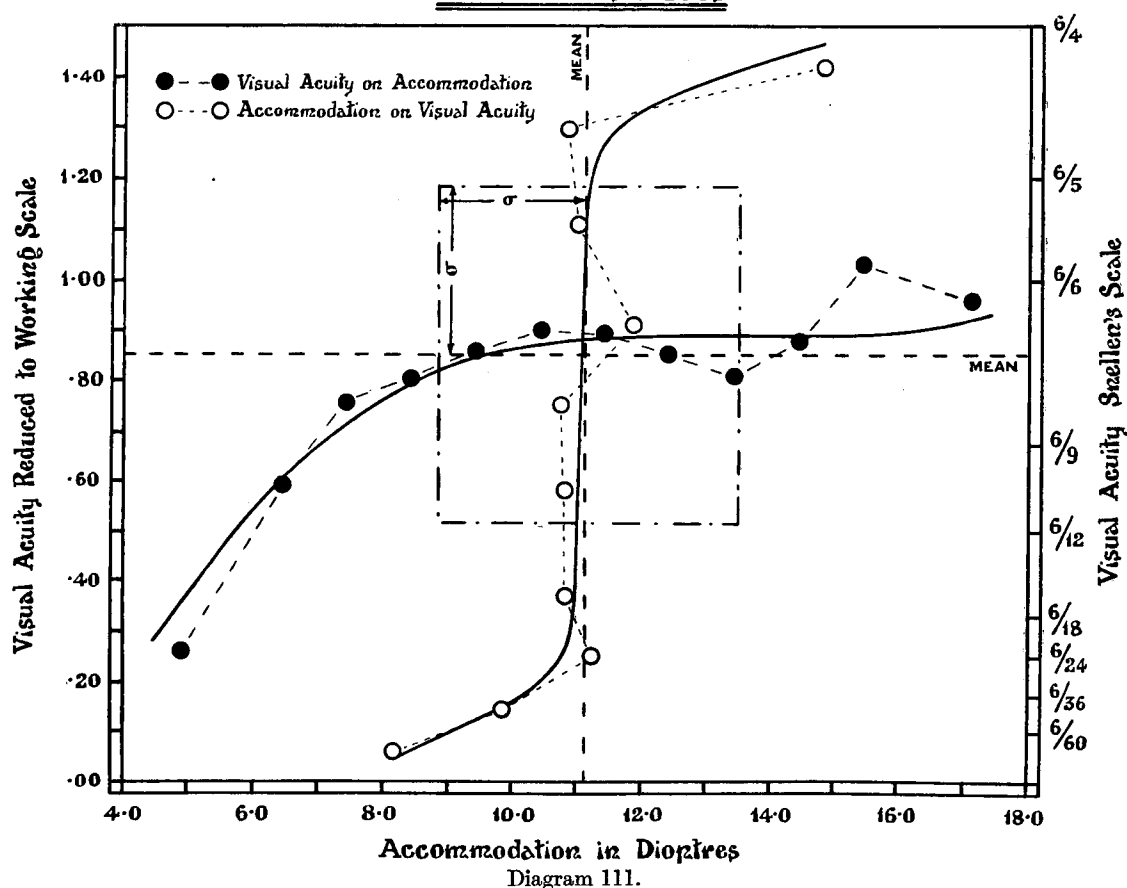
The large values of the correlation ratios relative to the value of the correlation coefficient suggest that the regression lines will be found to be skew. This is confirmed by the array-means:

Grade of Accommodation	Mean Visual Acuity	Grade of Visual Acuity	Mean Accommodation
4.904 D.	.2609	1.42	14.81 D.
6.45 D.	.5922	1.29	10.88 D.
7.45 D.	.7589	1.11	11.02 D.
8.45 D.	.8061	.91	11.88 D.
9.45 D.	.8575	.75	10.75 D.
10.45 D.	.9007	.58	10.82 D.
11.45 D.	.8936	.37	10.84 D.
12.45 D.	.8513	.25	11.24 D.
13.45 D.	.8163	.14	9.84 D.
14.45 D.	.8761	.06	8.17 D.
15.45 D.	1.0294		
17.13 D.	.9576		
General Population:	.8513	General Population:	11.1195 D.

It will be seen at once from these results and from Diagram 111\* that the two ocular characters

### VISUAL ACUITY & ACCOMMODATION

#### ALIEN JEWISH BOYS



\* Visual Acuity on Accommodation was graduated by the cubic

$$\bar{V}.A. = .87472 + .01868 (Acc. - 10.45) - .007,855 (Acc. - 10.45)^2 + .000,920 (Acc. - 10.45)^3.$$

Several attempts were made to get a reasonable algebraical graduation of Accommodation on Visual Acuity, but we were at last forced to use the spline to indicate the nature of the relationship.

Accommodation and Visual Acuity are not independent. As the Accommodation increases so does the Visual Acuity, although for the range of Accommodation 9 to 13 D. there is a check to this increase. In the same way with a very low Visual Acuity there is a low Accommodation, from Visual Acuity .25 to 1.30 there is an almost steady Accommodation, but with better vision than 1.30 (than Snellen 6/5, say) there is a rapid rise in Accommodation. It is usually asserted that Visual Acuity is a function of the nervous system of the eye and that the Accommodation depends on muscular action on the refracting power of the eye. It would seem from the present results that either there is an association between the nervous system and this muscular action, or what is more probable that the present method of determining the Near Point really depends on Visual Acuity. At the same time, our investigations have shown that there is a very close association between Visual Acuity and General Refraction. Hence it seemed *a priori* certain that the Far Point would be correlated with Visual Acuity, and so, as is actually the case, there would be a correlation between Accommodation, as it is now measured, and Visual Acuity. There may be two separate factors, Accommodation a purely muscular power, and Visual Acuity a function of the nervous system of the eye, absolutely independent, as some authors have suggested, one of the other, but unless we can suppose that our observations were badly made, we must assert that the quantities now measured under these names are not independent.

(i) *Influence of Pigmentation of the Fundus on Ocular Characters.* (i) *Relation of Fundus to Eye (Iris) and Hair Pigmentation.* The fundus was observed and classified in the four categories, Light, Medium Light, Medium Dark, Dark. In order to test the significance of this classification, i.e. to ascertain whether we were measuring something of anthropometric value, we asked ourselves first what association it would show with Iris and Hair Pigmentation. As we have already stated, the former was classified by aid of Rudolf Martin's Eye Scale, which provides 16 standard eyes\*, and the latter by the aid of Eugen Fischer's glass thread hair scale.

Table CCXVIII. *Fundus and Eye (Iris) Colour.*

Eye Shade on Martin's Scale																	Fourfold Table				
																	Eye Colour				
Fundus	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Totals	Fundus	Light (1-6)	Dark (7-16)	Totals
Light ...	—	—	—	—	1	1	2	1	—	3	—	0.5	1	1	2	—	12.5				
Medium Light	—	—	3	38	23.5	23	31	34	6	13	14	27.5	12	25.5	10	1	261.5				
Medium Dark	—	4	22	86	22.5	19	12	8	—	2	5	2	6	3.5	2	2	196				
Dark ...	—	1	5	18	3	1	—	1	1	—	—	—	—	1	—	—	31	Light Dark	89.5 181.5	184.5 45.5	274 227
Totals ...	—	5	30	142	50	44	45	44	7	18	19	30	19	31	14	3	501	Totals	271	230	501

It seemed adequate for our present purpose to condense the large table into a fourfold and then determine the tetrachoric coefficient of correlation. We found:

$$r_t = +.6831 \pm .0305.$$

There is thus a quite high association between the eye pigmentation as judged on Martin's scale and the ophthalmologist's judgment of the fundus pigmentation by aid of the ophthalmoscope.

Turning to the Fundus and Hair Colour our data will be found in Table CCXIX on p. 219. The fourfold table is obtained by taking one dichotomy between Medium Light and Medium Dark fundi, and grouping together 4, 5, 27, 28, 29 and 30 as Dark Hair, the remainder being treated as Light Hair.

\* It is not hard to criticise this scale, but it is not easy to provide a better. The two copies in the Galton Laboratory are not wholly in accord, and no deep blue eyes are represented. We are inclined to believe that the influence of light on the colour of glass eyes is not inappreciable.

Table CCXIX. *Fundus and Hair Colour.*

Fundus	1	2	3	4	5	6	7	8	9	10	11	..	14	....	25	26	27	28	29	30	Totals
Light ...	—	—	—	2·5	—	—	3	4	—	1	—	..	—	....	1	1	—	—	—	—	12·5
Medium Light	1	—	4	30·5	72	20·5	74	28	14	0·5	5	..	1	....	2	3	5	—	—	1	261·5
Medium Dark	1	1	1	54·5	57·5	17·5	31	15	2	1·5	1	..	—	....	1	1	10	1	—	—	196
Dark... ..	—	—	—	17	6	1	3	2	—	—	—	..	—	....	—	—	2	—	—	—	31
Totals ...	2	1	5	104·5	135·5	39	111	49	16	3	6	..	1	....	4	5	17	1	—	1	501

Fourfold Table  
Hair Colour

Fundus	Dark	Light	Totals
Light	111	163	274
Dark	148	79	227
Totals	259	242	501

The value of the tetrachoric correlation coefficient is:

$$r_t = + \cdot 3783 \pm \cdot 0426.$$

This value is in very good accord with what we might anticipate, i.e. a value somewhat less than the correlation between Hair and Eye Pigmentations which is about  $\cdot 46^*$ . The value of the correlation between Fundus and Hair Colour for constant Iris Colour is  $\cdot 0935$ , i.e. very slight, it would indeed be zero, if the correlation between Fundus and Hair Colour were  $\cdot 3178$  instead of  $\cdot 3783$ . Or we may conclude that the association of pigmentation in Hair and Fundus is in the main due to the relation of both to iris pigmentation.

These last two tables suggest that the Fundus has been reasonably classified and that its pigmentation might serve as a valuable anthropometric character.

(ii) *Pigmentation of Fundus and Visual Acuity.* The following table provides our data:

Table CCXX. *Appearance of Fundus and Visual Acuity.*

Fundus	1·50	1·40	1·29	1·11	·91	·75	·58	·37	·25	·14	·08	·04	Totals
Light	—	—	1	4	9	3	2·5	5	1·5	1	—	—	27
Medium Light	2	8	50	88	115	53	70	55	23·5	32·5	6	10	513
Medium Dark	1	7	32	97	91	47	34·5	32	20	12·5	10	4	388
Dark	—	—	1	7	21	10	5	6	5	3	—	—	58
Totals	3	15	84	196	236	113	112	98	50	49	16	14	986

Taking the dichotomy between Medium Light and Medium Dark, we can treat the table as biserial and we find:

$$r_b = \cdot 0756 \pm \cdot 0336,$$

a value hardly more than 2·2 times its probable error and accordingly of doubtful significance. Anyhow, it points to the association being of no real importance. It is worth while, however, to consider the array-means. We have:

Grade of Fundus	Mean Visual Acuity
Light ...	$\cdot 7402 \pm \cdot 0463$
Medium Light	$\cdot 7662 \pm \cdot 0106$
Medium Dark	$\cdot 8193 \pm \cdot 0122$
Dark... ..	$\cdot 7321 \pm \cdot 0316$

General Population†:  $\cdot 7843 \pm \cdot 0077$

\* See the earlier portion of this memoir: *Annals of Eugenics*, Vol. I, pp. 22, 23.

† Standard Deviation:  $\cdot 3563$ .

Here again, considering the probable errors, the results border on the non-significant. But as they point to a reasonable conclusion, namely, that the extremely light or extremely dark fundi have a somewhat less Visual Acuity, we considered that there might be a slight relationship and proceeded to find the correlation ratio, although this is a doubtful procedure with only four arrays. We have:

$$\eta'^2_{VA.FP} = \cdot006,822, \quad \bar{\eta}^2_{VA.FP} = \cdot003,043 \pm \cdot001,673.$$

Again, we see that  $\eta'^2_{VA.FP}$  only exceeds  $\bar{\eta}^2_{VA.FP}$  by about 2.2 times the probable error of  $\bar{\eta}^2_{VA.FP}$ . We cannot therefore definitely assert significance, the value of  $\eta'_{VA.FP}$  would be about  $\cdot0826$ , and raised to  $\eta_{VA.FP} = \cdot0926$  by the class index correction\*. We are inclined to think there is some slight influence of the Pigmentation of the Fundus on Visual Acuity, Medium Pigmented Fundi having very slightly superior vision, but we cannot demonstrate it unquestionably on our data.

(iii) *Pigmentation of Fundus and Refraction Class.* We can hardly anticipate that there should be a relation between the structure of the eye and the pigmentation of the fundus. There is, perhaps, little probability of such an association anatomically or physiologically, but if the lighter fundi arise from racial admixture, the result cannot be definitely excluded. Our material is contained in the following table:

Table CCXXI. *Appearance of the Fundus and Refraction Class.*

Fundus	Refraction Class						Totals
	Hypermetropic Astigmatism	Hypermetropia	Emmetropia	Myopia	Myopic Astigmatism	Mixed Astigmatism	
Light ...	1	1	15	7	1	—	25
Medium Light	37.5	28	294.5	60	25	8	453
Medium Dark	33.5	26	215.5	54	7	4	340
Dark... ..	2	3	34	16	3	—	58
Totals ...	74	58	559	137	36	12	876

The difficulty as usual occurs with the small class of Mixed Astigmatics. If we divide them between the Hypermetropic and Myopic Astigmatics we can condense our table thus:

Fundus	Hypermetropic Astigmatism	Hypermetropia	Emmetropia	Myopia	Myopic Astigmatism	Totals
Light	42.5	29	309.5	67	30	478
Dark	37.5	29	249.5	70	12	398
Totals	80	58	559	137	42	876

Determining a biserial Correlation Ratio, we have:

$$\eta'^2_{FP.RC} = \cdot013,724, \quad \bar{\eta}^2_{FP.RC} = \cdot004,566 \pm \cdot002,172.$$

This suggests that  $\eta'^2_{FP.RC}$  is significant, and  $\eta'_{FP.RC} = \cdot1171$ , although small, suggests further inquiry. We therefore returned to the original table and treated it as a  $6 \times 4$  contingency table. There resulted:  $\phi'^2 = \cdot024,688$ ,  $\bar{\phi}^2 = \cdot026,256$ , or on the basis of mean square contingency,  $\phi'^2$  is less than  $\bar{\phi}^2$  and no association can be predicted. The chief contributions to  $\phi'^2$  arise, however, in the Myopia and Myopic Astigmatism columns, there being an excess of Myopes with very dark Fundi and a defect of Myopic Astigmatics with dark Fundi. We can exhibit this by aid of percentages from our condensed table:

\* Class index correlation =  $\cdot8921$ .

Table CCXXII. *Percentages of Medium Dark and Dark Fundi in the several Refraction Classes.*

Hypermetropic Astigmatism	Hypermetropia	Emmetropia	Myopia	Myopic Astigmatism	General Population
46.9 % ±3.8 %	50.0 % ±4.4 %	44.6 % ±1.4 %	51.1 % ±2.9 %	28.6 % ±5.0 %	45.4 % ±1.3 %

Here, having regard to their probable errors, there is no significance in the deviations of the percentages in the first three categories from the General Population value. The last category, Myopic Astigmatism, and possibly the last but one, are significant. We might anticipate as the Jews are very Myopic that the darker the individual, i.e. the more typically racial, the more Myopia; but what about the defect of Myopic Astigmatism? We have already seen that the Jewish boys, while more Myopic, are less Astigmatic than the English boys\*. Can it be that Astigmatism is a result of racial crossing? The more mixed a race, the more Astigmatic because the inheritance has been particulate and not a perfect blend—can such hybridism be called into account to any extent for Astigmatism in general? If this be correct, the large percentage of Astigmatic Myopes in the Jewish boys with the Lighter Fundi may again† indicate that we are not dealing with a pure Jewish race. This is only thrown out as a suggestion, it may receive some confirmation when we come to deal with Astigmatism directly. The differences are too small to lay great stress at present on the point.

(iv) *Pigmentation of Fundus and General Refraction.* Our data are exhibited in Table CCXXIII.

Table CCXXIII. *Appearance of Fundus and General Refraction.*

General Refraction in Dioptres (Central Values)

Appearance of Fundus	+ 6.75	+ 6.00	+ 5.25	+ 4.50	+ 3.75	+ 3.00	+ 2.25	+ 1.50	+ 0.75	0.00	- 0.75	- 1.50	- 2.25	- 3.00	- 3.75	- 4.50	- 5.25	- 6.00	- 6.75	Totals
	—	—	—	—	—	—	0.5	0.5	7	10	5	1	1	—	—	—	—	—	—	—
Light ...	—	—	—	—	—	—	0.5	0.5	7	10	5	1	1	—	—	—	—	—	—	25
Medium Light	1	3	2	2	13	6.5	13.5	8	93	231.5	35.5	17	14	3	2	—	4	—	4	453
Medium Dark	—	1	2	1	2.5	3	17.5	14.5	70.5	168.5	28	14	9.5	3	3	—	2	—	—	340
Dark... ..	—	—	—	—	1	1	1	—	11.5	25.5	11	3	1	1	2	—	—	—	—	58
Totals ...	1	4	4	3	16.5	10.5	32.5	23	182	435.5	79.5	35	25.5	7	7	—	6	—	4	876

The constants of this table are the following:

General Refraction, Mean: + .1455 D.; Standard Deviation: 1.3962 D.

Biserial Coefficient of Correlation‡:  $r_b = - .0135 \pm .0360$ .

Accordingly, if we only separate into Light and Dark Fundi no association of Fundus and General Refraction is to be found. If we take the array-means we have:

Grade of Fundus Pigmentation	Mean General Refraction
Light ...	- .0150 D. ± .1883 D.
Medium Light	+ .1689 D. ± .0442 D.
Medium Dark	+ .1743 D. ± .0511 D.
Dark... ..	- .1358 D. ± .1236 D.
General Population:	+ .1455 D. ± .0318 D.

There results from these array-means:

$$\eta'^2_{GR.FP} = .003,374, \quad \bar{\eta}^2_{GR.FP} = .003,425 \pm .001,882,$$

or  $\eta'^2_{GR.FP}$  is not significant having regard to  $\bar{\eta}^2_{GR.FP}$ .

\* Although we suspect some mingling with another race in these Jews, they are taken as a whole probably of purer race than the English are. † See the earlier portion of this memoir: *Annals of Eugenics*, Vol. I, pp. 17-19.

‡ Dichotomy between Medium Light and Medium Dark.

We may therefore assert that there is no association of Fundus Pigmentation and General Refraction to be found on our data. This is true if we look at the probable errors of the array-means, even the mean for the "Dark" is not definitely significant. It is quite certain that there is no close association of Fundus and General Refraction. At the same time in studying the Fundus Pigmentation we have so frequently come across the fact that the Light and Dark Fundi *both* differ in the *same* direction from the medium value, that we should not be surprised if there was again a distinction of this kind, here only feebly adumbrated and indicating that the purer the race, the smaller the General Refraction.

(v) *Pigmentation of Fundus and General Astigmatism.* Our data are given in Table CCXXIV.

Table CCXXIV. *Appearance of Fundus and General Astigmatism.*

General Astigmatism in Dioptres (Central Values)

Fundus	+ 3.00	+ 2.25	+ 1.50	+ 0.75	0.00	- 0.75	- 1.50	- 2.25	- 3.00	- 3.75	- 4.50	- 5.25	Totals
Light ...	—	—	—	4	18	2	1	—	—	—	—	—	25
Medium Light	0.5	1.5	4	20	321	60	10	16.5	6.5	9	3	1	453
Medium Dark	—	—	1	25.5	266	29.5	10	8	—	—	—	—	340
Dark... ..	—	—	—	5.5	44	6.5	2	—	—	—	—	—	58
Totals ...	0.5	1.5	5	55	649	98	23	24.5	6.5	9	3	1	876

With a dichotomy between Medium Light and Medium Dark we find for the biserial correlation coefficient:

$$r_b = + \cdot 1712 \pm \cdot 0343^*.$$

There is thus a quite significant, if not very large, correlation between a Lighter Fundus and Astigmatism with the rule. This confirms the observation made in the section dealing with Refraction Class and the appearance of the Fundus. Turning to the array-means we find:

Appearance of Fundus	Mean General Astigmatism
Light ...	$\cdot 0000 \pm \cdot 1004$ D. }
Medium Light	$\cdot 3162 \pm \cdot 0236$ D. }
Medium Dark	$\cdot 1015 \pm \cdot 0272$ D. }
Dark... ..	$\cdot 0647 \pm \cdot 0659$ D. }
General Population†:	$\cdot 2072 \pm \cdot 0170$ D.

The differences are in the main significant, and appear to indicate that the Astigmatism is least when the Fundus is very Light or very Dark, i.e. probably with racial purity, but with Medium Pigmentation especially on the light side the General Astigmatism increases.

Determining the correlation ratio of Astigmatism on Pigmentation of Fundus we find:

$$\eta'^2_{GA.FP} = \cdot 023,582, \quad \bar{\eta}^2_{GA.FP} = \cdot 003,425 \pm \cdot 001,882.$$

Thus  $\eta'^2_{GA.FP}$  differs significantly from  $\bar{\eta}^2_{GA.FP}$  and we have  $\eta'_{GA.FP} = \cdot 1536$ , which corrected for the class index of Fundus Pigmentation ( $\cdot 8921$ ) gives  $\eta_{GA.FP} = \cdot 1722$ , a value just in excess of that of the biserial correlation coefficient  $r_b (= \cdot 1712)$ .

(vi) *Pigmentation of Fundus and Corneal Refraction.* Table CCXXV below provides the data for A and B only. The constants are as follows:

Corneal Refraction, Mean: 43.8756 D.; Standard Deviation: 1.5253 D.

Biserial Coefficient of Correlation with the dichotomy between Medium Light and Medium Dark is given by

$$r_b = \cdot 0048 \pm \cdot 0425.$$

\* If we disregard the sign of the Astigmatism we only raise  $r_b$  to  $+ \cdot 1720 \pm \cdot 0343$ , the Higher Astigmatism being with the Lighter Fundus. The array-means become Light:  $\cdot 2400$  D., M. Light:  $\cdot 4305$  D., M. Dark:  $\cdot 2228$  D., and Dark:  $\cdot 2069$  D.

† Standard Deviation:  $\cdot 7439$  D.

Thus no association between Fundus Pigmentation and Corneal Refraction is indicated. This is confirmed if we take the array-means when we find:

Grade of Pigmentation	Mean Corneal Refraction
Light ...	44.425 $\pm$ .230 D.
Medium Light	43.909 $\pm$ .073 D.
Medium Dark	43.902 $\pm$ .088 D.
Dark... ..	43.625 $\pm$ .167 D.
<hr/>	
General Population:	43.876 $\pm$ .052 D.

Table CCXXV. *Appearance of Fundus and Corneal Refraction (A and B only).*

Corneal Refraction in Dioptres (Central Values)

Fundus	38.125	38.625	39.125	39.625	40.125	40.625	41.125	41.625	42.125	42.625	43.125	43.625	44.125	44.625	45.125	45.625	46.125	46.625	47.125	47.625	Totals
Light ...	—	—	—	—	—	—	—	—	—	—	1	4	7	4	—	2	2	—	—	—	20
Medium Light	1	—	—	—	7	1	5	8	11	14	21	29	31	24	16	6	13.5	6.5	2	1	197
Medium Dark	—	—	—	—	2	3	3	3	11	15	15	15	11	14	15	14	12	2	1	1	137
Dark... ..	—	—	—	—	1	—	1	2	2	4	2	7	9	5	3	2	—	—	—	—	38
Totals ...	1	—	—	—	10	4	9	13	24	33	39	55	58	47	34	24	27.5	8.5	3	2	392

None of these means, having regard to their probable errors, can be considered to differ significantly from that of the General Population. We cannot therefore suppose  $r_b$  small because the regression line is curved, but must hold that Corneal Refraction is not associated with the Pigmentation of the Fundus as far as the observations of *A* and *B* are concerned.

We next considered whether this result would be modified if we included the observations of *C*. These are given in Table CCXXVI below added to those of *A* and *B*.

Corneal Refraction, Mean: 43.4518 D.; Standard Deviation: 1.5550 D.

Biserial Coefficient of Correlation\*:  $r_b = - .0280 \pm .0340$ .

This signifies that there is no significant relation between Light or Dark Pigmentation and Corneal Refraction. If we turn to the array-means we have:

Grade of Pigmentation	Mean Corneal Refraction
Light ...	44.1458 $\pm$ .2141 D.
Medium Light	43.4529 $\pm$ .0464 D.
Medium Dark	43.4022 $\pm$ .0534 D.
Dark... ..	43.4833 $\pm$ .1354 D.
<hr/>	
General Population:	43.4518 $\pm$ .0335 D.

These lead to:  $\eta^2_{CR.FP} = .005,304$ ,  $\bar{\eta}^2_{CR.FP} = .003,061 \pm .001,681$ ;

$\eta^2_{CR.FP}$  cannot therefore be asserted to be significant having regard to the value of  $\bar{\eta}^2_{CR.FP}$ . This is in general confirmed by the probable errors. Only the Light category in the array-means shows a possibly significant difference as corresponding to a cornea of less radius, i.e. possibly to a racial difference in the Jewish boys with a Light Fundus.

Table CCXXVI. *Appearance of Fundus and Corneal Refraction (A, B and C).*

Corneal Refraction in Dioptres

	38.125	38.625	39.125	39.625	40.125	40.625	41.125	41.625	42.125	42.625	43.125	43.625	44.125	44.625	45.125	45.625	46.125	46.625	47.125	47.625	Totals
Light ...	—	—	—	—	—	—	—	—	1	1	3	4	7	4	—	2	2	—	—	—	24
Medium Light	1	—	—	4	14	12	26	27	45	40	61	69	72	48	33	22	24.5	8.5	1	2	510
Medium Dark	—	—	4	3	5	9	20	19	39	42	50	49	39	35	24.5	24	17.5	4	2	—	386
Dark... ..	—	—	—	—	1	—	4	3	4	10	2	8	13	7	4	3	1	—	—	—	60
Totals ...	1	—	4	7	20	21	50	49	89	93	116	130	131	94	61.5	51	45	12.5	3	2	980

\* Dichotomy between Medium Light and Medium Dark.

(vii) *Pigmentation of the Fundus and Corneal Astigmatism.* Our data are exhibited in Table CCXXVII.

Table CCXXVII. *Appearance of the Fundus and Corneal Astigmatism (A, B and C).*

Fundus	Corneal Astigmatism in Dioptres												Totals
	-2.25	-1.50	-0.75	0.00	+0.75	+1.50	+2.25	+3.00	+3.75	+4.50	+5.25	+6.00	
Light ...	—	—	1	10	13	—	—	—	—	—	—	—	24
Medium Light	—	5	6	221	175.5	42	30.5	15	7	3	4	1	510
Medium Dark	1	2	4.5	167	161	27	18.5	4	1	—	—	—	386
Dark ...	—	—	3	22	30	4	1	—	—	—	—	—	60
Totals ...	1	7	14.5	420	379.5	73	50	19	8	3	4	1	980

The constants of this table are as follows:

Corneal Astigmatism (regarding sign):

Mean: + .6230 D.; Standard Deviation: .8596 D.

Corneal Astigmatism (disregarding sign):

Mean: + .6712 D.; Standard Deviation: .8505 D.

Biserial Correlation Coefficient (regarding sign):  $r_b = - .1156 \pm .0333$ .

Biserial Correlation Coefficient (disregarding sign):  $r_b = - .1161 \pm .0333$ .

Turning to the array-means we find:

Grade of Fundus Pigmentation	Mean Corneal Astigmatism	
	Regarding Sign	Disregarding Sign
Light ...	+ .3750 D. $\pm$ .1183	+ .4375 D. $\pm$ .1171
Medium Light	+ .7118 D. $\pm$ .0257	+ .7588 D. $\pm$ .0254
Medium Dark	+ .5440 D. $\pm$ .0295	+ .5887 D. $\pm$ .0292
Dark ...	+ .4750 D. $\pm$ .0749	+ .5500 D. $\pm$ .0741
General Population:	+ .6230 D. $\pm$ .0185	+ .6712 D. $\pm$ .0183

We deduce from these the correlation ratios:

Regarding sign:  $\eta'^2_{CA.FP} = .012,734$ ,  $\bar{\eta}^2_{CA.FP} = .003,061 \pm .001,681$ .

Disregarding sign:  $\eta'^2_{CA.FP} = .012,318$ ,  $\bar{\eta}^2_{CA.FP} = .003,061 \pm .001,681$ .

Both are clearly significant and we find regarding sign  $\eta'_{CA.FP} = .1128$ , and disregarding sign  $\eta'_{CA.FP} = .1110$ , or correcting for class index: regarding sign  $\eta_{CA.FP} = .1264$ , and disregarding sign  $\eta_{CA.FP} = .1246$ .

Thus as we have seen that for General Astigmatism there is a small but definite association with Fundus Pigmentation, so there is for Corneal Astigmatism but to a somewhat lesser extent.

The array-means indicate that the regression is hardly linear, it is least for the lightest and darkest Fundi but rises for the medium. This agrees with what we noted in the case of General Astigmatism, and may, as we have remarked, be due to there being less Astigmatism in the purer racial groups.

(viii) *Pigmentation of the Fundus and Distance of Near Point.* Our data are provided in Table CCXXVIII.

Table CCXXVIII. *Appearance of Fundus and Distance of Near Point.*

Distance of Near Point in mm.

Fundus	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175	180	..	200	Totals
Light...	—	—	—	—	1	1	—	2	2	7	2	1	1	1	—	2	—	—	—	—	—	—	—	—	—	1	—	—	—	—	..	—	22
Medium Light	1	1	2	2	3	9	9.5	19.5	31.5	46.5	39	45	47	32	17	16	15	10	4	10	8	3	3	2	—	—	—	1	—	—	..	1	378
Medium Dark	—	—	—	—	—	3	10	22	15.5	27.5	36	47	33	24	24	15	12	7	9	9	8	4	2	2	—	—	—	—	—	—	..	—	310
Dark ...	—	—	—	—	3	2	2	7	1	3	5	4	3	2	1	3	1	1	1	1	—	—	—	—	—	—	—	—	—	—	..	—	40
Totals ...	1	1	2	2	7	15	21.5	50.5	50	84	82	97	84	59	42	36	28	18	14	20	16	7	5	4	—	1	—	1	—	1	..	1	750

Treating this table as a biserial table with dichotomy between Medium Light and Medium Dark, we find:

$$r_b = \cdot 0514 \pm \cdot 0386,$$

a non-significant coefficient.

Testing this by array-means we find:

Grade of Fundus Pigmentation	Mean Distance of Near Point
Light ... ..	90.0000 $\pm$ 2.916 mm.
Medium Light	92.4338 $\pm$ 0.703 mm.
Medium Dark	95.0726 $\pm$ 0.777 mm.
Dark ... ..	85.3750 $\pm$ 2.163 mm.
General Population*:	93.0767 $\pm$ 0.499 mm.

It is the last two categories only which respectively approach and exceed the limit of non-significance. We have the result noticed before that those with specially Dark Fundi have a close Near Point, i.e. are probably myopic. The great distance of the Near Point of the Medium Dark group probably corresponds with the small excess of the Hypermetropic, as shown in the table of percentages on p. 221.

Proceeding to determine the correlation ratio we have:

$$\eta'^2_{NP.FP} = \cdot 128,795, \quad \bar{\eta}^2_{NP.FP} = \cdot 004,000 \pm \cdot 002,197.$$

Thus  $\eta'^2_{NP.FP}$  is significantly different from  $\bar{\eta}^2_{NP.FP}$  and  $\eta'_{NP.FP} = \cdot 3589$ .

This value is far more considerable than that suggested by  $r_b$ , which was non-significant. We conclude therefore that the skew form of the regression curve indicated by the array-means is real, and that the low values of the Distance of the Near Point are actually found when the Fundus is very Light or very Dark (? in pure races) and not to the same extent in the Medium range.

*Summary of Influence of Fundus Pigmentation on the Ocular Characters.* In general, there is very little association between the Fundus Pigmentation and Ocular Characters. This pigmentation is a racial character closely related to the iris pigmentation, and significantly, but to a less extent, with the hair pigmentation. The one ocular character which shows quite definite, if not very intense, association with the Fundus Pigmentation is the Astigmatism, whether General or Corneal; it is suggested, but not demonstrated, that this correlation results from racial admixture. Where two races with different sizes and shapes of the orbital cavities intercross, then we might anticipate that if the inheritance is particulate the curvatures of the eyes in the principal meridians might show signs of this hybridism, and Astigmatism be produced. If one of this race had a light, and the other a dark fundus racially, a correlation would be produced between appearance of Fundus and Astigmatism which has no origin in any link between the anatomical and physiological characters of the eye. This relation between Astigmatism and Fundus Pigmentation would be reflected in a weaker association of Visual Acuity and Distance of the Near Point with Fundus Pigmentation. It would undoubtedly be of interest to ascertain whether Eurasians, mulattoes or other human hybrids show any special degree of Astigmatism.

(j) *Interrelations of Various Refractions of the Eye.* We have already seen that in a rough sort of manner it is possible to obtain a measure of the relation between the refractive power of the cornea and the refractive power of the lens, and that the Astigmatism of the one must be highly related to that of the other: see pp. 170, 179-180 and 189. This suggests that there is a relation between the moulding of the cornea and the moulding of the lens. We cannot determine whether this interrelation is a growth phenomenon or due to muscular action, but the fact that the

\* Standard Deviation: 20.2780 mm.

refractive powers of cornea and lens are so highly correlated is of much significance. It may be said that the first thing to do is to establish a relation between the directions of axes for cornea and lens. But there are difficulties which rapidly come to sight when the problem is examined. In the first place we have not the direction of the axis of the lens at all, we have only the directions of the axes for General Astigmatism and for Corneal Astigmatism, and we must deduce our knowledge of the direction of the axis of the lens from these. In the next place only 26 %\* of the eyes we examined had sensible astigmatism, and of these only 28 % had direction of axis differing sensibly from the horizontal†. It will be seen therefore that the data to determine the relation of Corneal to General Axis of Astigmatism are very slender. We must be content therefore with the broad fact that in the main the Axes of Astigmatism for both the Corneal and General factors are horizontal and vertical. The correlation table for the directions to the horizontal of the principal meridians in the case of General and Corneal Astigmatism is given below.

Table CCXXIX. *Directions of Axes of General and Corneal Astigmatisms.*

Direction of Axis of General Astigmatism (Central Values)																					
Direction of Axis of Corneal Astigmatism (Central Values)	45°	40°	35°	30°	25°	20°	15°	10°	5°	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	Totals	
	—	—	—	—	—	—	—	—	—	—	+	+	+	+	+	+	+	+	+	—	
	45°	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2	
	40°	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2	
	35°	1	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2	
	30°	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	—	—	—	3	
	25°	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
	20°	—	—	—	1	—	—	—	—	3	—	—	—	—	—	—	—	—	—	6	
	15°	1	—	—	1	—	—	—	—	6	—	—	—	—	—	—	2	—	—	9	
	10°	1	—	—	1	—	—	—	3	7	—	—	—	—	—	—	—	—	—	13	
	5°	—	—	—	—	—	—	—	—	6	—	—	—	—	—	—	—	—	—	6	
	0°	6	1	—	8	—	2	5	4	720	—	3	3	1	1	1	17	—	—	9	780
	5°	—	—	—	1	—	—	—	—	8	1.5	—	—	—	—	—	—	—	—	—	10.5
	10°	—	—	—	—	—	—	—	1	11	0.5	1	1.5	1	1	2	—	—	—	19	
	15°	—	—	—	1	—	—	—	—	7	—	—	2.5	1	—	—	—	—	—	11.5	
	20°	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	2	
	25°	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	—	—	—	4	
	30°	—	—	—	1	—	—	—	—	2	—	—	—	—	—	—	—	—	—	3	
	35°	—	—	—	—	—	—	—	—	5	—	—	—	—	—	—	—	—	—	5	
	40°	—	—	—	1	—	—	—	—	2	—	—	—	—	—	—	—	—	—	3	
	45°	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1	
	Totals	10	1	—	15	—	2	5	10	—	788	2	5	7	3	2	22	—	—	10	882

The constants of this table after removing 279 cases which had neither General nor Corneal Astigmatism‡ are the following:

Axis of General Astigmatism, Mean:  $+0^{\circ} \cdot 3814$ ; Standard Deviation‡:  $11^{\circ} \cdot 7190$ .

Axis of Corneal Astigmatism, Mean:  $+0^{\circ} \cdot 3980$ ; Standard Deviation‡:  $8^{\circ} \cdot 3338$ .

Product Moment Correlation Coefficient:  $r = + \cdot 0799 \pm \cdot 0273$ .

It will be seen at once that the correlation is very low. We can approach the matter from

\* This is for General Astigmatism: see Table CCII, p. 194. For Corneal Astigmatism, however, the figure is 57 %: see Table CCVIII, p. 203. The difference probably depends on the greater ease with which Corneal Refractions can be ascertained and so small differences recorded. If we exclude the small positive astigmatisms centring at  $+ \cdot 75$ , we find the percentage reduced to 19 %, more in accord with the General Astigmatism figure.

† Of 900 eyes, 663 had no General Astigmatism. Of the 237 astigmatic eyes, 171 had the axis horizontal, this left 66 eyes or slightly over 7 % of all eyes with the axis of General Astigmatism differing from the horizontal.

‡ We are left with 442 out of 603 or 73.3 % of cases in which both sets of axes are horizontal and vertical. There are 59 cases in which the axis of Corneal Astigmatism is horizontal, but not that of General Astigmatism, and 68 cases in which the axis of General Astigmatism, but not that of Corneal Astigmatism is horizontal; this leaves only 34 cases in which both differ from zero.

another standpoint by taking the difference between the directions of the axes of the two astigmatisms for the 603 cases of astigmatism of both kinds.

*Angle between Axes (General minus Corneal).*

-50°	-45°	-40°	-35°	-30°	-25°	-20°	-15°	-10°	-5°	0	+5°	+10°	+15°	+20°	+25°	+30°	+35°	+40°	+45°	+50°	+55°	+60°	+65°	+70°
3	10	2	1	20	1	7	10	11	9.5	450	8.5	17	13	6	3	11	7	3	7	—	1	1	—	1

If the reader remembers that we are not taking the smallest angle between the principal axes, but the angle between the principal axes of each astigmatism nearer to the horizontal, he will see how angles like 70° can arise. He will notice that 509 cases lie between  $-17^{\circ}5$  and  $+17^{\circ}5$  or something like 84 % have this fairly close degree of agreement. If this be borne in mind, it will be clear that the bulk of  $R_1$ 's and  $C_1$ 's and again of  $R_2$ 's and  $C_2$ 's are approximately in the same directions and there is meaning in correlating them. The constants of the above frequency-distributions are: Mean:  $+0^{\circ}0166$ ; Standard Deviation:  $13^{\circ}8264^*$ . If the angles be  $\alpha_g$  and  $\alpha_c$ , then we have  $r_{\alpha_g \alpha_c} = \frac{\sigma^2_{\alpha_g} + \sigma^2_{\alpha_c} - \sigma^2_{\alpha_g - \alpha_c}}{2\sigma_{\alpha_g}\sigma_{\alpha_c}} = .0799$ , checking the former result.

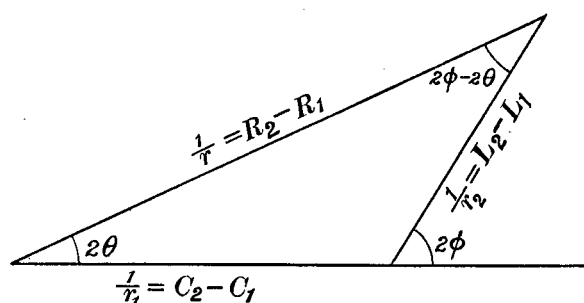
We shall now try and correlate the refractive powers of the cornea and of the lens in the principal meridians. In order to do this we will call the General Refractions  $R_1$  and  $R_2$ , the Corneal Refractions  $C_1$  and  $C_2$ , and the pseudo-Lenticular Refractions  $L_1$  and  $L_2$ †, where the subscripts 1 and 2

\* Sheppard's correction not used.

† The reader must bear in mind that  $L_1$  and  $L_2$  are not the true lenticular refractions, but obtained from the difference of the Total and the Corneal Refractions. These remainders are, of course, chiefly due to the lens but actually the position of the lens and the media in which it is placed would have to be taken into consideration. We have defined pseudo-lenticular refractions  $L_1$  and  $L_2$  to be the differences between  $R_1$  and  $C_1$  and  $R_2$  and  $C_2$ . They are not therefore the true lenticular refractions. The sources of error are twofold:

(i) the two refracting systems have not their lenses in contact. We have indicated something of the size of error introduced by the separation of crystalline lens and cornea on p. 179;

(ii) the principal axes nearer to the horizontal of  $R_1$  and  $C_1$  do not coincide in all cases. Actually the three astigmatisms can be shown to arrange themselves as the sides of a triangle, with the angles between the sides double the angles between the principal axes nearer to the horizontal. (Of course the same crude assumption has been made, i.e. of "thin" systems placed in contact.)



$$\text{Thus } L_2 - L_1 = \sqrt{(R_2 - R_1)^2 + (C_2 - C_1)^2 - 2(R_2 - R_1)(C_2 - C_1)\cos 2\theta} \dots\dots\dots(i),$$

$$\text{and } \sin 2(\phi - \theta) = \frac{C_2 - C_1}{L_2 - L_1} \sin 2\theta \dots\dots\dots(ii),$$

give the curvature  $\frac{1}{r_2}$  corresponding to the lenticular astigmatism and the angle  $\phi$  between the principal axes of lens and of cornea.

We have now to determine the principal lenticular refractions  $L_1$  and  $L_2$ . We deduce easily:

$$\left. \begin{aligned} R_1 &= L_1 + C_1 + (C_2 - C_1) \sin^2 \theta + (L_2 - L_1) \sin^2 (\phi - \theta) \\ R_2 &= L_1 + C_1 + (C_2 - C_1) \cos^2 \theta + (L_2 - L_1) \cos^2 (\phi - \theta) \end{aligned} \right\} \dots\dots\dots(iii).$$

$$\text{Accordingly } R_1 + R_2 = L_1 + L_2 + C_1 + C_2 \dots\dots\dots(iv).$$

Whence by aid of (i):

$$\left. \begin{aligned} L_1 &= \frac{1}{2}(R_1 + R_2) - \frac{1}{2}(C_1 + C_2) \pm \frac{1}{2}\sqrt{(R_2 - R_1)^2 + (C_2 - C_1)^2 - 2(R_2 - R_1)(C_2 - C_1)\cos 2\theta} \\ L_2 &= \frac{1}{2}(R_1 + R_2) - \frac{1}{2}(C_1 + C_2) \mp \frac{1}{2}\sqrt{(R_2 - R_1)^2 + (C_2 - C_1)^2 - 2(R_2 - R_1)(C_2 - C_1)\cos 2\theta} \end{aligned} \right\} \dots\dots\dots(v).$$

The difficulty of this solution is that, while (ii) gives this value of  $\phi$ , we do not know which value of the radical corresponds to which value of  $L$ .

$$\text{Returning to our triangle we have: } \frac{\sin 2(\phi - \theta)}{\sin 2\phi} = \frac{C_2 - C_1}{R_2 - R_1},$$

refer to the principal meridians nearer to the horizontal and vertical respectively. We desire to know how all these refractions are interrelated. We have already found the correlation coefficients of  $R_1$  and  $R_2$  (see p. 175), of  $C_1$  and  $C_2$  (see p. 201) and deduced approximate values for the correlations of  $R_1$  and  $L_1$  (see p. 177), of  $C_1$  and  $L_1$  and of  $R_1$  and  $C_1$  (see p. 180).

We have deduced directly for the present purpose the correlations of  $R_2$  and  $C_2$  for both  $A$  and  $B$  alone and for these observers combined with  $C$ . We have also deduced for both series the

Table CCXXX. *Correlation of Corneal and General Refractions in Principal Meridians nearer to Vertical (A, B and C).*

$R_2$ , General Refraction in Principal Meridian nearer to Vertical (Central Values in Dioptres)

$C_2$ , Corneal Refraction in Principal Meridian nearer to Vertical (Central Values in Dioptres)																					Totals		
	+ 6.00	+ 5.25	+ 4.50	+ 3.75	+ 3.00	+ 2.25	+ 1.50	+ 0.75	0.00	- 0.75	- 1.50	- 2.25	- 3.00	- 3.75	- 4.50	- 5.25	- 6.00	- 6.75	- 7.50	- 8.25	- 9.00	- 9.75	
38-125	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38-625	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
39-125	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	—	—	—	—	—	—	—	2
39-625	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
40-125	—	—	—	—	—	—	—	—	1	1	1	1	—	—	—	—	—	—	—	—	—	—	4
40-625	—	1	—	—	—	—	1	2	6	3	—	—	—	—	—	—	—	—	—	—	—	—	13
41-125	—	—	—	—	—	—	—	3	15	5	—	1	1	—	—	—	—	—	—	—	—	—	25
41-625	1	—	—	—	—	1	2	5	17	2	3	—	—	1	—	—	—	—	—	—	—	—	32
42-125	—	1	—	—	—	—	2	9	34	6	3	2	4	—	—	—	—	—	—	—	—	—	61
42-625	—	—	—	—	—	—	3	7	38	8	3	1	—	—	—	—	—	—	—	—	—	—	60
43-125	—	—	—	2	—	3	5	21	55	7	—	1	—	2	—	—	—	—	1	—	1	—	98
43-625	—	—	—	—	1	1	3	28	44	7	5	4	1	—	—	—	—	—	—	—	—	—	94
44-125	—	—	—	1	1	2	2	26	59	17	5	2	—	—	—	—	—	—	—	—	—	—	115
44-625	—	1	—	—	—	2	1	27	53	14	3	1	1	2	—	1	—	—	—	1	—	—	107
45-125	—	—	—	1	—	1	2	21	37	7	6	3	1	1	1	2	—	1	—	—	—	—	84
45-625	—	—	—	—	—	1	2	14	35	9	3	2	3	—	—	1	—	—	—	—	—	—	70
46-125	—	1	—	—	3	—	1	6	14	4	3	1	4	3	—	2	—	—	—	—	1	1	44
46-625	—	—	—	1	—	1	—	5	19	5	2	—	—	—	—	—	—	—	—	—	—	—	33
47-125	—	—	—	1	—	2	—	5	10	1	1	—	1	—	—	—	—	—	—	—	1	—	22
47-625	—	—	—	—	1	1	1	1	3	2	—	1	—	—	—	—	—	—	—	—	—	—	10
48-125	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
48-625	—	—	—	—	—	—	—	1	2	—	1	—	—	—	—	—	—	—	—	—	—	—	4
Totals	1	4	—	6	6	15	25	182	443	99	42	20	16	9	1	6	—	1	1	1	3	1	882

hence:  $\cot 2\phi = \tan 2\theta - \frac{C_2 - C_1}{R_2 - R_1} \operatorname{cosec} 2\theta \dots\dots\dots(\text{vi}).$

Let  $2\gamma$  be the smallest angle given by (vi), then the solutions of (v) are  $2\phi = 2\gamma$  and  $2\phi = \pi + 2\gamma$ . Thus:

$$\left. \begin{aligned} L_1 &= \frac{1}{2}(R_1 + R_2) - \frac{1}{2}(C_1 + C_2) - \frac{1}{2} \frac{\sin 2\theta}{\sin 2\gamma} (R_2 - R_1) \text{ in direction } \gamma \text{ with the axis of } C_2 - C_1 \\ L_2 &= \frac{1}{2}(R_1 + R_2) - \frac{1}{2}(C_1 + C_2) - \frac{1}{2} \frac{\sin 2\theta}{\sin (\pi + 2\gamma)} (R_2 - R_1) \text{ in direction } \frac{\pi}{2} + \gamma \text{ with the axis of } C_2 - C_1 \end{aligned} \right\} \dots(\text{vii}).$$

(vi) and (vii) should suffice to determine the "lenticular" refractions. It would not be very heavy work to determine  $L_1$  and  $L_2$  for the relatively few observations in which  $\theta$  differs from zero. Unfortunately we can only ascertain truly  $L_1$  and  $L_2$  from a knowledge of the absolute values of  $R_1$  and  $R_2$ . Such absolute values are not determined. The absolute curvatures, and so the refractive powers, are found in the case of the cornea, but in the case of General Refraction we are measuring the additive or subtractive refractive power needful to make an individual eye emmetropic, but we are not reducing that eye to an emmetropic eye of standard dimensions; our zero, so to speak, is individual to the eye observed, while the corneal refractive powers are absolute. We cannot really determine the refractive powers of the lens  $L_1$  and  $L_2$  in this manner, because we are adding an  $R$  relative to the individual to an absolute  $C$ , and the amount of error we should introduce by replacing the zero of  $R$  by an average constant appears at present unknown, and quite possibly of the same order as the corrections for  $\theta$ , since  $\theta$  is usually small. We have contented ourselves accordingly with pseudo-lenticular refractions,  $L_1 = R_1 - C_1$  and  $L_2 = R_2 - C_2$ , which would be approximately correct if the principal axes of General and Corneal Astigmatism coincided, which they do not in all, but in the great majority of cases. These pseudo-lenticular refractions will still be subject to the errors introduced by the absolute origin of the Corneal and the relative origin of the General Refraction. Actually in the above formulae  $R$  is the total refractive power of the eye in a given meridian and equals  $R_{em} - R$ , where  $R$  is the "General Refraction" of the ophthalmologist, and  $R_{em}$  the refractive power of the individual eye if rendered emmetropic in that meridian.

correlations of  $R_1$  and  $C_2$ , and for the larger series  $C_1$  and  $R_2$ . These are amply sufficient to enable us to obtain all the remaining correlations indirectly. We shall now proceed to discuss these additional tables, and indicate the indirect method by which the remaining correlation coefficients can be determined.

We have dealt with the correlation of  $R_1$  and  $C_1$  on p. 177 and seen how small it is. We now take  $R_2$  and  $C_2$ . Table CCXXX (p. 228) gives our data for  $A$ ,  $B$  and  $C$ 's observations.

The constants of this table are as follows:

General Refraction  $R_2$ , Mean:  $-.09524$  D.; Standard Deviation:  $1.38235$  D.

Corneal Refraction  $C_2$ , Mean:  $44.0530$  D.; Standard Deviation:  $1.60412$  D.

Product Moment Correlation Coefficient:  $r = -.0411 \pm .0227$ .

It is clear that the correlation does not differ sensibly from zero. The array-means have again non-significant differences when the arrays are taken either way and neither series shows a significant trend. See Diagram 112, p. 230).

In order to investigate the matter further we considered the data for  $A$  and  $B$  only.

Table CCXXX bis. *Correlation of Corneal and General Refractions in Principal Meridians nearer to Vertical (A and B).*

$R_2$ , General Refraction in Principal Meridian nearer to Vertical (Central Values in Dioptres)

$C_2$ , Corneal Refraction in Principal Meridian nearer to Vertical (Central Values in Dioptres)																							Totals
	+ 6.00	+ 5.25	+ 4.50	+ 3.75	+ 3.00	+ 2.25	+ 1.50	+ 0.75	0.00	- 0.75	- 1.50	- 2.25	- 3.00	- 3.75	- 4.50	- 5.25	- 6.00	- 6.75	- 7.50	- 8.25	- 9.00	- 9.75	
38.125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38.625	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
39.125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39.625	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—
40.125	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	2
40.625	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
41.125	—	—	—	—	—	—	—	1	4	—	—	—	—	—	—	—	—	—	—	—	—	—	5
41.625	—	—	—	—	—	1	1	3	1	3	1	—	—	—	—	—	—	—	—	—	—	—	7
42.125	—	1	—	—	—	—	2	1	3	1	—	—	—	—	—	—	—	—	—	—	—	—	10
42.625	—	—	—	—	—	—	3	5	2	2	1	—	—	—	—	—	—	—	—	—	—	—	13
43.125	—	—	—	—	—	1	4	10	8	1	—	—	—	—	—	—	—	—	—	—	—	—	26
43.625	—	—	—	—	1	1	3	21	6	1	1	2	1	—	—	—	—	—	—	—	—	—	37
44.125	—	—	—	—	—	1	2	20	15	4	2	—	—	—	—	—	—	—	—	—	—	—	44
44.625	—	1	—	—	—	1	1	17	18	5	—	1	1	1	—	—	—	—	—	—	—	—	47
45.125	—	—	—	—	—	1	2	17	15	4	2	2	—	1	—	1	—	—	—	—	—	—	46
45.625	—	—	—	—	—	1	2	12	16	4	2	2	3	—	—	1	—	1	—	—	—	—	43
46.125	—	—	—	—	2	—	1	4	4	2	2	—	4	1	—	—	—	—	—	—	—	1	21
46.625	—	—	—	1	—	—	—	4	11	3	1	—	—	—	—	—	—	—	—	—	—	—	20
47.125	—	—	—	1	—	2	—	4	7	1	1	—	1	—	—	—	—	—	—	—	—	—	17
47.625	—	—	—	—	1	—	1	1	2	2	—	1	—	—	—	—	—	—	—	—	—	—	8
48.125	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
48.625	—	—	—	—	—	—	—	1	2	—	1	—	—	—	—	—	—	—	—	—	—	—	4
Totals	—	3	—	2	4	9	22	122	114	31	16	9	12	5	—	2	—	1	—	1	—	1	354

This table again exhibits very little correlation. We have the following values of the constants:

General Refraction,  $R_2$ , Mean:  $+.0636$  D.; Standard Deviation:  $1.5345$  D.

Corneal Refraction,  $C_2$ , Mean:  $44.6589$  D.; Standard Deviation:  $1.5867$  D.

Product Moment Correlation Coefficient:  $r = -.0896 \pm .0356$ .

Correlation Ratios:

$$\eta'^2_{C_2.R_2} = .068,087, \quad \bar{\eta}^2_{C_2.R_2} = .042,373 \pm .010,199,$$

$$\eta'^2_{R_2.C_2} = .096,935, \quad \bar{\eta}^2_{R_2.C_2} = .053,672 \pm .011,409.$$

Probably  $r$  and  $\eta'^2_{C_2.R_2}$  are just, while  $\eta'^2_{R_2.C_2}$  is more definitely significant, having regard to their probable errors. We have:  $\eta'_{C_2.R_2} = .2609$  and  $\eta'_{R_2.C_2} = .3113$ .

The array-means were determined and are exhibited in Diagram 113; it will be seen that they do not suggest any regular series and we have only graduated them with the straight regression lines (not shown for clearness in the diagrams):

$$\tilde{C}_2 = 44.6648 - .09265R_2,$$

$$\tilde{R}_2 = 3.9333 - .08665C_2,$$

**CORRELATION OF  
GENERAL & CORNEAL REFRACTIONS  $R_2$  WITH  $C_2$   
(OBSERVERS A, B & C) ALIEN JEWISH BOYS**

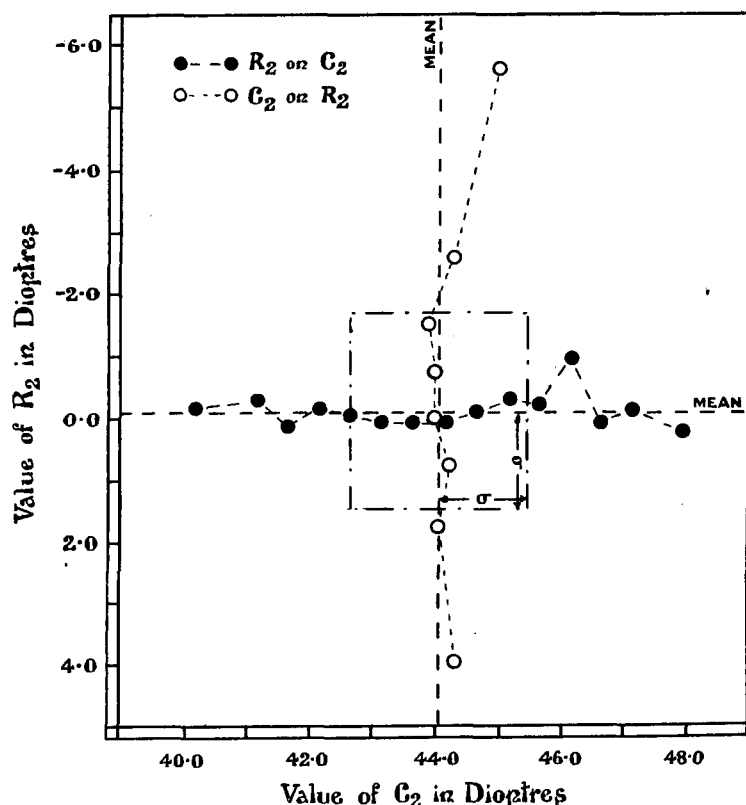


Diagram 112.

**CORRELATION OF  
GENERAL & CORNEAL REFRACTIONS  $R_2$  WITH  $C_2$   
(OBSERVERS A & B ONLY) ALIEN JEWISH BOYS**

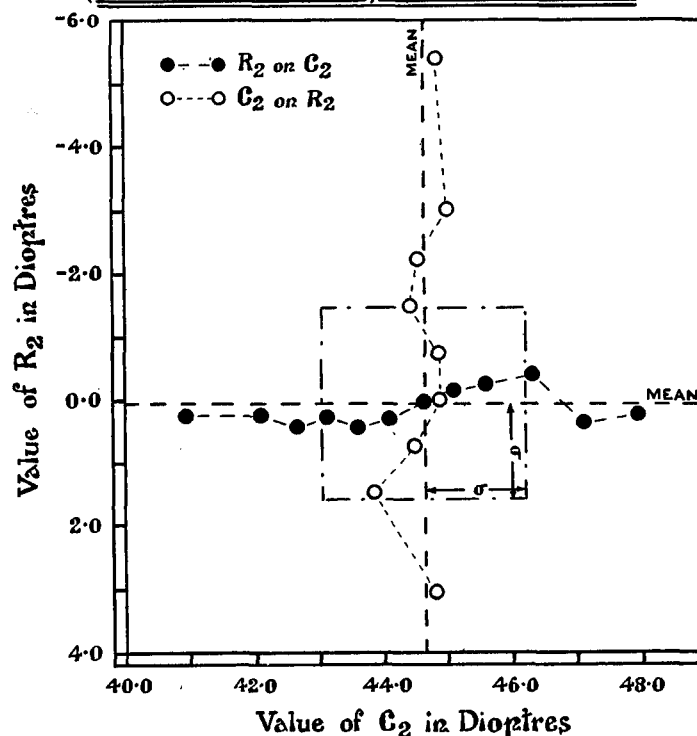


Diagram 113.

where  $\tilde{C}_2$  and  $\tilde{R}_2$  are the probable values of the "vertical" Corneal and General Refractions for given values  $R_2$  and  $C_2$  of the "vertical" General and Corneal Refractions respectively.

It will be clear that the refractions in these "vertical" directions are as little related as they were in the "horizontal" directions.

We now turn to the cross-correlations, namely, those between the Corneal and General Refractions in the cross meridians, i.e.  $R_1$  with  $C_2$  and  $R_2$  with  $C_1$ .

We take first  $R_1$  with  $C_2$  for A, B and C combined. Table CCXXXI (p. 231) provides the data. The constants of this table run as follows:

General Refraction  $R_1$ , Mean: + .1403 D.; Standard Deviation: 1.4121 D.

Corneal Refraction  $C_2$ , Mean: 44.0530 D.; Standard Deviation: 1.6255 D.

Product Moment Correlation Coefficient:  $r = + .0911 \pm .0225$ .

Correlation Ratios:

$$\eta'^2_{C_2.R_1} = .065,138,$$

$$\eta'^2_{R_1.C_2} = .065,992,$$

$$\bar{\eta}^2_{C_2.R_1} = .019,274 \pm .004,433,$$

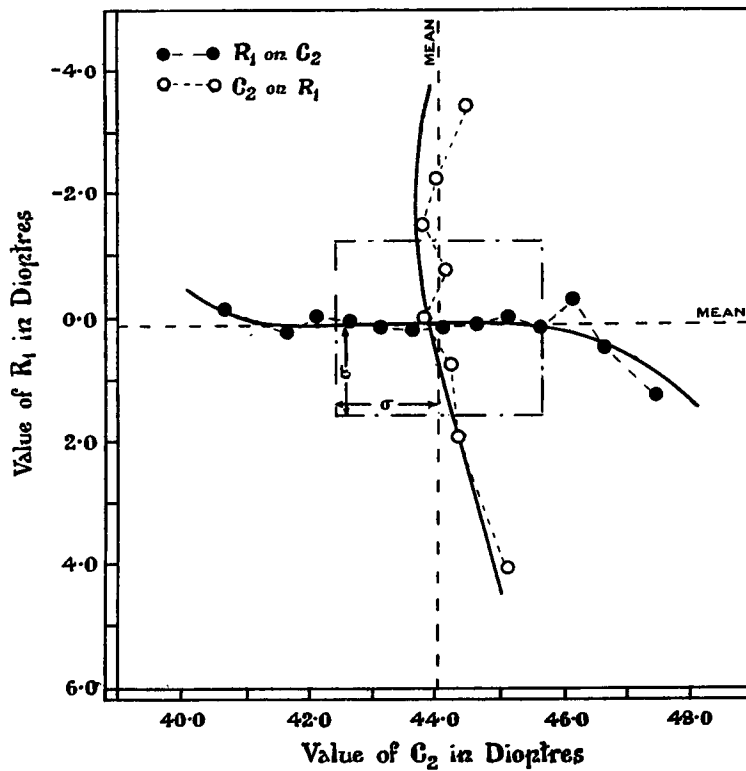
$$\bar{\eta}^2_{R_1.C_2} = .023,810 \pm .004,894.$$

Table CCXXXI. *Correlation of General Refraction in Meridian nearer the Horizontal with Corneal Refraction in Meridian nearer the Vertical (A, B and C).* $R_1$ , General Refraction in Meridian nearer the Horizontal (Central Values in Dioptres)

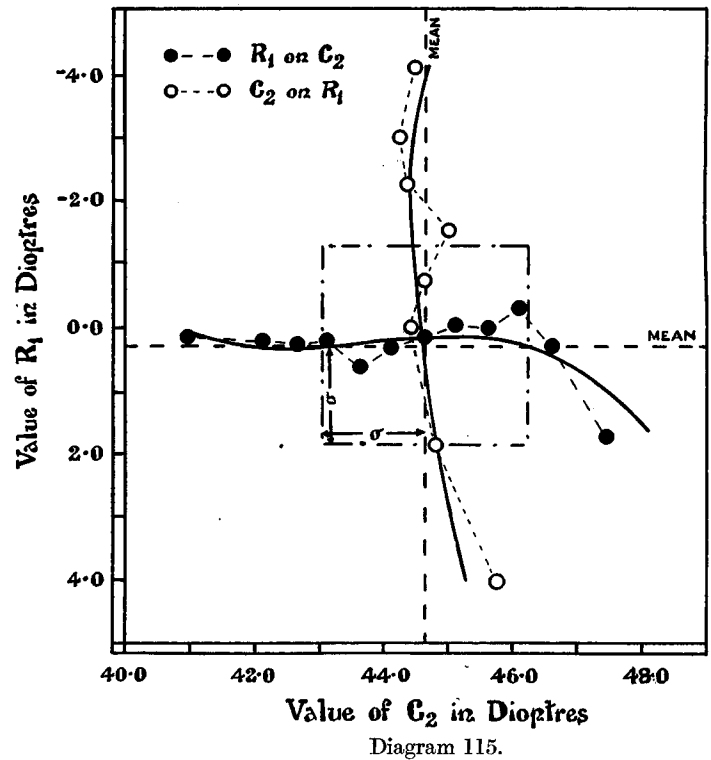
$C_2$ Corneal Refraction in Principal Meridian nearer the Vertical (Central Values in Dioptres)	+ 6.75	+ 6.00	+ 5.25	+ 4.50	+ 3.75	+ 3.00	+ 2.25	+ 1.50	+ 0.75	0.00	- 0.75	- 1.50	- 2.25	- 3.00	- 3.75	- 4.50	- 5.25	- 6.00	- 6.75	Totals
38.125	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
38.625	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
39.125	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	2
39.625	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	1
40.125	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	4
40.625	—	—	1	—	—	—	—	1	3	5	3	—	—	—	—	—	—	—	—	13
41.125	—	—	—	—	—	—	1	—	3	16	3	—	1	1	—	—	—	—	—	25
41.625	—	1	—	—	—	—	2	1	5	18	2	2	—	—	1	—	—	—	—	32
42.125	—	—	1	—	—	—	3	—	9	36	4	3	3	2	—	—	—	—	—	61
42.625	—	—	—	—	—	—	2	1	7	42	5	3	—	—	—	—	—	—	—	60
43.125	—	—	1	1	2	1	3	1	23	55	5	—	2	—	2	—	—	—	2	98
43.625	—	—	—	—	—	2	4	3	23	50	3	6	2	1	—	—	—	—	—	94
44.125	—	—	—	—	2	1	2	2	29	61	12	3	3	—	—	—	—	—	—	115
44.625	1	—	—	1	1	1	2	3	25	55	11	2	2	—	3	—	—	1	—	107
45.125	—	—	1	—	3	1	3	2	18	35	8	5	4	—	1	—	2	—	1	84
45.625	—	—	—	—	3	—	1	4	15	30	11	3	2	1	—	—	—	—	—	70
46.125	—	1	—	—	2	1	1	2	6	18	2	2	3	2	—	—	3	—	1	44
46.625	—	—	—	1	1	—	4	—	7	13	5	2	—	—	—	—	—	—	—	33
47.125	—	—	—	—	2	3	2	—	5	5	2	1	1	—	—	—	1	—	—	22
47.625	—	2	—	1	1	2	—	1	1	1	—	—	—	—	—	—	—	—	—	10
48.125	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
48.625	—	—	—	—	—	—	—	—	3	1	—	—	—	—	—	—	—	—	—	4
Totals	1	4	4	4	17	11	30	22	182	443	79	35	25	7	7	—	6	1	4	882

**CORRELATION OF****GENERAL & CORNEAL REFRACTIONS  $R_1$  WITH  $C_2$** 

(OBSERVERS A, B &amp; C) ALIEN JEWISH BOYS

**CORRELATION OF****GENERAL & CORNEAL REFRACTIONS  $R_1$  WITH  $C_2$** 

(OBSERVERS A &amp; B ONLY) ALIEN JEWISH BOYS



It will be seen that all these associations are significant if small, and we have:

$$\eta'_{C_2, R_1} = \cdot 2552, \quad \bar{\eta}_{R_1, C_2} = \cdot 2569.$$

The results appear to indicate that the regression curves differ somewhat from straight lines. They are represented in Diagram 114\*.

There appears in neither regression series a continuously changing curve. Rather we might fairly treat the regression of  $C_2$  on  $R_1$  as linear from  $R_1 = +2$  D. to  $-4$  D., but there appears to be a rise in the Corneal Refraction of the "vertical" Meridian, when the "horizontal" General Refraction is in excess of  $+2$  D. In the same way when the "vertical" Corneal Refraction rises from 40 D. to 46 D. there is no significant change in the "horizontal" General Refraction, but with further increase of the Corneal Refraction in the "vertical" we begin to get a definite rise in the "horizontal" General Refraction.

Table CCXXXI bis. Correlation of General Refraction in Meridian nearer the Horizontal with Corneal Refraction in Meridian nearer the Vertical (A and B only).

$R_1$ , General Refraction in Meridian nearer the Horizontal (Central Values in Dioptres)

$C_2$ , Corneal Refraction in Principal Meridian nearer the Vertical (Central Values in Dioptres)	$R_1$																		Totals	
	+6.75	+6.00	+5.25	+4.50	+3.75	+3.00	+2.25	+1.50	+0.75	0.00	-0.75	-1.50	-2.25	-3.00	-3.75	-4.50	-5.25	-6.00	-6.75	
38-625	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
39-125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39-625	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
40-125	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2
40-625	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
41-125	—	—	—	—	—	—	—	—	1	4	—	—	—	—	—	—	—	—	—	5
41-625	—	—	—	—	—	—	1	—	4	1	—	1	—	—	—	—	—	—	—	7
42-125	—	—	1	—	—	—	1	—	2	3	1	—	—	2	—	—	—	—	—	10
42-625	—	—	—	—	—	—	1	1	4	4	2	1	—	—	—	—	—	—	—	13
43-125	—	—	—	—	—	—	2	—	12	9	1	—	—	—	2	—	—	—	—	26
43-625	—	—	—	—	—	1	3	3	20	7	—	1	1	1	—	—	—	—	—	37
44-125	—	—	—	—	—	—	1	1	24	12	4	1	1	1	—	—	—	—	—	44
44-625	1	—	—	—	1	—	—	2	17	17	4	1	2	—	1	—	—	1	—	47
45-125	—	—	1	—	—	1	1	2	14	15	6	2	1	—	1	—	1	—	—	46
45-625	—	—	—	—	3	—	1	3	9	13	9	2	2	1	—	—	—	—	—	43
46-125	—	—	—	—	—	1	1	2	4	6	2	1	1	2	—	—	—	—	1	21
46-625	—	—	—	—	1	—	2	—	4	7	5	1	—	—	—	—	—	—	—	20
47-125	—	—	—	—	1	2	2	—	4	4	2	1	1	—	—	—	—	—	—	17
47-625	—	2	—	—	1	2	—	1	1	1	—	—	—	—	—	—	—	—	—	8
48-125	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
48-625	—	—	—	—	—	—	—	—	3	1	—	—	—	—	—	—	—	—	—	4
Totals	1	2	3	—	7	7	16	16	123	104	37	13	11	6	4	—	1	1	2	354

We controlled these results by taking  $A$  and  $B$ 's observations only: see Table CCXXXI bis.

The constants were found to be:

General Refraction  $R_1$ , Mean:  $+ \cdot 2945$  D.; Standard Deviation:  $1 \cdot 5688$  D.

Corneal Refraction  $C_2$ , Mean:  $44 \cdot 6589$  D.; Standard Deviation:  $1 \cdot 5867$  D.

Product Moment Correlation Coefficient:  $r = \cdot 0777 \pm \cdot 0356$ .

Correlation Ratios:

$$\eta'^2_{C_2, R_1} = \cdot 098,330, \quad \bar{\eta}^2_{C_2, R_1} = \cdot 045,197 \pm \cdot 010,517,$$

$$\eta'^2_{R_1, C_2} = \cdot 140,826, \quad \bar{\eta}^2_{R_1, C_2} = \cdot 053,672 \pm \cdot 011,409.$$

The associations are significant and give:

$$\eta'_{C_2, R_1} = \cdot 3136, \quad \eta'_{R_1, C_2} = \cdot 3753.$$

\* The cubics used for graduation are:

$$\tilde{C}_2 = 43 \cdot 9052 + \cdot 190,025 R_1 + \cdot 034,103 R_1^2 - \cdot 004,681 R_1^3,$$

$$\tilde{R}_1 = \cdot 064,755 - \cdot 03666 (C_2 - 44 \cdot 125) + \cdot 03294 (C_2 - 44 \cdot 125)^2 + \cdot 015,366 (C_2 - 44 \cdot 125)^3.$$

Thus while the correlation coefficient has decreased to probable insignificance, the correlation ratios have increased in significance and intensity.

Diagram 115 (p. 231) exhibits the regression lines; they are very similar to those of the previous diagram with shifted means.

These diagrams certainly cannot be graduated with straight lines. The graduating cubics for  $A$  and  $B$  only are:

$$\tilde{C}_2 = 44.57875 + .11387R_1 + .01910R_1^2 - .00215R_1^3,$$

$$\tilde{R}_1 = .20845 - .11714(C_2 - 44.125) + .02772(C_2 - 44.125)^2 + .023,034(C_2 - 44.125)^3,$$

the units in both cases being dioptries.

Lastly, we have correlated  $R_2$  the "vertical" General Refraction with  $C_1$  the "horizontal" Corneal Refraction. Table CCXXXII provides the data for  $A$ ,  $B$  and  $C$ 's combined observations. The constants of this table are:

General Refraction  $R_2$ , Mean:  $-.0952$  D.; Standard Deviation:  $1.3824$  D.

Corneal Refraction  $C_1$ , Mean:  $43.4889$  D.; Standard Deviation:  $1.5336$  D.

Product Moment Correlation Coefficient:  $r = -.0305 \pm .0227$ .

Correlation Ratios:

$$\eta'^2_{C_1.R_2} = .016,401, \quad \bar{\eta}^2 = .021,542 \pm .004,660,$$

$$\eta'^2_{R_2.C_1} = .012,828, \quad \bar{\eta}^2 = .020,408 \pm .004,538.$$

Thus no measure of the association is significant, and we fail to find any relation between the Corneal Refraction in the principal meridian nearer to the horizontal and the General Refraction in the principal meridian nearer to the vertical.

Table CCXXXII. *Correlation of General Refraction in Principal Meridian nearer the Vertical with Corneal Refraction in Principal Meridian nearer the Horizontal (A, B and C).*

$R_2$ , General Refraction in Principal Meridian nearer the Vertical (Central Values in Dioptries)

$C_1$ , Corneal Refraction in Principal Meridian nearer the Horizontal (Central Values in Dioptries)	6.00	5.25	4.50	3.75	3.00	2.25	1.50	0.75	0.00	0.75	1.50	2.25	3.00	3.75	4.50	5.25	6.00	6.75	7.50	8.25	9.00	9.75	Totals
	+	+	+	+	+	+	+	+	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
38.125	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
38.625	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39.125	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
39.625	—	—	—	—	—	—	1	1	3	1	—	—	—	—	—	—	—	—	—	—	—	—	6
40.125	—	—	—	—	—	3	1	3	3	1	1	—	—	—	—	—	—	—	—	—	—	—	16
40.625	—	—	—	—	—	—	—	2	11	3	—	—	—	—	—	—	—	—	—	—	—	—	17
41.125	—	2	—	—	—	1	1	1	22	10	1	1	2	1	—	—	—	—	—	—	—	—	42
41.625	—	—	—	—	—	1	1	8	23	4	3	2	—	—	—	—	—	—	—	—	—	—	42
42.125	—	—	—	1	1	—	2	17	39	6	3	3	2	1	—	—	—	—	—	—	—	—	75
42.625	1	—	—	—	—	2	7	18	47	8	2	—	—	1	1	1	—	—	—	1	—	—	89
43.125	—	—	—	2	1	2	—	35	53	9	2	4	—	—	—	1	—	—	1	—	1	—	111
43.625	—	1	—	1	—	1	3	25	65	11	6	2	2	1	—	—	—	—	—	—	—	—	118
44.125	—	—	—	1	1	1	5	22	56	19	6	3	2	3	—	—	—	—	—	—	1	1	121
44.625	—	1	—	—	1	2	1	18	41	5	6	1	1	1	—	1	—	—	—	—	—	—	80
45.125	—	—	—	—	—	—	1	9	29	5	3	2	2	—	—	1	—	—	—	—	1	—	54
45.625	—	—	—	—	—	—	2	11	23	6	2	—	2	1	—	1	—	—	—	—	—	—	48
46.125	—	—	—	—	1	1	—	9	22	4	3	1	1	—	—	1	—	—	—	—	—	—	43
46.625	—	—	—	1	1	—	—	—	5	4	1	—	—	—	—	—	—	—	—	—	—	—	12
47.125	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
47.625	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	1	4	—	6	6	15	25	182	443	99	42	20	16	9	1	6	—	1	1	1	3	1	882

The non-significance of these associations seems to need confirmation, so we give the table for  $A$  and  $B$  alone.

The data are given in Table CCXXXIII, p. 235. The constants are the following:

General Refraction  $R_2$ , Mean:  $+0.0636$  D.; Standard Deviation:  $1.5347$  D.

Corneal Refraction  $C_1$ , Mean:  $43.9470$  D.; Standard Deviation:  $1.4905$  D.

Product Moment Correlation Coefficient:  $r = -0.0456 \pm 0.0358$ .

Correlation Ratios:  $\eta^2_{C_1, R_2} = 0.065,031$ ,  $\bar{\eta}^2_{C_1, R_2} = 0.042,373 \pm 0.010,199$ ,

$\eta^2_{R_2, C_1} = 0.053,289$ ,  $\bar{\eta}^2_{R_2, C_1} = 0.039,548 \pm 0.009,866$ .

### CORRELATION OF

#### GENERAL & CORNEAL REFRACTIONS $R_2$ WITH $C_1$

(OBSERVERS A, B & C) ALIEN JEWISH BOYS

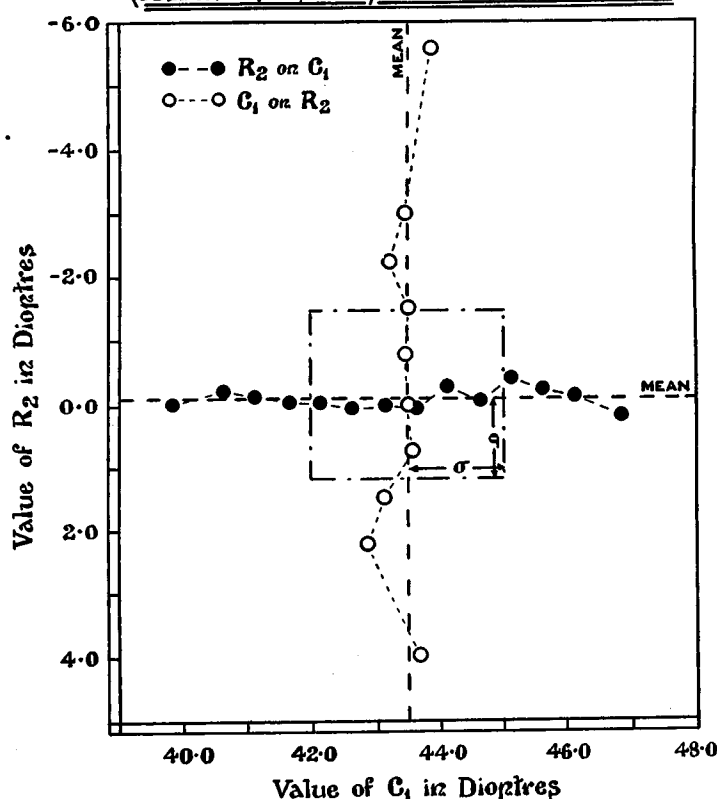


Diagram 116.

### CORRELATION OF

#### GENERAL & CORNEAL REFRACTIONS $R_2$ WITH $C_1$

(OBSERVERS A & B ONLY) ALIEN JEWISH BOYS

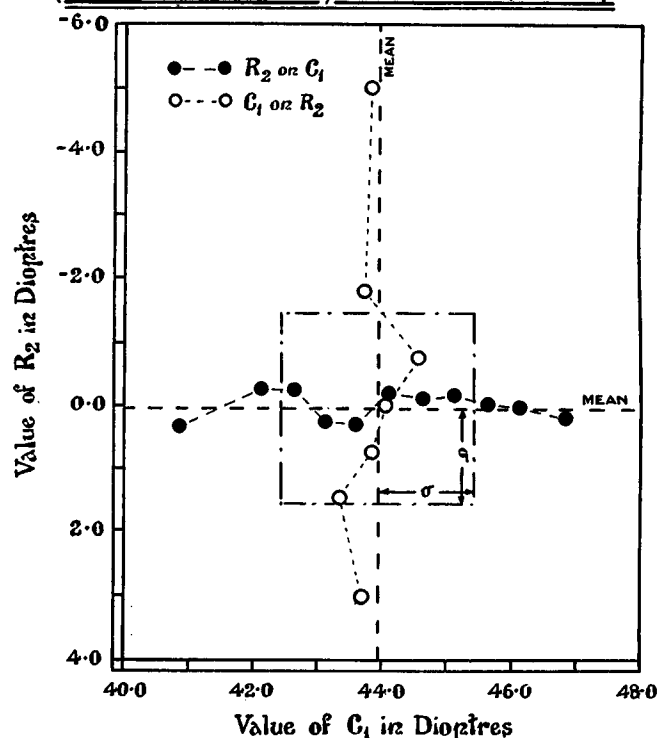


Diagram 117.

Neither the correlation coefficient nor the correlation ratios can be asserted to be definitely significant and the array-means either way do not indicate any ordered sequence. Hence the previous conclusion, that there is no significant association, is confirmed.

It will be clear from these results that there is very little association indeed between the General and Corneal Refractions, either in the same, or in cross-meridians. The correlations are of an entirely different and negligible order as compared with those of the principal refractions of the same category, whether Corneal or General with each other, or of the Lenticular with each other, or with the General and Corneal Refractions. It might be supposed that this result arose from the fact that there is very little correlation between the directions of the principal axes of the cornea and of the total ocular system. But this cannot be the true explanation, for we have seen that the great majority of eyes are not astigmatic at all, and of those that are, by far the larger number have the principal axes of both the Corneal and General Astigmatisms horizontal and vertical.

Table CCXXXIII. *Correlation of General Refraction in Principal Meridian nearer the Vertical with Corneal Refraction in Principal Meridian nearer the Horizontal (A and B only).* $R_2$ , General Refraction in Principal Meridian nearer the Vertical (Central Values in Dioptres)

$C_1$ , Corneal Refraction in Principal Meridian nearer the Horizontal (Central Values in Dioptres)	$R_2$																					Totals
	+ 5.25	+ 4.50	+ 3.75	+ 3.00	+ 2.25	+ 1.50	+ 0.75	0.00	- 0.75	- 1.50	- 2.25	- 3.00	- 3.75	- 4.50	- 5.25	- 6.00	- 6.75	- 7.50	- 8.25	- 9.00	- 9.75	Totals
38.125	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
40.125	—	—	—	—	1	1	1	1	1	1	1	1	—	—	—	—	—	—	—	—	—	8
40.625	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2
41.125	2	—	—	—	1	—	1	3	—	—	—	—	—	—	—	—	—	—	—	—	—	7
41.625	—	—	—	—	—	1	5	1	—	1	1	—	—	—	—	—	—	—	—	—	—	9
42.125	—	—	—	—	—	2	6	6	2	1	—	2	1	—	—	—	—	—	—	—	—	20
42.625	—	—	—	—	1	6	13	7	2	—	—	—	1	—	—	—	—	—	1	—	—	31
43.125	—	—	—	1	1	—	21	9	1	—	2	—	—	—	—	—	—	—	—	—	—	36
43.625	1	—	1	—	1	3	19	17	3	2	1	2	—	—	1	—	—	—	—	—	—	50
44.125	—	—	—	—	1	5	17	21	6	2	1	1	3	—	—	—	—	—	—	—	1	58
44.625	—	—	—	1	2	1	12	17	3	2	1	1	—	—	1	—	1	—	—	—	—	42
45.125	—	—	—	—	—	1	9	9	2	2	1	2	—	—	—	—	—	—	—	—	—	26
45.625	—	—	—	—	—	2	8	7	3	1	—	2	—	—	—	—	—	—	—	—	—	23
46.125	—	—	—	1	1	—	8	11	3	2	1	1	—	—	—	—	—	—	—	—	—	28
46.625	—	—	1	1	—	—	—	2	4	1	—	—	—	—	—	—	—	—	—	—	—	9
47.125	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
47.625	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	3	—	2	4	9	22	122	114	31	16	9	12	5	—	2	—	1	—	1	—	1	354

We shall now proceed to indicate how many of these correlations may be obtained indirectly. For this purpose we shall write  $A_G$  = General Astigmatism =  $R_2 - R_1$ ,  $A_C$  = Corneal Astigmatism =  $C_2 - C_1$ .

If a bar denote a mean, we have:

$$A_G - \bar{A}_G = R_2 - \bar{R}_2 - (R_1 - \bar{R}_1),$$

or

$$\delta A_G = \delta R_2 - \delta R_1.$$

If brackets denote a mean summation value:

$$\{\delta A_G \delta C_1\} = \{\delta R_2 \delta C_1\} - \{\delta R_1 \delta C_1\},$$

and accordingly:

$$r_{R_2 C_1} = (r_{A_G C_1} \sigma_{A_G} + r_{R_1 C_1} \sigma_{R_1}) / \sigma_{R_2}.$$

Again:

$$\{\delta A_C \delta R_1\} = \{\delta R_1 \delta C_2\} - \{\delta R_1 \delta C_1\},$$

$$r_{R_1 C_2} = (r_{A_C R_1} \sigma_{A_C} + r_{R_1 C_1} \sigma_{C_1}) / \sigma_{C_2}.$$

All the quantities on the right-hand sides of these two equations have been found if we remember that:

$$\sigma_{R_2}^2 = \sigma_{A_G}^2 + \sigma_{R_1}^2 + 2\sigma_{A_G} \sigma_{R_1} r_{A_G R_1},$$

and

$$\sigma_{C_2}^2 = \sigma_{A_C}^2 + \sigma_{C_1}^2 + 2\sigma_{A_C} \sigma_{C_1} r_{A_C C_1}.$$

We find:  $\sigma_{R_2} = 1.5655$  D. and  $\sigma_{C_2} = 1.6078$  D. compared with  $\sigma_{R_1} = 1.5822$  D. and  $\sigma_{C_1} = 1.6357$  D. found directly. These results are as close as we can anticipate from the two methods. We have also from the above equations:  $r_{R_2 C_1} = -.0297$  and  $r_{R_1 C_2} = +.0611$ , while found directly they are  $r_{R_2 C_1} = -.0305$  and  $r_{R_1 C_2} = +.0777$ . These are comparisons for  $A$  and  $B$ 's data only. The results justify us in believing that when it is not possible to find correlations directly the approximate methods will give good results\*. If we term  $L_1$  and  $L_2$  the Pseudo-Lenticular Refractions in the principal axes, i.e. in the great bulk of cases the horizontal and vertical,

\* The results would be closer, if Sheppard's corrections had been omitted. Thus in the two ways of approaching the correlation of directions of axes (see pp. 226-227) the agreement is perfect ( $-.0799$ ) if these corrections are omitted, but if used,  $.0818$  against  $.0709$ .

defining them by  $\delta L_1 = -\delta R_1 - \delta C_1$ ,  $\delta L_2 = -\delta R_2 - \delta C_2^\dagger$ , then  $L_2 - L_1 = -(R_2 - R_1) - (C_2 - C_1)$  will be the Pseudo-Lenticular Astigmatism. This terminology is not exact, for the General Refraction is not the sum of the Corneal and Lenticular Refractions. But the quantities we have called  $L_1$  and  $L_2$  are chiefly contributed by the crystalline lens, and for this reason the terminology is convenient if not wholly accurate.

In the following table the correlation coefficients marked with an asterisk have been found directly.

Table CCXXXIV. *Correlations of the Various Refraction Components.*

	$R_1$	$R_2$	$C_1$	$C_2$	$L_1$	$L_2$
$R_1$	—	+ .8965*	— .0994*	+ .0911*	— .6340	— .7553
$R_2$	—	(+ .9282)	(— .1358)*	(+ .0777)*	(— .6791)	(— .6790)
$C_1$	+ .8965*	—	— .0305*	— .0411*	— .6917	— .6347
$C_2$	(+ .9282)	—	(— .0456)*	(— .0896)*	(— .6149)	(— .6712)
$L_1$	— .0994*	— .0305*	—	+ .8552*	— .7065	— .6541
$L_2$	(— .1358)*	(— .0456)*	—	(+ .8184)*	(— .6350)	(— .5896)
	+ .0911*	— .0411*	+ .8552*	—	— .7392	— .7460
	(+ .0777)*	(— .0896)*	(+ .8184)*	—	(— .6807)	(— .6881)
	— .6340	— .6917	— .7065	— .7392	—	+ .9556
	(— .6791)	(— .6149)	(— .6350)	(— .6807)	—	(+ .9541)
	— .7553	— .6347	— .6541	— .7460	+ .9556	—
	(— .6790)	(— .6712)	(— .5896)	(— .6881)	(+ .9541)	—

The quantities in brackets are derived from  $A$  and  $B$ 's observations only. Those without brackets include  $C$ 's observations also. We have further the following standard deviations:

For  $A$ ,  $B$  and  $C$ :  $\sigma_{L_1} = 1.9750$  D.,  $\sigma_{L_2} = 2.0741$  D.,  $\sigma_{A_L} = .6111$  D.

For  $A$ ,  $B$  only:  $\sigma_{L_1} = 2.0146$  D.,  $\sigma_{L_2} = 2.1062$  D.,  $\sigma_{A_L} = .6301$  D.

Whence by aid of the formula:

$$r_{L_1 L_2} = (\sigma_{L_1}^2 + \sigma_{L_2}^2 - \sigma_{A_L}^2) / (2\sigma_{L_1}\sigma_{L_2});$$

we deduce:

for  $A$ ,  $B$  and  $C$ :  $r_{L_1 L_2} = +.9556$ ; for  $A$  and  $B$ :  $r_{L_1 L_2} = +.9541$ .

The following formulae were used to determine the values in Table CCXXXIV not found directly:

$$\begin{aligned} r_{R_2 L_1} &= -\frac{\sigma_{R_1} r_{R_1 R_2} + \sigma_{C_1} r_{C_1 R_2}}{\sigma_{L_1}}, & r_{C_1 L_1} &= -\frac{\sigma_{R_1} r_{R_1 C_1} + \sigma_{C_1} r_{C_1 C_1}}{\sigma_{L_1}}, & r_{C_1 L_1} &= -\frac{\sigma_{R_1} r_{R_1 C_1} + \sigma_{C_1}}{\sigma_{L_1}}, \\ r_{R_1 L_2} &= -\frac{\sigma_{R_2} r_{R_1 R_2} + \sigma_{C_2} r_{C_2 R_1}}{\sigma_{L_2}}, & r_{C_1 L_2} &= -\frac{\sigma_{R_2} r_{R_2 C_1} + \sigma_{C_2} r_{C_1 C_2}}{\sigma_{L_2}}, & r_{C_2 L_2} &= -\frac{\sigma_{R_2} r_{R_2 C_2} + \sigma_{C_2}}{\sigma_{L_2}}, \\ r_{R_2 L_2} &= -\frac{\sigma_{R_2} + \sigma_{C_2} r_{R_2 C_2}}{\sigma_{L_2}}, & r_{R_1 L_1} &= -\frac{\sigma_{R_1} + \sigma_{C_1} r_{R_1 C_1}}{\sigma_{L_1}}. \end{aligned}$$

The table allows of fairly easy interpretation:

(i) The direct correlations between "horizontal" and "vertical" refractions, e.g.  $R_1$  with  $R_2$ ,  $C_1$  with  $C_2$  and  $L_1$  with  $L_2$ , are the highest.

(ii) The correlations as found from the series  $A$ ,  $B$  and  $C$  in no case differ very substantially from those found from the series  $A$  and  $B$ . We should prefer the latter results, were it not that by using them only we much increase the probable errors of sampling, as  $C$ 's observations are nearly double the number of those of  $A$  and  $B$  alone.

(iii) Cross-correlations such as  $R_1$  with  $L_2$  or  $C_2$  with  $R_1$  are not always as one might *a priori*

$\dagger$  Actually  $R + L + C = R_{em}$ , and if we suppose  $R_{em}$ —the refractive power of the individual eye rendered emmetropic—to vary only in a second order degree, we have  $\delta L = -\delta R - \delta C$ .

suppose less than direct correlations, i.e. correlations between refractions in approximately the same direction. Compare  $r_{L_1C_2}$  and  $r_{L_2R_1}$  with  $r_{L_1C_1}$  and  $r_{L_2R_2}$  respectively.

(iv) The "Lenticular" refractions are highly correlated with both the General and the Corneal Refractions; in the case of the latter with a compensatory effect.

(v) The Corneal Refractions have a very low relationship with the General Refractions. Thus while it would be relatively accurate to predict the Lenticular Refractions from the General Refractions or from the Corneal Refractions, no reasonable estimate can be formed of the Corneal Refractions from the General Refractions.

We shall now conclude this discussion of the Refractions by forming the correlations between their differences, i.e. between the General  $A_G$ , the Corneal  $A_C$ , and the Lenticular  $A_L$  Astigmatisms.

	$A_G$	$A_C$	$A_L$
$A_G$	—	— .6947*	— .2794
		(— .7578)*	(— .3263)
$A_C$	— .6947*	—	— .4965
	(— .7578)*		(— .3695)
$A_L$	— .2794	— .4965	—
	(— .3263)	(— .3695)	

The correlations with an asterisk were found directly, those without indirectly (see pp. 189–190 above).

We see here that:

(i) The series (in brackets) for  $A$  and  $B$  generally give somewhat higher correlations than those for  $A$ ,  $B$  and  $C$  combined.

(ii) All the correlations are appreciable, some high. Thus the Astigmatism of the Cornea and of the Lens are both correlated with the General Astigmatism, but the correlations show that the Corneal Astigmatism is more closely associated with the General Astigmatism than the Astigmatism of the Lens is.

(iii) While the Corneal Refraction has little relation to the General Refraction, the Corneal Astigmatism has quite a close relation with the General Astigmatism. It may be asked how this can possibly be, since the Astigmatism is only the difference of two Refractions. We are inclined to believe, although at present we cannot prove it, that the absence of any significant correlation in the case of the General and Corneal Refractions is due to the arbitrary nature of the origin in the case of General Refraction. The Corneal Refraction is an absolute measure of the eye, i.e. of the curvature of the cornea, but the General Refraction is not a measure of the refractive power of the eye as a whole; it is merely a measure of what must be added or subtracted from that power to make the eye emmetropic in the given meridian. We must subtract  $R$  the General Refraction from a refractive power  $R_{em}$  to obtain the total refraction of the eye. This  $R_{em}$  is a quantity which depends on the individual eye, and accordingly in the usual method of measuring general refraction, we measure it relative to an origin, which varies from eye to eye. We suggest that this variation destroys the absolute value of the General Refraction and so weakens the correlation between it and the absolute Corneal Refraction. On the other hand, when we take the difference between the principal refractions, this arbitrary origin practically disappears and we get the same result as if we had subtracted the absolute refractions. The full correlation between the Corneal and General Refractions would, we suggest, only be obtained if we were able to correlate  $C$  with  $R_{em} - R$ . Unfortunately our data do not provide a means of determining  $R_{em}$ . We may know its average value for the human eye, but to put  $R_{em}$  equal to this value is of no service as the correlation depends on the *variations* of the variates, not on their mean values.

(k) *Conclusion of Section C.* Having found so little influence of Environment on Sight and so little influence of Sight on Intelligence and School Success (see results in the following Sections D and E), we frankly admit that we were once inclined to lose confidence in our ophthalmological data. We therefore turned with some anxiety to a more minute study of the interrelations of our ocular characters, but the high correlations which this study has provided have convinced us that we are dealing with organic measurements of reasonable accuracy. It is perfectly true that if we could start *de novo*, we now see how the technique could be improved, and the observations usefully extended. But there is probably no long observational research, especially one of a statistical nature, about which a conscientious investigator does not feel at its conclusion how much better he could do it, were he to start it afresh. But to do a thing "better," does not signify that the former doing was bad, it may well have been good. What it does signify is that in the course of a long research—in this case extending over many years—the investigators are not only learning the interrelationship of phenomena, but also learning thereby how more effectively to approach the riddles which those phenomena present.

Table CCXXXV. *Summary of the Intercorrelation of Ocular Characters.*

Second Ocular Character

First Ocular Character	Visual Acuity	Refraction Class	General Refraction	Corneal Refraction	General Astigmatism	Corneal Astigmatism	Near Point Distance	Pigmentation of Fundus
Visual Acuity	$r = .7763^*$ $r = .7997^\dagger$	$\eta' = .6839$	$\eta' = .7301$ $r = .2706$	$\eta' = .1870$ ( $\eta' = .2199$ ) $r = -.0275$	$\eta' = .5004$ $r = .3665$	$\eta' = .4238$ ( $\eta' = .3764$ )	$\eta' = .2131$	$\eta' = .0826$ $r_b = .0756$
Refraction Class	—	$C'_2 = .8182$ $r_t = .9249$	—	—	—	—	—	$\phi'^2$ insign.
General Refraction	$\eta' = .5740$ $r = .2706$	$\eta' = .7501$ —	$r = .9229$	$\eta' = .1612$ ( $= .2580$ ) $r = -.0994$ ( $= -.1358$ )	$\eta' = .3272$ $r = -.2490$	$\eta' = .4495$ ( $= .4445$ ) $r = .3664$ ( $= .3381$ )	$\eta' = .2523$ $r = -.1828$	$\eta' = .0581$ $r_b = -.0135$
Corneal Refraction	$\eta' = .2205$ $r = -.0275$	$\eta' = .1381$ ( $\eta' = .1957$ )	$\eta' = .2115$ $r = -.0994$ ( $= -.1358$ )	$r = .9124$ ( $= .8706$ )	$\eta' = .1718$ ( $= .2323$ ) $r = +.1293$ ( $= +.0726$ )	— ( $r = .2722$ )	$\eta' = .2313$ $r = -.1156$ ( $= -.0242$ )	$\eta'$ insign. $r_b = -.0280$ ( $= .0048$ )
General Astigmatism	$\eta' = .4231$ $r = .3665$	$\eta' = .6863$ —	$\eta' = .6356$ $r = -.2490$	$\eta' = .1981$ ( $= .2328$ ) $r = +.1293$ ( $= +.0726$ )	$r = .8593$	$\eta' = .7375$ ( $= .8074$ ) $r = -.6947$ ( $= -.7578$ )	— $r = -.1417$	$\eta = .1722$ $r_b = .1712$
Corneal Astigmatism	$\eta' = .4066$ —	( $\eta' = .5676$ ) —	$\eta' = .5724$ ( $= .5756$ ) $r = .3664$ ( $= .3381$ )	— ( $r = .2722$ )	$\eta' = .7466$ ( $= .7535$ ) $^\dagger$ $r = -.6947$ ( $= -.7578$ )	$r = .7171$ ( $= .7430$ )	$r = -.0190$ ( $= +.1106$ ) $\eta' = .2288$ ( $=$ insign.)	$\eta = .1264$ $r_b = -.1156$
Near Point Distance	$\eta' = .2456$ —	$\eta' = .1842$ —	$\eta' = .2415$ $r = -.1828$	$\eta' = .1995$ $r = -.1156$ ( $= -.0242$ )	— $r = -.1417$	$r = -.0190$ ( $= +.1106$ ) $\eta' = .2082$ ( $=$ insign.)	$r = .7828$	$\eta' = .3589$ $r_b = .0514$
Pigmentation of Fundus	$r_b = .0756$	$\eta' = .1171$ $\phi'^2$ insign.	$r_b = -.0135$	$r_b = -.0280$ ( $= .0048$ )	$r_b = .1712$	$r_b = -.1156$	$r_b = .0514$	Direction of Axis
Direction of Axis, General Astigmatism	—	—	—	—	$\eta'_{DA \cdot GA} = .2061$ $\eta'_{GA \cdot DA} = .2433$	—	—	Corneal Astigmatism $r = .0799$
Direction of Axis, Corneal Astigmatism	—	—	—	—	—	$\eta'_{DA \cdot CA} = .1765$ $\eta'_{CA \cdot DA} = .2674$ $r = -.0597$	—	General Astigmatism $r = .0799$

\* Special Examination.

† Medical Examination (Boys).

 $^\dagger$  Less than the corresponding  $r$ , only because  $\eta'$  is uncorrected.

We believe that there is a wide field of work open in the statistical study of the eye, especially in childhood. It needs, in the first place, the cooperation of ophthalmologist and statistician. But it needs something further, it requires the apparatus, the time and the energy which money alone can provide. In the present investigation the collection of ophthalmic data was only a part of a much wider inquiry, namely, the general fitness of a particular class of emigrant. The Galton Laboratory possesses no funds for carrying on essential work of this type, and in this case money had to be raised from external sources, the total amount falling short of £100. The reader who realises this fact will perhaps be less ready to ask why that much more which he seeks, and we also want, especially in the ophthalmic inquiry, was not obtained! We needed more time, more subjects, more ophthalmic aid, and the war came and shattered any chance of carrying on further with replenished funds and revised technique. All we wish to claim is, that the results drawn from our data form a good first approximation to the interrelationship of ocular characters, and that we should like to see them, as they are for a special race, controlled by a similar investigation with better technique on a larger number of Gentile children.

Meanwhile, to the ophthalmologist who is not a mathematician, the diagrams of this section may prove the more interesting and readily interpreted part of it. To those acquainted with the modern mathematical theory of statistics the results of this section summarised in the accompanying Table CCXXXV (p. 238) will give a concise statement of the interocular relationships, which we have found in our special material.

This table must be read a little differently from some of the earlier tables. They read either in the rows or columns indifferently. This still holds for the coefficients of correlation, but the correlation ratios must be read *horizontally*.  $\eta'$  the correlation ratio in any cell is the uncorrected correlation ratio of the ocular character in the left-hand column on the ocular character at the top of the column in which  $\eta'$  occurs. As before the values in brackets are for *A* and *B*'s data only, those without brackets are for *A*, *B* and *C*'s observations combined. It will be seen at a glance that the ocular characters form a complex correlated system. In the diagonal cells we have placed the coefficients which measure the relationships of Right Eye to Left Eye.

#### NOTE ON LENTICULAR ASTIGMATISM.

IN dealing with this topic we have been content to consider only the crudest measurements of the influence of the lens on total astigmatism. We have not entered into the sources of this astigmatism. According to Hess in Graefe-Saemisch\* astigmatism other than corneal has not been sufficiently investigated at present to justify definite statements. Possibly observations by aid of the slit lamp, or even Young's method of investigating the astigmatism with the eye under water ("Mechanism of the Eye," *Miscellaneous Works*, Vol. I, pp. 26, 41), would throw light on the nature of lenticular astigmatism. The source of it has been variously attributed to (i) lamination of the lens, (ii) dislocation of the lens, i.e. to the centre of the lens not being on the optic axis, (iii) to a tilting of the lens, as by Young, and (iv) to the refracting power of the lens being different in different planes—especially the sagittal and equatorial†. In the case of (ii) and (iii) we can find an equivalent thin lens, with differentiated refracting powers in the principal

\* *Handbuch der gesamten Augenheilkunde*, Bd. VIII, Abt. 2, S. 420, Leipzig, 1903.

† It is convenient to term the principal meridian nearer the vertical, in accordance with cranial terminology, the sagittal plane, and the principal meridian nearer the horizontal, the equatorial plane. Unfortunately the Germans (and Southall) call the former the meridional plane and the latter the sagittal plane. Our sagittal plane, as it will usually contain the optical axes of the eye and the eccentric ray, coincides with the primary plane of Coddington and Denis Taylor and our equatorial with their secondary plane.

planes, which would produce the same astigmatism. This would also be true in the case of (i), but in the present stage of our knowledge it might be difficult to give the equivalent lens even by rough determinations. The problem of the equivalent lens in cases (ii) and (iii) deserves some consideration and we may discuss it in three ways: (a) by treating the lens as thick which the crystalline lens certainly is, and considering the formulae for a thick lens tilted or dislocated, (b) by treating the lens as thin and considering the full formulae for a thin lens tilted or dislocated, and (c) by neglecting cubic powers of small quantities in the formulae obtained in (b).

(a) (i) *On Lenticular Astigmatism when we take into account the Thickness of the Lens.* The general formulae for a convexo-convex lens of thickness  $d$ , radii  $r_1$  and  $r_2$ , refractive index  $n_2$  in a medium of refractive index  $n_0$ , the lens being supposed of uniform non-laminated material, have been given by L. Hermann\*. They cover both the cases in which we are for the time specially interested, i.e. the tilted and dislocated lens, or indeed the combination of tilt and dislocation.

The angles and points are indicated in the accompanying figure.

$$G_1 G_2 = d, \quad PQ = T, \quad C_2 C_1 = r_1 + r_2 - d.$$

$$n_0 \sin \phi_1 = n_2 \sin \alpha,$$

$$n_0 \sin \phi_2 = n_2 \sin \beta.$$

$$\text{Let } A = \sin(\phi_1 - \alpha)/\sin \alpha,$$

$$B = \sin(\phi_2 - \beta)/\sin \beta,$$

$$T = r_1 \cos \alpha - (r_1 + r_2 - d) \cos \theta + r_2 \cos \beta,$$

$$0 = r_1 \sin \alpha - (r_1 + r_2 - d) \sin \theta + r_2 \sin \beta.$$

Then the distances of the image points from the emergent point  $Q$  are given by

$$\frac{\cos^2 \phi_2}{F_1} = \frac{B}{r_2} + \frac{\cos^2 \beta}{\frac{r_1}{A} \cos^2 \alpha - \frac{T}{n_2/n_0}},$$

$$\frac{1}{F_2} = \frac{B}{r_2} + \frac{1}{\frac{r_1}{A} - \frac{T}{n_2/n_0}}.$$

(a) (ii) *Case of Dislocation through a distance  $y = PM$ , the incident Ray being parallel to the Axis  $C_1 C_2$ .* Fig. I now takes the form of Fig. II (p. 241) and for this special case  $\phi_1 = \theta + \alpha$ ,  $y = r_1 \sin(\theta + \alpha)$ . For the eye focussed on a distant object  $r_1 = 10$  mm.,  $r_2 = 6$  mm., while  $n_0 = 1.3365$ ,  $n_2 = 1.4371$ . We consider two cases:

Dislocation of 2 mm.

$$F_1 = 42.0486 \text{ mm.},$$

$$F_2 = 47.0287 \text{ mm.},$$

$$\theta + \phi_2 - \beta = 2^\circ 23' 0384,$$

Dislocation of 3 mm.

$$F_1 = 32.9894 \text{ mm.},$$

$$F_2 = 43.9504 \text{ mm.},$$

$$\theta + \phi_2 - \beta = 3^\circ 51' 1147,$$

$$f_1 = N'i = F_1 \cos(\theta + \phi_2 - \beta) = 42.0122 \text{ mm.}, \quad f_1 = N'i = F_1 \cos(\theta + \phi_2 - \beta) = 32.9149 \text{ mm.},$$

$$f_2 = N'i' = F_2 \cos(\theta + \phi_2 - \beta) = 46.9880 \text{ mm.}, \quad f_2 = N'i' = F_2 \cos(\theta + \phi_2 - \beta) = 43.8511 \text{ mm.},$$

where  $N'$  is the point at which  $NQ$  produced meets the optic axis.

\* *Pflügers Archiv für die gesamte Physiologie*, Bd. xviii, S. 453, Bonn, 1878. We have to acknowledge our indebtedness to Mr R. C. Stevenson of the Department of Applied Mathematics, University College, London, for a reference to this paper of Hermann and for checking our numerical results by a duplicate working.

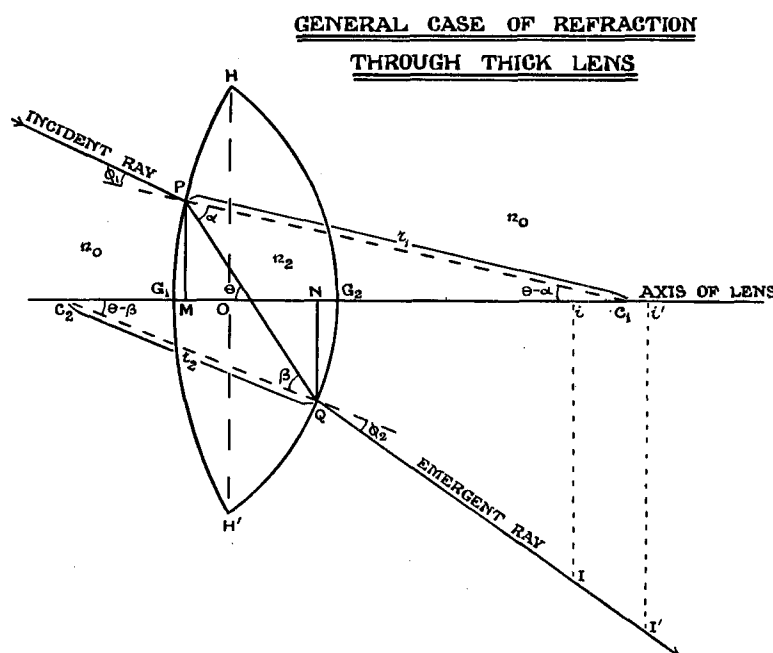


Fig. I.

Now if we termed  $\left(\frac{1}{f_1} - \frac{1}{f_2}\right) \times 1000$  the astigmatism we should have for dislocation of

$$2 \text{ mm.} \\ R_E - R_S = 2.521 \text{ D.}$$

$$3 \text{ mm.} \\ R_E - R_S = 7.577 \text{ D.}$$

where  $R_E$  and  $R_S$  are the refractive powers in the equatorial and sagittal planes respectively.

These values are sensibly identical with those we should obtain by using the  $F$ 's instead of the  $f$ 's. But we do not think these values correct for a thin lens placed at  $Q$

with refractive powers of  $\frac{1}{f_1}$  and  $\frac{1}{f_2}$  would have to be met by a ray parallel to its axis to give images at axial distances  $N'i$  and  $N'i'$ . The equivalent thin lens must be placed at  $P$ , or we must add  $MN = T \cos \theta$  to both  $f_1$  and  $f_2$ . These values are 3.0700 and 2.3652 for dislocations of 2 mm. and 3 mm. respectively. We have therefore

$$\begin{aligned} &\text{Dislocation of 2 mm.} \\ f_1' &= Pi = 45.0822 \text{ mm.}, \\ f_2' &= Pi' = 50.0580 \text{ mm.}, \\ \frac{1}{f_1'} - \frac{1}{f_2'} &= R_E - R_S = 2.205 \text{ D.}, \end{aligned}$$

### DISLOCATION OF THICK LENS

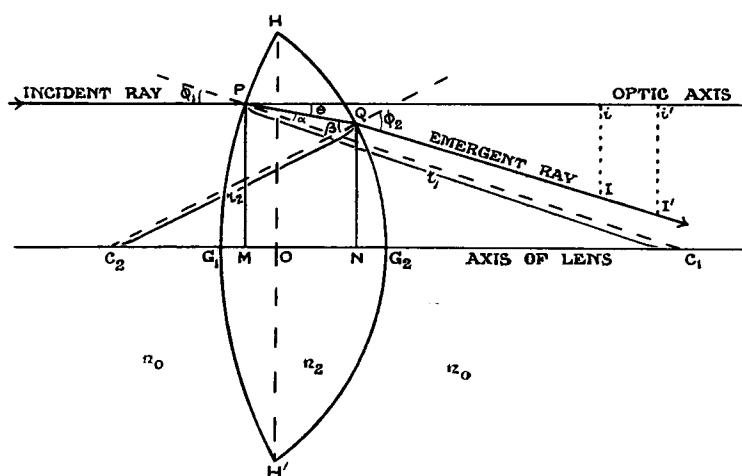


Fig. II.

$$\begin{aligned} &\text{Dislocation of 3 mm.} \\ f_1' &= Pi = 35.2801 \text{ mm.}, \\ f_2' &= Pi' = 46.2163 \text{ mm.}, \\ \frac{1}{f_1'} - \frac{1}{f_2'} &= R_E - R_S = 6.707 \text{ D.} \end{aligned}$$

The radius of the crystalline lens is about 4.8 mm., and a dislocation of 3 mm. would bring the edge of the lens up to the border of the iris and any greater dislocation would be accompanied by a double image and so be pathologically recognisable. It is indeed unlikely that the crystalline lens could be dislocated 3 mm. or even 2 mm. without penetrating the choroid. However, the results indicate degrees of lenticular astigmatism such as actually occur.

(a) (iii) *Tilting of Lens through an Angle  $\chi$ .* We shall suppose the incident ray to be so directed that if it were not refracted at entering the anterior surface of the lens it would pass through  $O$  the bisector of the chord. The accompanying figure indicates the points and angles.

In this case

$$\begin{aligned} \theta - \alpha &= \chi - \phi_1, \\ \sin \phi_1 &= \frac{OC_1}{r_1} \sin (\pi - \chi), \end{aligned}$$

and

$$\frac{OC_1}{r_1} = \cos HC_1O = \frac{C_1C_2^2 + HC_1^2 - HC_2^2}{2HC_1 \times C_1C_2} = \frac{2(r_1 - d)(r_1 + r_2) + d^2}{2r_1(r_1 + r_2 - d)}.$$

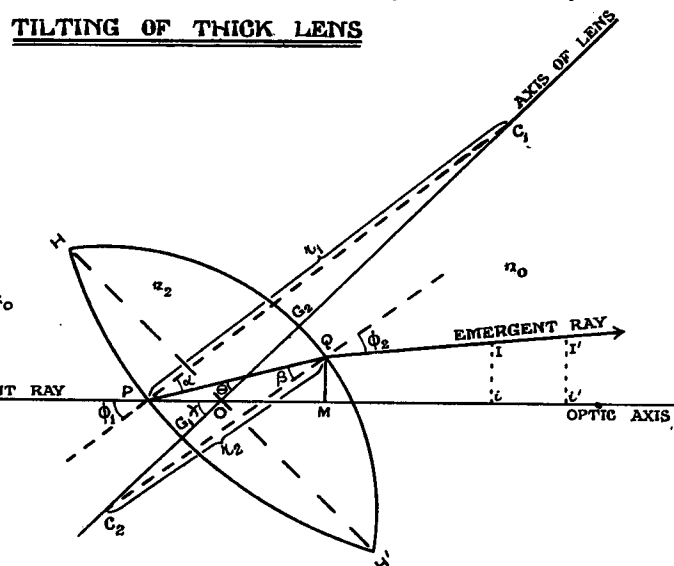


Fig. III.

This determines  $\frac{OC_1}{r_1}$  and therefore  $\sin \phi_1$  from the second equation. But  $\sin \alpha = \frac{n_0}{n_2} \sin \phi_1$ , hence  $\alpha$  is known and then from the first equation  $\theta$ . The remaining angles are determined as before. We have worked out the results for tilts of  $8^\circ$  and  $15^\circ$  and find the following values:

Tilt of $8^\circ$ .		Tilt of $15^\circ$ .	
$F_1 = 48.6017$ mm.,		$F_1 = 46.7887$ mm.,	
$F_2 = 49.0973$ mm.,		$F_2 = 48.9273$ mm.,	
$T = 3.6220$ mm.,		$T = 3.6780$ mm.,	
$T \cos \theta = 3.5909$ mm.,		$T \cos \theta = 3.5677$ mm.,	
$f_1 = F_1 \cos (\theta + \phi_2 - \beta) = 48.1457$ mm.,		$f_1 = F_1 \cos (\theta + \phi_2 - \beta) = 45.2529$ mm.,	
$f_2 = F_2 \cos (\theta + \phi_2 - \beta) = 48.6366$ mm.,		$f_2 = F_2 \cos (\theta + \phi_2 - \beta) = 47.3214$ mm.,	
$f_1' = 51.7366$ mm., $f_2' = 52.2275$ mm.,		$f_1' = 48.8206$ mm., $f_2' = 50.8891$ mm.,	
$R_S - R_E = \frac{1}{f_1'} - \frac{1}{f_2'} = .182$ D.,		$R_S - R_E = \frac{1}{f_1'} - \frac{1}{f_2'} = .832$ D.	

The  $R_S - R_E$  is the astigmatism of a lens placed at  $P$  where the optic axis of the eye, or the incident ray, meets the anterior surface of the lens. Had we placed this lens at  $Q$  the astigmatism would have been  $\frac{1}{f_1} - \frac{1}{f_2} = .210$  D. and  $.966$  D. respectively. A tilt of even  $8^\circ$  in the lens seems in itself very improbable without producing pathological effects other than mere astigmatism. On the whole, therefore, we are inclined to think that small tilts of the lens would produce too small effects to be a main source of lenticular astigmatism.

(b) (i) We shall now proceed to consider the formulae which we obtain if the lens be treated as "thin." In this case  $C_2 C_1 = r_1 + r_2$ ,  $n_0 \sin \phi_1 = n_2 \sin \alpha$ ,  $n_0 \sin \phi_2 = n_2 \sin \beta$  as before.  $A$  and  $B$  are defined as for the thick lens, but  $T = 0$ . The formulae\* on p. 240 reduce to

$$\frac{1}{F_1} = \frac{B}{r_2 \cos^2 \phi_2} + \frac{A \cos^2 \beta}{r_1 \cos^2 \alpha \cos^2 \phi_2},$$

$$\frac{1}{F_2} = \frac{B}{r_2} + \frac{A}{r_1}.$$

(b) (ii) *Dislocation of Lens through a distance  $y = PM$ , the incident Ray being parallel to the Axis.* Here  $PM = QN = pO$  say, because the lens is thin;  $y = r_1 \sin \phi_1 = r_2 \sin (\beta + \alpha - \phi_1)$ .

We obtain the following values for dislocations of 2 mm. and 3 mm. respectively:

Dislocation of 2 mm.		Dislocation of 3 mm.	
$F_1 = 42.1319$ ,		$F_1 = 32.6151$ ,	
$F_2 = 47.3577$ ,		$F_2 = 43.9593$ ,	
$\phi_1 - \alpha + \phi_2 - \beta = 2^\circ 25' .2237$ ,		$\phi_1 - \alpha + \phi_2 - \beta = 3^\circ 54' .7908$ ,	
$f_1 = F_1 \cos (\phi_1 - \alpha + \phi_2 - \beta) = 42.0944$ ,		$f_1 = F_1 \cos (\phi_1 - \alpha + \phi_2 - \beta) = 32.5391$ ,	
$f_2 = F_2 \cos (\phi_1 - \alpha + \phi_2 - \beta) = 47.3155$ ,		$f_2 = F_2 \cos (\phi_1 - \alpha + \phi_2 - \beta) = 43.8568$ ,	

$$\text{Astigmatism} = 1000 \left( \frac{1}{f_1} - \frac{1}{f_2} \right) = R_E - R_S,$$

hence:  $R_E - R_S = 2.62$ ,

$$R_E - R_S = 7.93.$$

\* If we substitute the values of  $A$  and  $B$  and  $\sin \phi_1 = \frac{n_2}{n_0} \sin \alpha$ ,  $\sin \phi_2 = \frac{n_2}{n_0} \sin \beta$ , these formulae reduce to those given by J. P. C. Southall, *The Principles and Methods of Geometrical Optics*, § 251, pp. 363-364, New York, 1910.

(b) (iii) *Tilting of Lens through an Angle  $\chi$* . Here we have  $\phi_1 = \chi$ , and therefore  $\theta = \alpha$  in the general formulae; also  $T = 0$ . From the equation for  $T$  we get  $\cos \theta = \cos \beta$  or  $\theta = \beta = \alpha$ , and hence

$$\phi_1 = \phi_2 \text{ and } A = B = \frac{\sqrt{n_2^2 - n_0^2 \sin^2 \chi} - n_0 \cos \chi}{n_0}.$$

The fundamental equations reduce to\*

$$\frac{1}{F_1} = \frac{\sqrt{n_2^2 - n_0^2 \sin^2 \chi} - n_0 \cos \chi}{n_0 \cos^2 \chi} K,$$

$$\frac{1}{F_2} = \frac{\sqrt{n_2^2 - n_0^2 \sin^2 \chi} - n_0 \cos \chi}{n_0} K,$$

where

$$K = \frac{1}{r_1} + \frac{1}{r_2}.$$

Hence the difference of the refractive powers, remembering to project, is

$$R_S - R_E = \frac{1000K}{n_0 \cos \chi} (\sqrt{n_2^2 - n_0^2 \sin^2 \chi} - n_0 \cos \chi) \tan^2 \chi.$$

We find astigmatisms of .40 D. and 1.49 D. for tilts of  $8^\circ$  and  $15^\circ$  respectively.

(c) (i) For the case of dislocation of a thin lens the following approximate formula was obtained by neglecting cubic and higher powers of small quantities:

$$R_S - R_E = \frac{1000(n_2 - n_0)}{n_0} y^2 K \left\{ \left( \frac{n_2}{n_0} \right)^2 K^2 - \left( 1 + \frac{2n_2}{n_0} \right) \frac{K}{r_1} + \left( 1 + \frac{2n_0}{n_2} \right) \frac{1}{r_1^2} \right\},$$

where

$$K = \frac{1}{r_1} + \frac{1}{r_2}.$$

For the eye focussed on a distant object  $r_1 = 10$  mm.,  $r_2 = 6$  mm., and so  $K = .26667$ , while  $n_0 = 1.3365$ ,  $n_2 = 1.4371$ . We then have

$$R_S - R_E = .5380y^2.$$

For a dislocation of 2 mm. we find  $R_S - R_E = 2.15$  D., and for a dislocation of 3 mm. we find  $R_S - R_E = 4.84$  D.

(c) (ii) For the tilting of a thin lens the corresponding approximate formula is

$$R_S - R_E = .006114 (\text{angle of tilt in degrees})^2.$$

This gives us astigmatisms of .39 D. and 1.38 D. for tilts of  $8^\circ$  and  $15^\circ$  respectively.

### Summary of Results.

Astigmatism produced

Dislocation of lens	Thick lens	Thin lens	Approximate formula for thin lens
Dislocation of 2 mm.	2.205 D.	2.62 D.	2.15 D.
Dislocation of 3 mm.	6.707 D.	7.93 D.	4.84 D.

It is clear from these results that the approximate formula cannot be applied to obtain even rough results for dislocations of more than 1.5 to 2 mm. at most. The thickness of the lens reduces

\* These formulae are the same as those given by Coddington, who appears to have been the first to investigate the problem of a centric pencil passing obliquely to the axis of a lens (*A System of Optics, Part I*, p. 120, Cambridge, 1823). We owe the reference to these formulae to the kindness of Professor L. N. G. Filon.

the amount of the astigmatism. The values of the astigmatisms produced by dislocation might account for the observed lenticular astigmatism.

Astigmatism produced

Tilting of lens	Thick lens	Thin lens	Approximate formula for thin lens
Tilt of $8^{\circ}$ Tilt of $15^{\circ}$	.182 D. .832 D.	.40 D. 1.49 D.	.39 D. 1.38 D.

The approximate formula gives fairly reliable results compared with the full formula for the thin lens for values of  $8^{\circ}$  and under. The thickness of the lens tends to reduce the amount of the astigmatism as it did in the case of dislocation of the lens. The results obtained for the astigmatism, even for such large tilts as  $8^{\circ}$  and  $15^{\circ}$  (which correspond to separations of the iris from the lens of 1.5 to 2.5 mm.), seem incapable of accounting for the observed lenticular astigmatisms. We therefore conclude that while dislocation might, it is improbable that tilting can be the source of lenticular astigmatism. With regard to dislocation, while it is optically admissible as a source of the required astigmatism, it may be questioned whether a dislocation of 2 to 3 mm. is anatomically possible without a change in the size of the lens or a modification of the choroid. It seems an easier explanation to suppose the curvatures of the lens to be different in the equatorial and sagittal planes. It must anyhow be borne in mind that a tilt or dislocation may be looked upon as optically equivalent to a properly placed thin lens with different refractive powers in its principal planes. For the purposes of the present memoir it is indifferent what the source of the lenticular astigmatism and the lenticular refractions may be. They are measures of the contributions of the crystalline lens to the errors of refraction, whether they arise from dislocation, tilting or other causes.

*(To be continued)*